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Assessing Local Influences in a Multivariate t-Model with Uniform Structure*

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Abstract

The present paper is devoted to the problem of assessing local influences in a multivariate t-model with uniform structure. The effects of some minor perturbation on the statistical inference are considered based on Cook's curvature measure. This leads to the largest curvature direction which is the statistic mainly concerned in the local influence analysis. As an application, a common covariance-weighted perturbation is thoroughly discussed.

Keywords: Multivariate t-model, uniform structure, ω-model, likelihood displacement, curvature.

AMS Subject Classification: 62H.

§ 1. Introduction

It is well known that, if there exists the minor perturbation in some factors of the statistical model, then it is the influence analysis to demonstrate how to evaluate the effects of the minor perturbation on the statistical inference. Hampel (1974) reduced the above perturbation to originate from the relevant distribution function, changing from F to $F + \Delta F$, this resulted in a series of essential statistics such as the influence function et al. Furthermore, if the perturbations is explicit, regardless of the distribution, we often start from the likelihood function. In the statistical inference, to describe concretely the influence of all kinds of perturbation schemes are vital. Cook (1986) introduced the concept of curvature measure into the local influence analysis, which proclaimed that the differential geometry had permeate into some aspects in the statistics. By applying Cook's method, some authors studied some statistical model including the classical linear regression and growth curve model etc. (Bai, 1999). The random error in the above model is usually assumed to be normal. However, the assumption of normality is no longer reasonable in some situations. For example, as pointed out by Joarder and Ali (1997), a lot of economic and business data, e.g., stock return data, exhibit fat tailed distribution, and the multivariate t-model accommodates thin failed as well as fat tailed distributions. These models have attracted considerable attention in the recent literature (Joarder etc., 1997, Lange etc., 1989). In this paper, the

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model considered is a multivariate t-model which can be given as follows:

$$Y = XBZ^T + E, \tag{1.1}$$

where X is a $q \times p$ known design matrix, B is a $p \times m$ unknown parameter matrix, Y is a $q \times n$ response matrix, $rk(X) = p$ and $rk(Z) = m$, respectively. Furthermore, the $q \times n$ error matrix E is assumed to have qn-variate t-distribution $t_{qn}(\gamma, 0, I_n \otimes \Sigma)$, i.e., the density of E is determined by

$$\begin{aligned} f(e_1, e_2, \cdots, e_n) &= \frac{\Gamma((\gamma + nq)/2)}{\Gamma(\gamma/2)\pi^{nq/2}} |\Sigma|^{-n/2} \\ &\quad \cdot \Big(1 + \sum_{i=1}^{n} e_i^T \Sigma^{-1} e_i\Big)^{-(\gamma+nq)/2}, \qquad e_i \in R^q,\ i = 1, \cdots, n, \end{aligned} \tag{1.2}$$

where $\gamma > 0$ is a known degree of freedom and Σ is an unknown $q \times q$ positive definite matrix (Wang, 1987). Among the all structures of Σ, this paper focuses attention on the following special structure:

$$\Sigma = \sigma^2[(1-\rho)I_q + \rho \mathbf{1}_q \mathbf{1}_q^T], \tag{1.3}$$

where $\sigma^2 > 0$ and $\rho \in (-1/(q-1), 1)$ are unknown, $\mathbf{1}_q = (1, \cdots, 1)^T \in \mu(X) = \{Xt : t \in R^p\}$. The structure (1.3) is referred to as the uniform structure (US). The main objective of this paper is to use Cook's curvature measure to assess the influences of minor perturbation on the statistical inferences in the multivariate t-model (1.1) and (1.2) with US (1.3). The remainders in this paper is organized as follows. In section 2, we briefly restate some preliminaries which are needed in the sequel. The main results obtained in this paper will be provided in section 3. As an application, the consequences are used to discuss special covariance-weighted perturbation in section 4 and a simulation calculation is made in section 5.

§ 2. Some Definitions and Lemmas

In this section, some definitions and lemmas will be present which are needed in the sequel.

Definition 2.1 (Wei etc., 1991) The perturbation model satisfied the following regulations is referred to as ω-model.

a. Suppose that the log-likelihood function of random matrix Y corresponding to the postulated model M is denoted by $L(\theta)$, where $\theta \in \Theta$ is an unknown distribution parameter vector and $\Theta \subset R^s$ is open.

b. Suppose that $\omega = (\omega_1, \omega_2, \cdots, \omega_t)^T \in \Omega$ is a vector describing the perturbation factors, where Ω is open subset of R^t. Let $M(\omega)$ stand for the perturbed model and $L(\theta|\omega)$ be the log-likelihood function corresponding to $M(\omega)$. Further, assume that $L(\theta|\omega)$ has the continuous partial derivatives of the second degree in $\Theta \times \Omega$.

c. There is a point $\omega_0 \in \Omega$ such that $M(\omega_0) = M$, thus, $L(\theta|\omega_0) = L(\theta)$ for all $\theta \in \Theta$.

d. Suppose that $\widehat{\theta}$ and $\widehat{\theta}(\omega)$ are the MLEs of $\theta \in \Theta$ corresponding to M and $M(\omega)$, respectively, so that $\widehat{\theta}(\omega_0) = \widehat{\theta}$.

For the ω-model in the Definition 2.1, Cook (1986) proposed the following definition.

Definition 2.2 If θ can be partitioned as $\theta = (\theta_1^T, \theta_2^T)^T \in \Theta \subset R^s$, where $\theta_1 \in R^{s_1}$ is interest and $\theta_2 \in R^{s-s_1}$ is nuisance, then the likelihood displacement on the interested vector θ_1 is defined as follows:

$$LD_s(\omega) = 2[L(\widehat{\theta}) - L(\widetilde{\theta}(\omega))], \tag{2.1}$$

where $\widehat{\theta}(\omega) = (\widehat{\theta}_1(\omega)^T, \widehat{\theta}_2(\omega)^T)^T$ ($\widehat{\theta}_1(\omega) \in R^{s_1}$, $\widehat{\theta}_2(\omega) \in R^{s-s_1}$), $\widehat{\theta} = \widehat{\theta}(\omega_0) = (\widehat{\theta}_1^T, \widehat{\theta}_2^T)^T$, $\widetilde{\theta}(\omega) = (\widehat{\theta}_1(\omega)^T, \widetilde{\theta}_2(\widehat{\theta}_1(\omega))^T)^T$ and $\widetilde{\theta}_2(\theta_1)$ is the MLE of θ_2 with θ_1 given corresponding to the postulated model M, i.e.

$$L((\theta_1^T, \widetilde{\theta}_2(\theta_1)^T)^T) = \max_{\theta_2 \in \Theta_2} L((\theta_1^T, \theta_2^T)^T). \tag{2.2}$$

It is clear the likelihood displacement function $z = LD_s(\omega)$ versus ω (so-called influence graph) contains the essential information about the influence of the minor perturbation scheme on the inference of θ_1. It can be shown that $z = LD_s(\omega)$ attains its local minimum value 0 at ω_0 and its first derivatives along every direction at ω_0 vanishes (Cook, 1986). Therefore, we choose its second derivatives along every direction evaluated at ω_0, i.e., its curvatures along every direction evaluated at ω_0, to measure the sensitivities to the minor perturbation scheme. According to Definition 2.1, we easily establish the following lemma.

