应用概率统计　第二十二卷　　　　Chinese Journal of Applied Probability
第二期　2006 年 5 月　　　　and Statistics Vol. 22 No. 2 May 2006

Explicit Asymptotics for the Ruin Probability with Risky Investment Included*

CHEN YIQING

(*Department of Statistics and Actuarial Science, The University of Hong Kong, Hong Kong*)

DONG XIAOJU　　XIE XIANGSHENG

(*School of Economics and Management, Guangdong University of Technology, Guangzhou, 510090*)

Abstract

In this paper, we investigate the ruin probability of a discrete-time risk model, in which the surplus of an insurance business is currently invested into a risky asset. Using a purely probabilistic treatment, we establish explicit asymptotic relations for the infinite-time ruin probabilities, hence we extend a recent result of Tang and Tsitsiashvili (2003) to the infinite-time case.

Keywords: Asymptotics, regular variation, ruin probability, stochastic equation.

AMS Subject Classification: 62E20, 60G50, 60G70.

§ 1. Introduction

Following Nyrhinen (1999), Tang and Tsitsiashvili (2003), and Chen and Xie (2005), we consider a discrete-time risk model, in which the surplus of the insurance company is currently invested into a risky asset which may lead to a negative return in each year. Denote by $A_n \in (-\infty, \infty)$ the net income (the total incoming premium minus the total claim amount) within year n and by $r_n \in (-1, \infty)$ the return rate at year n, $n = 1, 2, \cdots$. Let the initial surplus be $x \geq 0$. Hence, if we assume that the net income A_n is calculated at the beginning of year n, then the surplus accumulated till the end of year n, characterized by S_n, satisfies the recurrence equation

$$S_0 = x \geq 0, \quad S_n = (1 + r_n)(S_{n-1} + A_n), \qquad n = 1, 2, \cdots; \tag{1.1}$$

alternatively, if we assume that the net income A_n is calculated at the end of year n, then the surplus accumulated till the end of year n, characterized by T_n, satisfies the recurrence equation

$$T_0 = x \geq 0, \quad T_n = (1 + r_n)T_{n-1} + A_n, \qquad n = 1, 2, \cdots. \tag{1.2}$$

Throughout the paper, we assume that the net incomes A_n, $n = 1, 2, \cdots$, constitute a sequence of independent, identically distributed (i.i.d.) random variables (r.v.'s), that the return rates r_n, $n = 1, 2, \cdots$, also constitute a sequence of i.i.d. r.v.'s, and that the two sequences $\{A_n, n = 1, 2, \cdots\}$ and $\{r_n, n = 1, 2, \cdots\}$ are independent.

*This work was supported by the Guangdong Natural Science Foundation (Project No. 980415).

Received 2004. 12. 10. Revised 2005. 7. 1.

Write

$$X_n = -A_n, \quad Y_n = \frac{1}{1+r_n}, \qquad n = 1, 2, \cdots. \tag{1.3}$$

The r.v. X_n is the net payout during year n and the r.v. Y_n is the discount factor from year n to year $n-1$, $n = 1, 2, \cdots$. In what follows, we write by X and Y the generic r.v.'s of the sequences $\{X_n, n = 1, 2, \cdots\}$ and $\{Y_n, n = 1, 2, \cdots\}$ and write by F and G the distribution functions (d.f.'s) of the r.v.'s X and Y, respectively. Clearly, the r.v. Y is strictly positive. We assume that the tail probability $\overline{F}(x) = 1 - F(x) = \mathsf{P}(X > x)$ satisfies $\overline{F}(x) > 0$ for any real number x. In the terminology of Norberg (1999) and Tang and Tsitsiashvili (2003), we call X the insurance risk and Y the financial risk.

Corresponding to the surplus processes (1.1) and (1.2), we define the ruin probabilities within finite time $n = 1, 2, \cdots$ and infinite time as

$$\psi_S(x, n) = \mathsf{P}\Big(\min_{0\le k\le n} S_k < 0 | S_0 = x\Big), \qquad \psi_T(x, n) = \mathsf{P}\Big(\min_{0\le k\le n} T_k < 0 | T_0 = x\Big), \tag{1.4}$$

respectively,

$$\psi_S(x) = \mathsf{P}\Big(\min_{0\le k<\infty} S_k < 0 | S_0 = x\Big), \qquad \psi_T(x) = \mathsf{P}\Big(\min_{0\le k<\infty} T_k < 0 | T_0 = x\Big). \tag{1.5}$$

In this paper we are interested in the asymptotic behavior of these probabilities.