Lemma 2.1 (Cook, 1986) For the ω-model, the curvature C_d of the influence graph $z = LD_s(\omega)$ along the unit direction $d \in R^t$ at ω_0 can be written as

$$C_d = 2|d^T \ddot{F} d|, \tag{2.3}$$

where $\ddot{F} = G^T H^T \ddot{L} H G$ which is known as Hessian matrix,

$$G = \frac{\partial \widehat{\theta}_1(\omega)}{\partial \omega^T}\Big|_{\omega=\omega_0}, \qquad H = \begin{pmatrix} I_{k_1} \\ \dfrac{\partial \widetilde{\theta}_2(\theta_1)}{\partial \theta_1^T} \end{pmatrix}_{\theta_1 = \widehat{\theta}_1}$$

and

$$\ddot{L} = \frac{\partial^2 L(\theta)}{\partial \theta \partial \theta^T}\Big|_{\theta=\widehat{\theta}} = \begin{pmatrix} \dfrac{\partial^2 L((\theta_1^T, \theta_2^T)^T)}{\partial \theta_1 \partial \theta_1^T} & \dfrac{\partial^2 L((\theta_1^T, \theta_2^T)^T)}{\partial \theta_1 \partial \theta_2^T} \\ \dfrac{\partial^2 L((\theta_1^T, \theta_2^T)^T)}{\partial \theta_2 \partial \theta_1^T} & \dfrac{\partial^2 L((\theta_1^T, \theta_2^T)^T)}{\partial \theta_2 \partial \theta_2^T} \end{pmatrix}_{\theta=\widehat{\theta}}.$$

From Lemma 2.1, we know that the maximum influence curvature is

$$C_{\max} = 2 \max_{\|d\|=1} |d^T \ddot{F} d|, \tag{2.4}$$

and the direction $d_{\max} \in R^t$ corresponding to $C_{\max}$ is the unit eigenvector corresponding to the largest absolute eigenvalue of $\ddot{F}$, which is the statistic we concern mainly in local influence analysis (Cook, 1986) and referred to as the maximum influence curvature direction (MICD).

Corollary 2.1 (Bai, 1999) If $\theta_1 \in \Theta_1 \subset R$, then

$$d_{\max} \Big\| \frac{\partial \widehat{\theta}_1(\omega)}{\partial \omega} \Big|_{\omega=\omega_0}. \tag{2.5}$$

Unless stated otherwise, for an interested object ϕ in the ω-model, the notation $\ddot{F}_\phi$, G_ϕ, H_ϕ and $\ddot{L}_\phi$ stand for the matrices $\ddot{F}$, G, H and $\ddot{L}$ defined by Lemma 2.1 corresponding to ϕ, respectively.

The following lemma establishes an important property of the Hessian matrix $\ddot{F}$ defined by Lemma 2.1.

Lemma 2.2 (Bai, 1999) Suppose that $\eta = f(\theta_1)$ is a one-to-one measurable transformation from θ_1 to η, where $\eta \in H \subset R^{s_1}$, then $\ddot{F}_{\theta_1} = \ddot{F}_\eta$.

It is remarked that Lemma 2.2 implies the Hessian matrix $\ddot{F}$ defined by Lemma 2.1 and the direction $d_{\max}$ corresponding to the largest absolute eigenvalue of $\ddot{F}$ are invariant under an one-to-one measurable transformation of the interested parameter vector θ_1.

§ 3. Application to Multivariate t-Model

The main results obtained in this paper will be presented in this section.

Theorem 3.1 For the multivariate t-model (1.1) and (1.2) with US (1.3), if there exists some perturbation such that $\mathbf{1}_q \in \mu(X(\omega)) = \{X(\omega)t : t \in R^p\}$ and B, σ^2 and ρ are respectively of interest, then the Hessian matrix $\ddot{F}$ in Lemma 2.1 can be expressed as

$$\ddot{F}_B = -\gamma \frac{\partial \mathrm{Vec}(\widehat{B}(\omega))^T}{\partial \omega} \{(Z^T Z) \otimes [X^T(\widehat{\xi}^{-1} P_{\mathbf{1}_q} + \widehat{\eta}^{-1} Q_{\mathbf{1}_q}) X]\} \frac{\partial \mathrm{Vec}(\widehat{B}(\omega))}{\partial \omega^T} \Big|_{\omega=\omega_0}, \tag{3.1}$$

while the MICD $d_{\max \sigma^2}$ and $d_{\max \rho}$ in Corollary 2.1 are given by

$$d_{\max \sigma^2} \Big\| \frac{\partial \widehat{\sigma}^2(\omega)}{\partial \omega} \Big|_{\omega=\omega_0}, \qquad d_{\max \rho} \Big\| \frac{\partial \widehat{\rho}(\omega)}{\partial \omega} \Big|_{\omega=\omega_0}, \tag{3.2}$$

where

$$\begin{aligned}
&\widehat{B}(\omega) = (X(\omega)^T X(\omega))^{-1} X(\omega)^T Y(\omega) Z(\omega) [Z(\omega)^T Z(\omega)]^{-1}, \\
&\widehat{\xi}(\omega) = \frac{\gamma}{n} \mathrm{tr}(P_{\mathbf{1}_q} S(\omega)), \\
&\widehat{\eta}(\omega) = \frac{\gamma}{(q-1)n} \mathrm{tr}[(I_q - P_{X(\omega)}) Y(\omega) Y(\omega)^T + (P_{X(\omega)} - P_{\mathbf{1}_q}) S(\omega)], \\
&\widehat{\sigma}^2(\omega) = \frac{1}{q}[\widehat{\xi}(\omega) + (q-1)\widehat{\eta}(\omega)], \qquad \widehat{\rho}(\omega) = \frac{1}{q-1}\Big[\frac{q\widehat{\xi}(\omega)}{\widehat{\xi}(\omega) + (q-1)\widehat{\eta}(\omega)} - 1\Big], \\
&S(\omega) = Y(\omega)(I_n - P_{Z(\omega)}) Y(\omega)^T, \qquad \widehat{\xi} = \widehat{\xi}(\omega_0), \qquad \widehat{\eta} = \widehat{\eta}(\omega_0),
\end{aligned}$$

$X(\omega)$, $Y(\omega)$ and $Z(\omega)$ are the matrices in the ω-model corresponding to X, Y and Z in the multivariate t-model, respectively. While the notation P_A denotes the $p \times p$ orthogonal projection matrix on the space spanned by the columns of the $p\times q$ matrix $A=(a_{ij})_{p\times q}$ and $\mathrm{Vec}(A)$ stand for a pq-dimensional column vector with the $(j-1)p+i$-th component be a_{ij}, $i=1,\cdots,p$, $j=1,\cdots,q$.

Proof It follows from (1.1) and (1.2) that the log-likelihood function of Y is given by

$$\begin{aligned} L(B,\Sigma) &= \ln\Big[\frac{\Gamma((\gamma+nq)/2)}{\Gamma(\gamma/2)\pi^{nq/2}}\Big]-\frac{n}{2}\ln(|\Sigma|) \\ &\quad -\frac{\gamma+nq}{2}\ln\{1+\mathrm{tr}[(Y-XBZ^T)^T\Sigma^{-1}(Y-XBZ^T)]\}, \end{aligned} \tag{3.3}$$

where Σ is of the US (1.3). Note that Σ can also be rewrite as

$$\Sigma=\xi P_{\mathbf{1}_q}+\eta Q_{\mathbf{1}_q}, \tag{3.4}$$

where ξ and η are determined by

$$\begin{cases} \xi=\sigma^2[1+(q-1)\rho], \\ \eta=\sigma^2(1-\rho). \end{cases} \tag{3.5}$$

It is easy to from (3.4) that

$$\Sigma^{-1}=\xi^{-1}P_{\mathbf{1}_q}+\eta^{-1}Q_{\mathbf{1}_q},\qquad |\Sigma|=\xi\eta^{q-1}. \tag{3.6}$$

Substituting (3.6) into (3.3) to obtain the log-likelihood function of Y as

$$\begin{aligned} L(B,\xi^{-1},\eta^{-1}) &= \ln\Big[\frac{\Gamma((\gamma+nq)/2)}{\Gamma(\gamma/2)\pi^{nq/2}}\Big]+\frac{n}{2}[\ln(\xi^{-1})+(q-1)\ln(\eta^{-1})] \\ &\quad -\frac{\gamma+nq}{2}\ln\{1+\mathrm{tr}[(Y-XBZ^T)^T(\xi^{-1}P_{\mathbf{1}_q}+\eta^{-1}Q_{\mathbf{1}_q})(Y-XBZ^T)]\}, \end{aligned} \tag{3.7}$$

which shows that the first derivatives of Y with respect to $\mathrm{Vec}(B)$, ξ^{-1} and η^{-1} are given by

$$\begin{cases} \dfrac{\partial L}{\partial \mathrm{Vec}(B)}=(\gamma+nq)\dfrac{\mathrm{Vec}[X^T(\xi^{-1}P_{\mathbf{1}_q}+\eta^{-1}Q_{\mathbf{1}_q})(Y-XBZ^T)Z]}{1+\mathrm{tr}[(Y-XBZ^T)^T(\xi^{-1}P_{\mathbf{1}_q}+\eta^{-1}Q_{\mathbf{1}_q})(Y-XBZ^T)]}, \\ \dfrac{\partial L}{\partial\xi^{-1}}=\dfrac{n}{2}\xi-\dfrac{\gamma+nq}{2}\dfrac{\mathrm{tr}[(Y-XBZ^T)^TP_{\mathbf{1}_q}(Y-XBZ^T)]}{1+\mathrm{tr}[(Y-XBZ^T)^T(\xi^{-1}P_{\mathbf{1}_q}+\eta^{-1}Q_{\mathbf{1}_q})(Y-XBZ^T)]}, \\ \dfrac{\partial L}{\partial\eta^{-1}}=\dfrac{(q-1)n}{2}\eta-\dfrac{\gamma+nq}{2}\dfrac{\mathrm{tr}[(Y-XBZ^T)^TQ_{\mathbf{1}_q}(Y-XBZ^T)]}{1+\mathrm{tr}[(Y-XBZ^T)^T(\xi^{-1}P_{\mathbf{1}_q}+\eta^{-1}Q_{\mathbf{1}_q})(Y-XBZ^T)]}. \end{cases} \tag{3.8}$$

Solving the equation group with respect to $\mathrm{Vec}(B)$, ξ^{-1} and η^{-1},

$$\begin{cases} \dfrac{\partial L}{\partial \mathrm{Vec}(B)}=0, \\ \dfrac{\partial L}{\partial\xi^{-1}}=0, \\ \dfrac{\partial L}{\partial\eta^{-1}}=0, \end{cases}$$

to obtain the MLEs of $\mathrm{Vec}(B)$, ξ^{-1} and η^{-1} as

$$\begin{cases} \mathrm{Vec}(\widehat{B})=\mathrm{Vec}[(X^TX)^{-1}X^TYZ(Z^TZ)^{-1}], \\ \widehat{\xi}^{-1}=\dfrac{n}{\gamma}\dfrac{1}{\mathrm{tr}(P_{\mathbf{1}_q}S)}, \\ \widehat{\eta}^{-1}=\dfrac{(q-1)n}{\gamma}\dfrac{1}{\mathrm{tr}[(I_q-P_X)YY^T+(P_X-P_{\mathbf{1}_q})S]}, \end{cases} \tag{3.9}$$

where $S = Y(I_n - P_Z)Y^T$. Note that, from (3.5) we have

$$\begin{cases} \sigma^2 = \dfrac{1}{q}[\xi + (q-1)\eta], \\ \rho = \dfrac{1}{q-1}\Big[\dfrac{q\xi}{\xi + (q-1)\eta} - 1\Big], \end{cases} \tag{3.10}$$

hence, the MLEs of σ^2 and ρ is given by

$$\begin{cases} \widehat{\sigma}^2 = \dfrac{1}{q}[\widehat{\xi} + (q-1)\widehat{\eta}], \\ \widehat{\rho} = \dfrac{1}{q-1}\Big[\dfrac{q\widehat{\xi}}{\widehat{\xi} + (q-1)\widehat{\eta}} - 1\Big]. \end{cases} \tag{3.11}$$

Further, the second derivatives of L with respect to $\mathrm{Vec}(B)$, ξ^{-1} and η^{-1} can be derive from (3.8) as

$$\begin{cases} \dfrac{\partial^2 L}{\partial \mathrm{Vec}(B) \partial \mathrm{Vec}(B)^T} = -(\gamma + nq)\dfrac{(Z^T Z) \otimes [X^T(\xi^{-1}P_{\mathbf{1}_q} + \eta^{-1}Q_{\mathbf{1}_q})X]}{1 + \mathrm{tr}(W)} + \dfrac{2(\gamma + nq)}{[1 + \mathrm{tr}(W)]^2} \\ \qquad \cdot \mathrm{Vec}[X^T(\xi^{-1}P_{\mathbf{1}_q} + \eta^{-1}Q_{\mathbf{1}_q})UZ]\mathrm{Vec}[X^T(\xi^{-1}P_{\mathbf{1}_q} + \eta^{-1}Q_{\mathbf{1}_q})UZ]^T, \\ \dfrac{\partial^2 L}{\partial \mathrm{Vec}(B) \partial \xi^{-1}} = (\gamma + nq)\dfrac{\mathrm{Vec}(X^T P_{\mathbf{1}_q} UZ)}{1 + \mathrm{tr}(W)} - \dfrac{\gamma + nq}{[1 + \mathrm{tr}(W)]^2} \\ \qquad \cdot \mathrm{tr}[U^T P_{\mathbf{1}_q} U]\mathrm{Vec}[X^T(\xi^{-1}P_{\mathbf{1}_q} + \eta^{-1}Q_{\mathbf{1}_q})UZ], \\ \dfrac{\partial^2 L}{\partial \mathrm{Vec}(B) \partial \eta^{-1}} = (\gamma + nq)\dfrac{\mathrm{Vec}(X^T Q_{\mathbf{1}_q} UZ)}{1 + \mathrm{tr}(W)} - \dfrac{\gamma + nq}{[1 + \mathrm{tr}(W)]^2} \\ \qquad \cdot \mathrm{tr}[U^T Q_{\mathbf{1}_q} U]\mathrm{Vec}[X^T(\xi^{-1}P_{\mathbf{1}_q} + \eta^{-1}Q_{\mathbf{1}_q})UZ], \\ \dfrac{\partial^2 L}{\partial \xi^{-1} \partial \xi^{-1}} = -\dfrac{n}{2}\xi^2 + \dfrac{\gamma + nq}{2}\Big[\dfrac{\mathrm{tr}(U^T P_{\mathbf{1}_q} U)}{1 + \mathrm{tr}(W)}\Big]^2, \\ \dfrac{\partial^2 L}{\partial \xi^{-1} \partial \eta^{-1}} = \dfrac{\gamma + nq}{2}\dfrac{\mathrm{tr}[U^T P_{\mathbf{1}_q} U]\mathrm{tr}[U^T Q_{\mathbf{1}_q} U]}{[1 + \mathrm{tr}(W)]^2}, \\ \dfrac{\partial^2 L}{\partial \eta^{-1} \partial \eta^{-1}} = -\dfrac{(q-1)n}{2}\eta^2 + \dfrac{\gamma + nq}{2}\Big[\dfrac{\mathrm{tr}(U^T Q_{\mathbf{1}_q} U)}{1 + \mathrm{tr}(W)}\Big]^2, \end{cases} \tag{3.12}$$