Hereafter, all limit relationships are for $x \to \infty$ unless stated otherwise; for two positive functions $a(x)$ and $b(x)$, we write $a(x) \lesssim b(x)$ if $\limsup a(x)/b(x) \le 1$, write $a(x) \gtrsim b(x)$ if $\liminf a(x)/b(x) \ge 1$, and write $a(x) \sim b(x)$ if both.

Nyrhinen (1999) investigated the asymptotic behavior of the ruin probability $\psi_S(x)$. In terms of the model described above, we can obtain a combination of Theorems 3.3 and 3.4 of Nyrhinen (1999) as follows: if (1) $w = \sup\{t | \mathsf{E}Y^t \le 1\} \in (0, \infty)$, (2) $\mathsf{E}Y^t$ and $\mathsf{E}|X|^t$ are finite for some $t > w$, (3) $\overline{F}(0) > 0$, and (4) some of the convolution powers of the distribution of $\log Y$ has a non-trivial absolutely continuous component, then the relation

$$\psi_S(x) \sim Cx^{-w} \tag{1.6}$$

holds for some positive, but implicit, constant C.

The objective of the present paper is to establish explicit asymptotic relations for the ruin probabilities $\psi_S(x)$ and $\psi_T(x)$ under some other assumptions on the tails of the r.v.'s X and Y. In the proof, we will use a recent result of Tang and Tsitsiashvili (2003), who investigated the finite-time ruin probabilities $\psi_S(x, n)$ and $\psi_T(x, n)$ under the assumptions that F is heavy tailed and that the tail $\overline{G}$ is dominated by the tail $\overline{F}$. We will also apply some results obtained by Vervaat (1979) in the literature of stochastic difference equations.

The rest of this paper consists of three sections: Section 2 presents the main results after recalling an important class of heavy-tailed distributions, Section 3 collects some lemmas, and Section 4 proves the theorems.

§ 2. Main Results

We will assume that the d.f. F of the insurance risk X has a regularly varying tail, denoted by $F \in \mathcal{R}$. By definition, a d.f. F concentrated on $(-\infty, \infty)$ belongs to the class $\mathcal{R}$ if there is some $\alpha \geq 0$ such that

$$\lim_{x \to \infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = y^{-\alpha} \tag{2.1}$$

for any $y > 0$. For simplicity, we denote by $F \in \mathcal{R}_{-\alpha}$ the regularity property in (2.1). In this case we have

$$\overline{F}(x) = x^{-\alpha} c(x) \exp\Big\{ \int_a^x \frac{\varepsilon(y)}{y} \mathrm{d}y \Big\}, \qquad x > a, \tag{2.2}$$

for some $a > 0$, where $c(x) \to c \in (0, \infty)$ and $\varepsilon(x) \to 0$; see Bingham et al. (1987, page 21). The corresponding r.v. X satisfies

$$\mathsf{E}(X^+)^p < \infty \ \text{ for } \ 0 \leq p < \alpha, \qquad \mathsf{E}(X^+)^p = \infty \ \text{ for } \ p > \alpha, \tag{2.3}$$

where $x^+ = \max\{x, 0\}$.

Under the assumptions that F belongs to a heavy-tailed distribution class, which is slightly larger than the class $\mathcal{R}$, and that the tail $\overline{G}$ is dominated by the tail $\overline{F}$ (to be precise, $\mathsf{E}Y^p < \infty$ for some p larger than the upper Matuszewska index of the d.f. F), Tang and Tsitsiashvili (2003, Theorem 5.1 and Remark 5.1) obtained precise asymptotic estimates for the finite-time ruin probabilities $\psi_S(x, n)$ and $\psi_T(x, n)$ for each fixed $n = 1, 2, \cdots$. The really applicable results of their paper were obtained in the case that

$$F \in \mathcal{R}_{-\alpha} \quad \text{and} \quad \mathsf{E}Y^p < \infty \qquad \text{for some } \ p > \alpha > 0. \tag{2.4}$$

In this case, it holds for each fixed $n = 1, 2, \cdots$ that

$$\psi_S(x, n) \sim \frac{1 - (\mathsf{E}Y^\alpha)^n}{1 - \mathsf{E}Y^\alpha} \overline{F}(x); \tag{2.5}$$

see Tang and Tsitsiashvili (2003, Theorem 5.2(3) and Remark 5.1).