where $U = Y - XBZ^T$, $W = (Y - XBZ^T)^T(\xi^{-1}P_{\mathbf{1}_q} + \eta^{-1}Q_{\mathbf{1}_q})(Y - XBZ^T)$, which implies from (3.9) that the $(pm+2) \times (pm+2)$ matrix $\ddot{L}$ in Lemma 2.1 can be written as

$$\begin{aligned} \ddot{L}_B &= \begin{pmatrix} \ddot{L}_{11} & \ddot{L}_{12} & \ddot{L}_{13} \\ \ddot{L}_{21} & \ddot{L}_{22} & \ddot{L}_{23} \\ \ddot{L}_{31} & \ddot{L}_{32} & \ddot{L}_{33} \end{pmatrix} \\ &= -\begin{pmatrix} \gamma(Z^T Z) \otimes [X^T(\widehat{\xi}^{-1}P_{\mathbf{1}_q} + \widehat{\eta}^{-1}Q_{\mathbf{1}_q})X] & 0 & 0 \\ 0 & \dfrac{n[\gamma + (q-1)n]}{2(\gamma + qn)}\widehat{\xi}^2 & -\dfrac{(q-1)n^2}{2(\gamma + qn)}\widehat{\xi}\widehat{\eta} \\ 0 & -\dfrac{(q-1)n^2}{2(\gamma + qn)}\widehat{\xi}\widehat{\eta} & \dfrac{(q-1)n(\gamma + n)}{2(\gamma + qn)}\widehat{\eta}^2 \end{pmatrix}. \end{aligned} \tag{3.13}$$

If B (i.e., $\mathrm{Vec}(B)$) is of interest, then let $\partial L / \partial \xi^{-1} = 0$ and $\partial L / \partial \eta^{-1} = 0$ in (3.8) we can get the MLEs of ξ^{-1} and η^{-1} with B given as

$$\begin{cases}\widetilde{\xi}^{-1}(B)=\dfrac{n}{\gamma}\dfrac{1}{\mathrm{tr}[(Y-XBZ^T)^TP_{\mathbf{1}_q}(Y-XBZ^T)]},\\ \widetilde{\eta}^{-1}(B)=\dfrac{(q-1)n}{\gamma}\dfrac{1}{\mathrm{tr}[(Y-XBZ^T)^TQ_{\mathbf{1}_q}(Y-XBZ^T)]},\end{cases}$$

which shows that

$$\begin{cases}\dfrac{\partial\widetilde{\xi}^{-1}(B)}{\partial\mathrm{Vec}(B)}=\dfrac{2n}{\gamma}\dfrac{\mathrm{Vec}(X^TP_{\mathbf{1}_q}(Y-XBZ^T)Z)}{\{\mathrm{tr}[(Y-XBZ^T)^TP_{\mathbf{1}_q}(Y-XBZ^T)]\}^2},\\ \dfrac{\partial\widetilde{\eta}^{-1}(B)}{\partial\mathrm{Vec}(B)}=\dfrac{2(q-1)n}{\gamma}\dfrac{\mathrm{Vec}(X^TQ_{\mathbf{1}_q}(Y-XBZ^T)Z)}{\{\mathrm{tr}[(Y-XBZ^T)^TQ_{\mathbf{1}_q}(Y-XBZ^T)]\}^2}.\end{cases}$$

Thus, from (3.9), the matrix H_B in Lemma 2.1 can be expressed as

$$H_B=\left(\begin{array}{c}I_{pm}\\ \dfrac{\partial\widetilde{\xi}^{-1}(B)}{\partial\mathrm{Vec}(B)^T}\\ \dfrac{\partial\widetilde{\eta}^{-1}(B)}{\partial\mathrm{Vec}(B)^T}\end{array}\right)_{\mathrm{Vec}(B)=\mathrm{Vec}(\widehat{B})}=\left(\begin{array}{c}I_{pm}\\ 0\\ 0\end{array}\right). \tag{3.14}$$

Note that

$$G_B=\frac{\partial\mathrm{Vec}(\widehat{B}(\omega))}{\partial\omega^T}\Big|_{\omega=\omega_0}, \tag{3.15}$$

where $\mathrm{Vec}(\widehat{B}(\omega))$ is the MLEs of $\mathrm{Vec}(B)$ in the ω-model corresponding to the model (1.1) and (1.2) with US (1.3). It is similar to get (3.9), we have

$$\mathrm{Vec}(\widehat{B}(\omega))=\mathrm{Vec}[(X(\omega)^TX(\omega))^{-1}X(\omega)^TY(\omega)Z(\omega)(Z(\omega)^TZ(\omega))^{-1}],$$

i.e.,

$$\widehat{B}(\omega)=(X(\omega)^TX(\omega))^{-1}X(\omega)Y(\omega)Z(\omega)(Z(\omega)^TZ(\omega))^{-1}. \tag{3.16}$$

It follows from (3.13), (3.14) and (3.15) that the Hessian matrix $\ddot{F}$ in Lemma 2.1 can be written as

$$\begin{aligned}\ddot{F}_B &= G_B^TH_B^T\ddot{L}_BH_BG_B=G_B^T\ddot{L}_{11}G_B\\ &= -\gamma\frac{\partial\mathrm{Vec}(\widetilde{B}(\omega))^T}{\partial\omega}\{(Z^TZ)\otimes[X^T(\widehat{\xi}^{-1}P_{\mathbf{1}_q}+\widehat{\eta}^{-1}Q_{\mathbf{1}_q})X]\}\frac{\partial\mathrm{Vec}(\widehat{B}(\omega))}{\partial\omega^T}\Big|_{\omega=\omega_0},\end{aligned} \tag{3.17}$$

which implies that (3.1) holds.