If further we assume that

$$\mathsf{E}Y^\alpha < 1, \tag{2.6}$$

then the term $(\mathsf{E}Y^\alpha)^n$ in relation (2.5) vanishes as n increases. Though relation (2.5) was only proved by Tang and Tsitsiashvili (2003) to hold for fixed n, we intuitively believe that the infinite-time ruin probability $\psi_S(x)$ should satisfy

$$\psi_S(x) \sim \frac{1}{1 - \mathsf{E}Y^\alpha} \overline{F}(x). \tag{2.7}$$

The following result proves that this is true.

Theorem 2.1 Under assumptions (2.4) and (2.6), the asymptotic relation (2.7) holds.

Theorem 2.1 successfully establishes an asymptotic relation for the ruin probability $\psi_S(x)$ in a fully explicit form.

As for the ruin probability $\psi_T(x)$, we have a similar explicit asymptotic relation below.

Theorem 2.2 Under assumptions (2.4) and (2.6), it holds that

$$\psi_T(x) \sim \frac{\mathsf{E}Y^\alpha}{1-\mathsf{E}Y^\alpha}\overline{F}(x). \tag{2.8}$$

§ 3. Some Lemmas

We first point out a simple relationship between the ruin probabilities indexed by S and those indexed by T.

Lemma 3.1 For the risk model introduced in Section 1, the ruin probabilities defined by (1.4) and (1.5) satisfy the relations

$$\psi_T(x,n) = \int_0^\infty \psi_S(x/y,n)G(\mathrm{d}y), \qquad n=1,2,\cdots, \tag{3.1}$$

and

$$\psi_T(x) = \int_0^\infty \psi_S(x/y)G(\mathrm{d}y). \tag{3.2}$$

Proof Iterating the recurrence equation (1.1) yields that

$$S_0 = x, \quad S_n = x\prod_{j=1}^n(1+r_j) + \sum_{i=1}^n A_i\prod_{j=i}^n(1+r_j), \qquad n=1,2,\cdots. \tag{3.3}$$

We write the discounted value of the surplus S_n in (3.3) as

$$\widetilde{S}_0 = x, \qquad \widetilde{S}_n = S_n\prod_{j=1}^n Y_j = x + \sum_{i=1}^n A_i\prod_{j=1}^{i-1}Y_j = x - \sum_{i=1}^n X_i\prod_{j=1}^{i-1}Y_j,$$

where $\prod\limits_{j=1}^{0} = 1$ by convention. It is clear that for each $n=1,2,\cdots$,

$$\psi_S(x,n) = \mathsf{P}\Big(\max_{1\le k\le n}\sum_{i=1}^k X_i\prod_{j=1}^{i-1}Y_j > x\Big), \qquad \psi_S(x) = \mathsf{P}\Big(\max_{1\le k<\infty}\sum_{i=1}^k X_i\prod_{j=1}^{i-1}Y_j > x\Big). \tag{3.4}$$

Similarly, it holds that

$$\psi_T(x,n) = \mathsf{P}\Big(\max_{1\le k\le n}\sum_{i=1}^k X_i\prod_{j=1}^{i}Y_j > x\Big), \qquad \psi_T(x) = \mathsf{P}\Big(\max_{1\le k<\infty}\sum_{i=1}^k X_i\prod_{j=1}^{i}Y_j > x\Big). \tag{3.5}$$

Hence by the i.i.d. assumptions made in Section 1, relations (3.1) and (3.2) holds. #

The lemma below is a combination of several results of Vervaat (1979).

Lemma 3.2 Let $\{(\widetilde{X}_n,\widetilde{Y}_n), n=1,2,\cdots\}$ be a sequence of i.i.d. random pairs with generic random pair $(\widetilde{X},\widetilde{Y})$. Consider the stochastic difference equation

$$\widetilde{V}_n = \widetilde{Y}_n\widetilde{V}_{n-1} + \widetilde{X}_n, \qquad n=1,2,\cdots. \tag{3.6}$$

If $-\infty \le \mathsf{E}\log\widetilde{Y} < 0$ and $\mathsf{E}(\log|\widetilde{X}|)^+ < \infty$, then the r.v.'s $\widetilde{V}_n$ converges in distribution to some real-valued r.v. V_∞, which is invariant in distribution for all initial r.v.'s $\widetilde{V}_0$.

Proof By Theorem 1.6(b, c) of Vervaat (1979), the stochastic equation

$$\widetilde{V} =^d \widetilde{Y}\widetilde{V} + \widetilde{X}, \qquad \widetilde{V} \text{ is independent of } (\widetilde{X}, \widetilde{Y}), \tag{3.7}$$

has a solution, where $=^d$ denotes equality in distribution. Then, by Theorem 1.5(i) of Vervaat (1979), this solution is unique in distribution and $\widetilde{V}_n$ defined by (3.6) converges in distribution to some $V_\infty(\widetilde{V}_0)$, say, for any initial r.v. $\widetilde{V}_0$. Since, by Lemma 1.1 of Vervaat (1979), any limit r.v. $V_\infty(\widetilde{V}_0)$ should be a solution of equation (3.7), we prove that for all $\widetilde{V}_0$ the r.v.'s $V_\infty(\widetilde{V}_0)$ are identical in distribution. #

The following lemma is well known and is from Proposition of Feller (1971, p. 278) or Lemma 1.3.1 of Embrechts et al. (1997).

Lemma 3.3 Let F_1 and F_2 be two d.f.'s concentrated on $[0, \infty)$. If $F_1 \in \mathcal{R}_{-\alpha}$ and $F_2 \in \mathcal{R}_{-\alpha}$ for some $\alpha \geq 0$, then their convolution $F_1 * F_2 \in \mathcal{R}_{-\alpha}$ and $\overline{F_1 * F_2}(x) \sim \overline{F}_1(x) + \overline{F}_2(x)$.

The following lemma is from Breiman (1965).

Lemma 3.4 Let X and Y be two independent r.v.'s distributed by F and G, respectively, where Y is nonnegative. If d.f. $F \in \mathcal{R}_{-\alpha}$ for some $0 < \alpha < \infty$ and $\mathsf{E}Y^p < \infty$ for some $p > \alpha$, then

$$\lim_{x\to\infty} \frac{\mathsf{P}(XY > x)}{\mathsf{P}(X > x)} = \mathsf{E}Y^\alpha.$$

§ 4. Proofs of the Main Results

4.1 Proof of Theorem 2.1

From (3.4) and (2.5), it is clear that for each $n = 1, 2, \cdots$,

$$\psi_S(x) \geq \psi_S(x, n) \sim \frac{1 - (\mathsf{E}Y^\alpha)^n}{1 - \mathsf{E}Y^\alpha} \overline{F}(x).$$

It follows that for each $n = 1, 2, \cdots$,

$$\liminf_{x\to\infty} \frac{\psi_S(x)}{\overline{F}(x)} \geq \frac{1 - (\mathsf{E}Y^\alpha)^n}{1 - \mathsf{E}Y^\alpha}.$$

Letting $n \to \infty$ on the right-hand side,

$$\psi_S(x) \gtrsim \frac{1}{1 - \mathsf{E}Y^\alpha} \overline{F}(x). \tag{4.1}$$

It remains to derive a corresponding asymptotic upper bound for the ruin probability $\psi_S(x)$. To this end, from (3.4) we derive that

$$\psi_S(x) \leq \mathsf{P}\Big(\sum_{i=1}^{\infty} X_i^+ \prod_{j=1}^{i-1} Y_j > x\Big). \tag{4.2}$$