If σ^2 or ρ is of interest, then note that it is similar to (3.9) and (3.11), the MLEs of σ^2 and ρ in the above ω-model can be obtained as

$$\begin{cases}\widehat{\sigma}^2(\omega)=\dfrac{1}{q}[\widehat{\xi}(\omega)+(q-1)\widehat{\eta}(\omega)],\\ \widehat{\rho}^2(\omega)=\dfrac{1}{q-1}\Big[\dfrac{q\widehat{\xi}(\omega)}{\widehat{\xi}(\omega)+(q-1)\widehat{\eta}(\omega)}-1\Big],\end{cases}$$

where

$$\widehat{\xi}(\omega) = \frac{\gamma}{n}\mathrm{tr}[P_{\mathbf{1}_q}S(\omega)],$$
$$\widehat{\eta}(\omega) = \frac{\gamma}{(q-1)n}\mathrm{tr}[(I_q - P_{X(\omega)})Y(\omega)Y(\omega)^T + (P_{X(\omega)} - P_{\mathbf{1}_q})S(\omega)],$$
$$S(\omega) = Y(\omega)(I_n - P_{Z(\omega)})Y(\omega)^T,$$

by which and Corollary 2.1, we known that (3.2) is true. The proof is completed. #

Theorem 3.2 Under the assumptions of Theorem 3.1, if (σ^2,ρ) and (B,σ^2,ρ) are respectively interested, then the Hessian matrices $\ddot{F}_{(\sigma^2,\rho)}$ and $\ddot{F}_{(B,\sigma^2,\rho)}$ can be written as

$$\begin{aligned}\ddot{F}_{(\sigma^2,\rho)} \;=\; & -\frac{n}{2(\gamma+qn)}\Big\{\frac{\gamma+(q-1)n}{\widehat{\xi}^2}\frac{\partial\widehat{\xi}(\omega)}{\partial\omega}\frac{\partial\widehat{\xi}(\omega)}{\partial\omega^T} - \frac{(q-1)n}{\widehat{\xi}\widehat{\eta}}\Big(\frac{\partial\widehat{\xi}(\omega)}{\partial\omega}\frac{\partial\widehat{\eta}(\omega)}{\partial\omega^T} + \frac{\partial\widehat{\eta}(\omega)}{\partial\omega}\frac{\partial\widehat{\xi}(\omega)}{\partial\omega^T}\Big)\\ & + \frac{(q-1)(\gamma+n)}{\widehat{\eta}^2}\frac{\partial\widehat{\eta}(\omega)}{\partial\omega}\frac{\partial\widehat{\eta}(\omega)}{\partial\omega^T}\Big\}\Big|_{\omega=\omega_0},\end{aligned} \tag{3.18}$$

$$\ddot{F}_{(B,\sigma^2,\rho)} = \ddot{F}_B + \ddot{F}_{(\sigma^2,\rho)}, \tag{3.19}$$

where $\ddot{F}$ is given by (3.1).

Proof If (ξ^{-1},η^{-1}) is of interest, then let $\partial L/\partial\mathrm{Vec}(B) = 0$ in (3.8), we can obtain the MLE of $\mathrm{Vec}(B)$ with ξ^{-1} and η^{-1} given as

$$\mathrm{Vec}(\widetilde{B}(\xi^{-1},\eta^{-1})) = \mathrm{Vec}[(X^TX)^{-1}X^TYZ(Z^TZ)^{-1}], \tag{3.20}$$

which implies that the matrix $H_{(\xi^{-1},\eta^{-1})}$ in Lemma 2.1 is given by

$$H_{(\xi^{-1},\eta^{-1})} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \dfrac{\partial\mathrm{Vec}(\widetilde{B}(\xi^{-1},\eta^{-1}))}{\partial\xi^{-1}} & \dfrac{\partial\mathrm{Vec}(\widetilde{B}(\xi^{-1},\eta^{-1}))}{\partial\eta^{-1}}\end{pmatrix}_{(\xi^{-1},\eta^{-1})=(\widehat{\xi}^{-1},\widehat{\eta}^{-1})} = \begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}. \tag{3.21}$$

Further, note that the matrix $G_{(\xi^{-1},\eta^{-1})}$ and $\ddot{L}_{(\xi^{-1},\eta^{-1})}$ in Lemma 2.1 are given by

$$G_{(\xi^{-1},\eta^{-1})} = \begin{pmatrix}\dfrac{\partial\widehat{\xi}^{-1}(\omega)}{\partial\omega^T}\\ \dfrac{\partial\widehat{\eta}^{-1}(\omega)}{\partial\omega^T}\end{pmatrix}_{\omega=\omega_0} = -\begin{pmatrix}\dfrac{1}{\widehat{\xi}(\omega)^2}\dfrac{\partial\widehat{\xi}(\omega)}{\partial\omega^T}\\ \dfrac{1}{\widehat{\eta}(\omega)^2}\dfrac{\partial\widehat{\eta}(\omega)}{\partial\omega^T}\end{pmatrix}_{\omega=\omega_0}, \tag{3.22}$$

and

$$\begin{aligned}\ddot{L}_{(\xi^{-1},\eta^{-1})} &= \begin{pmatrix}\ddot{L}_{22} & \ddot{L}_{23} & \ddot{L}_{21}\\ \ddot{L}_{32} & \ddot{L}_{33} & \ddot{L}_{31}\\ \ddot{L}_{12} & \ddot{L}_{13} & \ddot{L}_{11}\end{pmatrix}\\ &= -\begin{pmatrix}\dfrac{n(\gamma+(q-1)n)}{2(\gamma+qn)}\widehat{\xi}^2 & -\dfrac{(q-1)n^2}{2(\gamma+qn)}\widehat{\xi}\widehat{\eta} & 0\\ -\dfrac{(q-1)n^2}{2(\gamma+qn)}\widehat{\xi}\widehat{\eta} & \dfrac{(q-1)n(\gamma+n)}{2(\gamma+qn)}\widehat{\eta}^2 & 0\\ 0 & 0 & \gamma(Z^TZ)\otimes(X^T(\widehat{\xi}^{-1}P_{\mathbf{1}_q}+\widehat{\eta}^{-1}Q_{\mathbf{1}_q})X)\end{pmatrix}.\end{aligned} \tag{3.23}$$

Thus, from (3.21), (3.22) and (3.23), we know that the matrix $\ddot{F}_{(\xi^{-1},\eta^{-1})}$ in Lemma 2.1 is given by

$$\begin{aligned}
&\ddot{F}_{(\xi^{-1},\eta^{-1})}\\
=\;& G^T_{(\xi^{-1},\eta^{-1})}H^T_{(\xi^{-1},\eta^{-1})}\ddot{L}_{(\xi^{-1},\eta^{-1})}H_{(\xi^{-1},\eta^{-1})}G_{(\xi^{-1},\eta^{-1})}\\
=\;& G^T_{(\xi^{-1},\eta^{-1})}\begin{pmatrix}\ddot{L}_{22} & \ddot{L}_{23}\\ \ddot{L}_{32} & \ddot{L}_{33}\end{pmatrix}G_{(\xi^{-1},\eta^{-1})}\\
=\;& -\frac{n}{2(\gamma+qn)}\Big\{\frac{\gamma+(q-1)n}{\widehat{\xi}^2}\frac{\partial\widehat{\xi}(\omega)}{\partial\omega}\frac{\partial\widehat{\xi}(\omega)}{\partial\omega^T}-\frac{(q-1)n}{\widehat{\xi}\widehat{\eta}}\Big(\frac{\partial\widehat{\xi}(\omega)}{\partial\omega}\frac{\partial\widehat{\eta}(\omega)}{\partial\omega^T}+\frac{\partial\widehat{\eta}(\omega)}{\partial\omega}\frac{\partial\widehat{\xi}(\omega)}{\partial\omega^T}\Big)\\
&+\frac{(q-1)(\gamma+n)}{\widehat{\eta}^2}\frac{\partial\widehat{\eta}(\omega)}{\partial\omega}\frac{\partial\widehat{\eta}(\omega)}{\partial\omega^T}\Big\}\Big|_{\omega=\omega_0}.
\end{aligned}\tag{3.24}$$

Note that the transformation from (ξ^{-1},η^{-1}) to (σ^2,ρ) defined by (3.5) is one-to-one measurable, hence by Lemma 2.2, we have

$$\ddot{F}_{(\sigma^2,\rho)}=\ddot{F}_{(\xi^{-1},\eta^{-1})},$$

which and (3.24) mean that (3.18) holds.