For each $n = 1, 2, \cdots, \infty$, set

$$U_n = \sum_{i=1}^{n} X_i^+ \prod_{j=1}^{i-1} Y_j.$$

By the i.i.d. assumptions, it is easy to see that for each $n = 1, 2, \cdots$,

$$U_n =^d \sum_{i=1}^{n} X_i^+ \prod_{j=i+1}^{n} Y_j = V_n, \tag{4.3}$$

where $\prod_{j=n+1}^{n} Y_j$ is equal to 1 by convention. Clearly, with $V_0 = 0$, the sequence $\{V_n, n = 1, 2, \cdots\}$ satisfies the recurrence equation

$$V_n = Y_n V_{n-1} + X_n^+ \qquad \text{for } n = 1, 2, \cdots. \tag{4.4}$$

Now we check the convergence in distribution of the sequence $\{V_n, n = 1, 2, \cdots\}$. By assumption (2.6) we easily understand that $-\infty \le \mathsf{E}\log Y < 0$ should hold since the function $f(t) = \mathsf{E}Y^t$ is convex in $t \in [0, \alpha]$ and $f'_+(0) = \mathsf{E}\log Y$. Hence by Lemma 3.2, V_n converges in distribution to a r.v. V_∞, say, which is invariant for all V_0. In view of (4.3), this actually proves that U_∞ is finite almost surely and is equal to V_∞ in distribution. Specifically, we choose V_∞ to be independent of the sequences $\{X_n, n = 1, 2, \cdots\}$ and $\{Y_n, n = 1, 2, \cdots\}$.

Knowing $F \in \mathcal{R}_{-\alpha}$, we announce that

$$\mathsf{P}(U_\infty > x) = \mathsf{P}(V_\infty > x) \le \mathsf{P}(V_0 > x), \qquad x \ge 0, \tag{4.5}$$

for some nonnegative initial r.v. V_0 with a d.f. from the class $\mathcal{R}_{-\alpha}$. For this purpose, we choose a nonnegative r.v. Z independent of the sequences $\{X_n, n = 1, 2, \cdots\}$ and $\{Y_n, n = 1, 2, \cdots\}$ such that $\mathsf{P}(Z > x) \sim c\overline{F}(x)$ for some positive constant c, which will be specified later. By Lemma 3.4 we know that the d.f. of the r.v. $Y_1 Z$ belongs to the class $\mathcal{R}_{-\alpha}$ and $\mathsf{P}(Y_1 Z > x) \sim c\mathsf{E}Y^\alpha \overline{F}(x)$; by this and Lemma 3.3 we further know that $\mathsf{P}(Y_1 Z + X_1^+ > x) \sim (c\mathsf{E}Y^\alpha + 1)\overline{F}(x)$. Hence, if we choose $c > 0$ sufficiently large such that $c\mathsf{E}Y^\alpha + 1 < c$, then there is some constant $x_0 > 0$ such that for all $x > x_0$, $\mathsf{P}(Y_1 Z + X_1^+ > x) \le \mathsf{P}(Z > x)$. By this inequality we can prove that for all $x \ge 0$,

$$\mathsf{P}(Y_1 Z + X_1^+ > x | Z > x_0) \le \mathsf{P}(Z > x | Z > x_0). \tag{4.6}$$

In fact, for $0 \le x \le x_0$, inequality (4.6) trivially holds since $\mathsf{P}(Z > x | Z > x_0) = 1$; for $x > x_0$, inequality (4.6) can be verified in the following way:

$$\begin{aligned} \mathsf{P}(Y_1 Z + X_1^+ > x | Z > x_0) &= \frac{\mathsf{P}(Y_1 Z + X_1^+ > x, Z > x_0)}{\mathsf{P}(Z > x_0)} \le \frac{\mathsf{P}(Y_1 Z + X_1^+ > x)}{\mathsf{P}(Z > x_0)} \\ &\le \frac{\mathsf{P}(Z > x)}{\mathsf{P}(Z > x_0)} = \mathsf{P}(Z > x | Z > x_0). \end{aligned}$$

We identify the initial r.v. V_0 such that it is independent of the sequences $\{X_n, n = 1, 2, \cdots\}$ and $\{Y_n, n = 1, 2, \cdots\}$ and is equal in distribution to the r.v. Z conditional on $(Z > x_0)$. Hence,

$$\mathsf{P}(V_0 > x) = \mathsf{P}(Z > x | Z > x_0) \sim \frac{c}{\mathsf{P}(Z > x_0)} \overline{F}(x). \tag{4.7}$$