Next, if (B,ξ^{-1},η^{-1}) is of interest, then the matrices $\ddot{L}_{(B,\xi^{-1},\eta^{-1})}$, $H_{(B,\xi^{-1},\eta^{-1})}$ and $G_{(B,\xi^{-1},\eta^{-1})}$ in Lemma 2.1 are respectively

$$\ddot{L}_{(B,\xi^{-1},\eta^{-1})}=\ddot{L}_B,\qquad H_{(B,\xi^{-1},\eta^{-1})}=I_{pm+2},\qquad G_{(B,\xi^{-1},\eta^{-1})}=\begin{pmatrix}G_B\\ G_{(\xi^{-1},\eta^{-1})}\end{pmatrix},$$

which shows from (3.17) and (3.24) that the matrix $\ddot{F}_{(B,\xi^{-1},\eta^{-1})}$ in Lemma 2.1 can be written as

$$\begin{aligned}
\ddot{F}_{(B,\xi^{-1},\eta^{-1})} &= G^T_{(B,\xi^{-1},\eta^{-1})}H^T_{(B,\xi^{-1},\eta^{-1})}\ddot{L}_{(B,\xi^{-1},\eta^{-1})}H_{(B,\xi^{-1},\eta^{-1})}G_{(B,\xi^{-1},\eta^{-1})}\\
&= G^T_B\ddot{L}_{11}G_B+G^T_{(\xi^{-1},\eta^{-1})}\begin{pmatrix}\ddot{L}_{22} & \ddot{L}_{23}\\ \ddot{L}_{32} & \ddot{L}_{33}\end{pmatrix}G_{(\xi^{-1},\eta^{-1})}\\
&= \ddot{F}_B+\ddot{F}_{(\xi^{-1},\eta^{-1})}.
\end{aligned}\tag{3.25}$$

Note that the transformation from (B,ξ^{-1},η^{-1}) to (B,σ^2,ρ) is one-to-one measurable, hence by Lemma 2.2, we can obtain

$$\ddot{F}_{(B,\sigma^2,\rho)}=\ddot{F}_{(B,\xi^{-1},\eta^{-1})},$$

Thus, the proof is completed. #

§ 4. Covariance-Weighted Perturbation Scheme

In this section, we focus our attention on assessing the local influence of some special perturbation on the multivariate t-model (1.1) and (1.2) with US (1.3). Without loss of generality, we consider only the covariance-weighted perturbation scheme, which is made up of the follow form

$$E\sim t_{qn}(\gamma,0,\Omega\otimes\Sigma),\tag{4.1}$$

where $\Omega = \mathrm{diag}(\omega_1, \omega_2, \cdots, \omega_n)$, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T \in R^n$ $(\omega_i > 0, i = 1, \cdots, n)$. It is equivalent to

$$E\Omega^{-1/2} \sim t_{qn}(\gamma, 0, I_n \otimes \Sigma), \tag{4.2}$$

where $\Omega^{-1/2} = \mathrm{diag}(\omega_1^{-1/2}, \cdots, \omega_n^{-1/2})$. Obviously, $\omega_0 = \mathbf{1}_n \in R^n$ stands for that there is no any perturbation in the model (4.2). For the covariance-weighted perturbation model (4.1) and (4.2), since $Y\Omega^{-1/2} = XB(\Omega^{-1/2}Z)^T + E\Omega^{-1/2}$, hence

$$\widehat{B}(\omega) = (X^TX)^{-1}X^TY\Omega^{-1/2}Z(Z^T\Omega^{-1}Z)^{-1}, \tag{4.3}$$

and

$$\begin{cases} \widehat{\xi}(\omega) = \dfrac{\gamma}{n}\mathrm{tr}[P_{\mathbf{1}_q}S(\omega)], \\ \widehat{\eta}(\omega) = \dfrac{\gamma}{(q-1)n}\mathrm{tr}[(I_q - P_X)Y\Omega^{-1}Y^T + (P_X - P_{\mathbf{1}_q})S(\omega)], \end{cases} \tag{4.4}$$

where $S(\omega) = Y[\Omega^{-1} - \Omega^{-1}Z(Z^T\Omega^{-1}Z)^{-1}Z^T\Omega^{-1}]Y^T$.

It follows from (4.3) and (4.4) that

$$\frac{\partial \mathrm{Vec}(\widehat{B}(\omega))}{\partial \omega^T}\Big|_{\omega=\omega_0} = -\{[(Z^TZ)^{-1}Z^T] \otimes [(X^TX)^{-1}X^TY(I_n - P_Z)]\}D_n, \tag{4.5}$$

$$\frac{\partial \widehat{\xi}(\omega)}{\partial \omega}\Big|_{\omega=\omega_0} = -\frac{\gamma}{n}\mathrm{diag}(M_1), \qquad \frac{\partial \widehat{\eta}(\omega)}{\partial \omega}\Big|_{\omega=\omega_0} = -\frac{\gamma}{(q-1)n}\mathrm{diag}(M_2), \tag{4.6}$$

where D_n is a $n^2 \times n$ matrix whose the $((j-1)n+i, k)$-th elements is $\delta_{ik}\delta_{jk}$ $(i, j, k = 1, 2, \cdots, n)$ (Bai, 1999), and δ_{ij} denotes the Keronecker sign, $M_1 = (I_n - P_Z)Y^TP_{\mathbf{1}_q}Y(I_n - P_Z)$, $M_2 = Y^T(I_q - P_X)Y + (I_n - P_Z)Y^T(P_X - P_{\mathbf{1}_q})Y(I_n - P_Z)$ and $\mathrm{diag}(A) = (a_{11}, a_{22}, \cdots, a_{nn})^T \in R^n$ $(A = (a_{ij})_{n\times n})$, which imply that

$$\frac{\partial \widehat{\sigma}^2(\omega)}{\partial \omega}\Big|_{\omega=\omega_0} = \frac{1}{q}\Big\{\frac{\partial \widehat{\xi}(\omega)}{\partial \omega} + (q-1)\frac{\partial \widehat{\eta}(\omega)}{\partial \omega}\Big\}_{\omega=\omega_0} = -\frac{\gamma}{qn}\mathrm{diag}(M_1 + M_2), \tag{4.7}$$

and

$$\begin{aligned} \frac{\partial \widehat{\rho}(\omega)}{\partial \omega}\Big|_{\omega=\omega_0} &= \frac{q}{q-1}\frac{\partial}{\partial \omega}\Big(\frac{\widehat{\xi}(\omega)}{\widehat{\xi}(\omega) + (q-1)\widehat{\eta}(\omega)}\Big)_{\omega=\omega_0} \\ &= -\frac{1}{qn\widehat{\sigma}^2}\mathrm{diag}\Big[(1-\widehat{\rho})M_1 - \frac{1+(q-1)\widehat{\rho}}{q-1}M_2\Big]. \end{aligned} \tag{4.8}$$

Following Theorems 3.1 and 3.2 and (4.5) $\sim$ (4.8), we can obtain the following conclusions.

Theorem 4.1 For the covariance-weighted perturbation scheme (4.1) or (4.2), if B, σ^2 and ρ are respectively of interest, then

$$\ddot{F}_B = -\gamma P_Z \odot \{(I_n - P_Z)Y^T[\widehat{\xi}^{-1}P_{\mathbf{1}_q} + \widehat{\eta}^{-1}(P_X - P_{\mathbf{1}_q})]Y(I_n - P_Z)\}, \tag{4.9}$$

and

$$d_{\max\sigma^2} \,\|\, \mathrm{diag}(M_1 + M_2), \qquad d_{\max\rho} \Big\| \,\mathrm{diag}\Big((1-\widehat{\rho})M_1 - \frac{1+(q-1)\widehat{\rho}}{q-1}M_2\Big). \tag{4.10}$$

Theorem 4.2 For the covariance-weighted perturbation scheme (4.1) or (4.2), if (σ^2,ρ) and (B,σ^2,ρ) are respectively of interest, then

$$\begin{aligned}\ddot{F}_{(\sigma^2,\rho)} \quad = \quad & -\frac{\gamma}{2(\gamma+qn)n}\Big\{\frac{\gamma+(q-1)n}{\xi^2}\mathrm{diag}(M_1)\mathrm{diag}(M_1)^T+\frac{\gamma+n}{(q-1)\widehat{\eta}^2}\mathrm{diag}(M_2)\mathrm{diag}(M_2)^T\\ & -\frac{2n}{\widehat{\xi}\widehat{\eta}}(\mathrm{diag}(M_1)\mathrm{diag}(M_2)^T+\mathrm{diag}(M_2)\mathrm{diag}(M_1)^T)\Big\},\end{aligned} \tag{4.11}$$

and

$$\ddot{F}_{(B,\sigma^2,\rho)}=\ddot{F}_B+\ddot{F}_{(\sigma^2,\rho)}. \tag{4.12}$$

where $A\odot B$ denotes the Hadamard product of the matrices A and B (Bai, 1999). Theorems 4.1 and 4.2 show how to calculate the Hessian matrix $\ddot{F}$ corresponding to the interest parameter in a covariance-weighted multivariate t-model with US. Consequently, we need to calculate the MICD of $\ddot{F}$, which indicates the most sensitive direction of the multivariate t-model for the covariance-weighted perturbation.

§ 5. Simulation Study

In this section, a simulative calculation is made based on the results obtained above.

Theorem 5.1 For the model (1.1) and (1.2) with the US (1.3). Let

$$\mathrm{Vec}(E)=(I_n\otimes\Sigma^{1/2})\begin{pmatrix}\tan\Theta_1\\ \dfrac{\tan\Theta_2}{\cos\Theta_1}\\ \vdots\\ \dfrac{\tan\Theta_{nq}}{\prod\limits_{i=1}^{nq-1}\cos\Theta_i}\end{pmatrix},\qquad |\Theta_i|<\frac{\pi}{2},\ \ i=1,2,\cdots,nq, \tag{5.1}$$

where $\Sigma^{1/2}$ denotes the unique symmetric root of $\Sigma(Ni,1982)$, then, $\Theta_1,\cdots,\Theta_{nq}$ is determined uniquely by E and are independent, and Θ_i has the density

$$h_i(\theta_i)=\frac{\Gamma((\gamma+i)/2)}{\Gamma((\gamma+i-1)/2)\pi^{1/2}}\cos^{\gamma+i-2}\theta_i,\qquad |\theta_i|<\frac{\pi}{2},\ \ i=1,\cdots,nq. \tag{5.2}$$

Proof It is obvious that the transformation from $\Theta=(\Theta_1,\cdots,\Theta_{nq})^T$ to $\mathrm{Vec}(E)$ is one-to-one. Note that the Jacobian $J_{(\mathrm{Vec}(E)\to\Theta)}$ of the transformation is given by

$$\begin{aligned}J_{(\mathrm{Vec}(B)\to\Theta)} \quad &= \left|(I_n\otimes\Sigma^{1/2})\begin{pmatrix}\dfrac{1}{\cos^2\theta_1} & 0 & \cdots & 0\\ \dfrac{\tan\theta_1\tan\theta_2}{\cos\theta_1} & \dfrac{1}{\cos\theta_1\cos^2\theta_2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ \dfrac{\tan\theta_1\tan\theta_{nq}}{\prod\limits_{i=1}^{nq-1}\cos\theta_i} & \dfrac{\tan\theta_2\tan\theta_{nq}}{\prod\limits_{i=1}^{nq-1}\cos\theta_i} & \cdots & \dfrac{1}{\cos^2\theta_{nq}\prod\limits_{i=1}^{nq-1}\cos\theta_i}\end{pmatrix}\right|\\ &= |\Sigma|^{n/2}\prod_{i=1}^{nq}\cos^{-(nq+2-i)}\theta_i,\qquad |\theta_i|<\frac{\pi}{2},\ \ i=1,\cdots,nq.\end{aligned} \tag{5.3}$$

On the other hand,

$$\frac{1}{1+\sum\limits_{i=1}^{n} e_i^T \Sigma^{-1} e_i} = \prod_{i=1}^{nq} \cos^2 \theta_i. \tag{5.4}$$

By combination (1.2), (5.3) and (5.4) to yield the density of $(\Theta_1, \cdots, \Theta_{nq})^T$ as

$$h(\theta_1, \cdots, \theta_{nq}) = \frac{\Gamma((\gamma+nq)/2)}{\Gamma(\gamma/2)\pi^{nq/2}} \prod_{i=1}^{nq} \cos^{\gamma+i-2} \theta_i, \qquad |\theta_i| < \frac{\pi}{2}, \ \ i=1,\cdots,nq,$$

which implies that $\Theta_1, \cdots, \Theta_{nq}$ are independent and the density of Θ_i is given by (5.2).

This theorem shows that, once generating nq independent random variable Θ_i, $i = 1, \cdots, nq$ in (5.1) which have the same type of distribution, we can simulate the multivariate t-model (1.1) and (1.2) with the US (1.3).

Note that, if the density of random variable $\Theta(\in(-\pi/2, \pi/2))$ is given by

$$g(\theta) = \frac{\Gamma((\gamma+1)/2)}{\Gamma(\gamma/2)\pi^{1/2}} \cos^{\gamma-1} \theta, \qquad |\theta| < \frac{\pi}{2}, \tag{5.5}$$

then making the transformation $T = \sin^2((\Theta+\pi/2)/2)$, which is easy know that $T \sim \mathrm{Beta}(\gamma/2, \gamma/2)$.

Therefore, in order to obtain $\Theta_1, \cdots, \Theta_{nq}$ in (5.1), we need only generate nq independent random variables which have the Beta distribution (Mao, etc., 1998).

In the simulative calculation, let $n = 30$, $m = 1$, $q = 4$, $p = 2$, $\gamma = 2$, $\sigma^2 = 3$, $\rho = 2/3$ and

$$X_{q\times p} = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 1 & 1 & 1 & 1 \end{pmatrix}^T, \qquad B_{p\times m} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \qquad Z_{n\times m} = \begin{pmatrix} \mathbf{1}_{20} \\ 2 \\ \mathbf{1}_9 \end{pmatrix},$$

we obtain some data in following tables.

Table 5.1 The response matrix Y in the model (1.1) generated by simulation

i	1	2	3	4	i	1	2	3	4
1	8.4640	10.4582	12.3123	14.2050	16	7.0669	9.0669	11.0665	13.0665
2	7.6227	9.4913	11.4903	13.4837	17	7.0629	9.0628	11.0623	13.0619
3	7.3384	9.3378	11.3344	13.3301	18	7.0596	9.0595	11.0595	13.0595
4	7.2967	9.2966	11.2750	13.2723	19	7.0575	9.0572	11.0564	13.0564
5	7.2337	9.2264	11.2261	13.2260	20	7.0554	9.0554	11.0553	13.0553
6	7.2041	9.2041	11.2022	13.2005	21	11.0540	15.0533	19.0533	23.0532
7	7.1783	9.1687	11.1684	13.1681	22	7.0491	9.0470	11.0469	13.0469
8	7.1436	9.1364	11.1346	13.1336	23	7.0456	9.0455	11.0455	13.0454
9	7.1192	9.1155	11.1143	13.1129	24	7.0433	9.0432	11.0432	13.0431
10	7.1051	9.1051	11.1043	13.1031	25	7.0401	9.0400	11.0400	13.0400
11	7.0959	9.0935	11.0933	13.0928	26	7.0397	9.0397	11.0396	13.0396
12	7.0904	9.0904	11.0903	13.0902	27	7.0390	9.0388	11.0388	13.0385
13	7.0896	9.0896	11.0894	13.0892	28	7.0378	9.0378	11.0378	13.0378
14	7.0849	9.0849	11.0839	13.0804	29	7.0372	9.0370	11.0370	13.0369
15	7.0719	9.0711	11.0703	13.0703	30	7.0358	9.0357	11.0357	13.0357

Some of the measurements developed in the preceding sections are calculated and displayed in Table 5.2.

Table 5.2 The diagnostic statistics $d_{\max}$ for the unknown parameters

i	$d_{\max B}$	$d_{\max \sigma^2}$	$d_{\max \rho}$	$d_{\max(\sigma^2,\rho)}$	$d_{\max(B,\sigma^2,\rho)}$
1	0.2369	0.2333	0.2218	-0.2118	0.0789
2	0.0952	0.0378	0.0346	-0.0317	-0.0065
3	0.0626	0.0167	0.0167	-0.0168	-0.0074
4	0.0545	0.0126	0.0125	-0.0125	-0.0079
5	0.0447	0.0085	0.0085	-0.0086	-0.0078
6	0.0405	0.0070	0.0070	-0.0070	-0.0076
7	0.0352	0.0053	0.0053	-0.0053	-0.0072
8	0.0296	0.0037	0.0037	-0.0037	-0.0065
9	0.0259	0.0029	0.0029	-0.0029	-0.0060
10	0.0240	0.0025	0.0025	-0.0025	-0.0057
11	0.0223	0.0021	0.0021	-0.0021	-0.0054
12	0.0217	0.0020	0.0020	-0.0020	-0.0053
13	0.0215	0.0020	0.0020	-0.0019	-0.0053
14	0.0206	0.0018	0.0018	-0.0018	-0.0051
15	0.0184	0.0015	0.0014	-0.0014	-0.0047
16	0.0177	0.0014	0.0013	-0.0013	-0.0046
17	0.0170	0.0013	0.0012	-0.0012	-0.0045
18	0.0165	0.0012	0.0011	-0.0011	-0.0044
19	0.0161	0.0011	0.0011	-0.0011	-0.0043
20	0.0158	0.0011	0.0011	-0.0010	-0.0042
21	<u>-0.9569</u>	<u>0.9713</u>	<u>0.9742</u>	<u>-0.9765</u>	<u>0.9965</u>
22	0.0145	0.0009	0.0009	-0.0009	-0.0039
23	0.0142	0.0009	0.0008	-0.0008	-0.0039
24	0.0138	0.0008	0.0008	-0.0008	-0.0038
25	0.0133	0.0008	0.0007	-0.0007	-0.0036
26	0.0132	0.0008	0.0007	-0.0007	-0.0036
27	0.0131	0.0007	0.0007	-0.0007	-0.0036
28	0.0129	0.0007	0.0007	-0.0007	-0.0036
29	0.0128	0.0007	0.0007	-0.0007	-0.0035
30	0.0125	0.0007	0.0006	-0.0006	-0.0035
$\lambda_{\max}$	-2.9167	–	–	-6.3971	-8.8315

It is obvious from Table 5.2 that the 21-th individual is influential on B, σ^2, ρ, (σ^2,ρ) and (B,σ^2,ρ). It may be caused by the the fact that the observations of the 21-th individual are relatively larger than those of others.

References

[1] Bai, P., Assessment of Local Influence in a growth curve model with Rao's simple covariance structure, *Acta Mathematatica Scientia*, **19(5)**(1999), 555–563.

[2] Cook, P.D., Assessment of local influence, *JR statist SOC*, **B48**(1986), 133–169.

[3] Hampel, F.R., The influence curve and its role in robust estimation, *JASA*, **69**(1974), 384–389.

[4] Joarder, A.H. and Ali, M.M., Estimation of the scale matrix of a multivariate t-model under entropy loss, *Metrika*, **46**(1997), 21–32.

[5] Lange, R.J. and Toylor, M.G., Robust statistical modelling using the t-distribution. *J. Amer. Statist. Assoc.*, **84**(1989), 881–896.

[6] Mao, S.S., Wang, J.L., Pu, X.L., *Advanced Mathematical Statistics*, Advanced Education Press (in Chinese), 1998.

[7] Ni, G.X., *Common Theory and Methods of Matrix*, Shanghai Science and Technology Press (in Chinese), 1982.

[8] Wang, S.G., *The Theory and Application of the Linear Model*, Anhui Education Press (in Chinese), 1987.

[9] Wei, B.C., Lu, G.B., Shi, J.Q., *An Introduction to Statistical Diagnotics* (in Chinese), Nanjing: Dongnan University Press, 1991.

具有均匀结构的多元 t - 模型的局部影响分析

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本文研究具有均匀结构的多元 t - 模型的局部影响分析问题．依据 Cook 的曲率度量，我们考虑了微小扰动对统计推断的影响，由此导出了局部影响分析中最为关心的统计量 — 最大曲率方向．作为一种应用，本文还祥细讨论了常见的协方差加权扰动形式．

关 键 词: 多元 t - 模型，均匀结构， ω - 模型，似然距离，曲率.

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