It follows from (4.6) that for all $x \ge 0$,

$$\mathsf{P}(V_1 > x) = \mathsf{P}(Y_1 V_0 + X_1^+ > x) = \mathsf{P}(Y_1 Z + X_1^+ > x | Z > x_0) \le \mathsf{P}(V_0 > x).$$

Successively,

$$\begin{aligned}\mathsf{P}(V_2>x) &= \mathsf{P}(Y_2V_1+X_2^+>x)\le \mathsf{P}(Y_2V_0+X_2^+>x)=\mathsf{P}(Y_1V_0+X_1^+>x)\\ &= \mathsf{P}(V_1>x)\le \mathsf{P}(V_0>x).\end{aligned}$$

Applying the mathematical induction method we know that for all $n=1,2,\cdots$ and all $x\ge 0$,

$$\mathsf{P}(V_n>x)\le\cdots\le \mathsf{P}(V_2>x)\le \mathsf{P}(V_1>x)\le \mathsf{P}(V_0>x).$$

Since V_n converges in distribution to V_∞, taking $n\to\infty$ yields that for all $x\ge 0$,

$$\mathsf{P}(U_\infty>x)=\mathsf{P}(V_\infty>x)\le \mathsf{P}(V_0>x),$$

as announced in (4.5).

We continue the proof of Theorem 2.1. For any $\varepsilon\in(0,1)$ and any $n=1,2,\cdots$, we split the probability on the right-hand side of inequality (4.2) into two parts as

$$\begin{aligned}\psi_S(x) &\le \mathsf{P}\Big(\sum_{i=1}^{n}X_i^+\prod_{j=1}^{i-1}Y_j>(1-\varepsilon)x\Big)+\mathsf{P}\Big(\sum_{i=n+1}^{\infty}X_i^+\prod_{j=1}^{i-1}Y_j>\varepsilon x\Big)\\ &= I_1(x,\varepsilon,n)+I_2(x,\varepsilon,n).\end{aligned}$$

Clearly, from (2.5) and (3.4) with X_i being replaced by X_i^+, we obtain that

$$I_1(x,\varepsilon,n)\sim\frac{1-(\mathsf{E}Y^\alpha)^n}{1-\mathsf{E}Y^\alpha}\overline{F}((1-\varepsilon)x).$$

Since V_∞ is independent of the sequences $\{X_n,n=1,2,\cdots\}$ and $\{Y_n,n=1,2,\cdots\}$,

$$\begin{aligned}I_2(x,\varepsilon,n) &= \mathsf{P}\Big(\Big(\sum_{i=n+1}^{\infty}X_i^+\prod_{j=n+1}^{i-1}Y_j\Big)\prod_{j=1}^{n}Y_j>\varepsilon x\Big)=\mathsf{P}\Big(V_\infty\prod_{j=1}^{n}Y_j>\varepsilon x\Big)\\ &\le \mathsf{P}\Big(V_0\prod_{j=1}^{n}Y_j>\varepsilon x\Big)\sim\frac{c(\mathsf{E}Y^\alpha)^n}{\mathsf{P}(Z>x_0)}\overline{F}(\varepsilon x),\end{aligned}$$

where in the last step we applied relation (4.7) and Lemma 3.4. Thus, for any $\varepsilon\in(0,1)$ and any $n=1,2,\cdots$,

$$\psi_S(x)\lesssim\frac{1-(\mathsf{E}Y^\alpha)^n}{1-\mathsf{E}Y^\alpha}\overline{F}((1-\varepsilon)x)+\frac{c(\mathsf{E}Y^\alpha)^n}{\mathsf{P}(Z>x_0)}\overline{F}(\varepsilon x).$$

It follows from $F\in\mathcal{R}_{-\alpha}$ that

$$\begin{aligned}\limsup_{x\to\infty}\frac{\psi_S(x)}{\overline{F}(x)} &\le \frac{1-(\mathsf{E}Y^\alpha)^n}{1-\mathsf{E}Y^\alpha}\limsup_{x\to\infty}\frac{\overline{F}((1-\varepsilon)x)}{\overline{F}(x)}+\frac{c(\mathsf{E}Y^\alpha)^n}{\mathsf{P}(Z>x_0)}\limsup_{x\to\infty}\frac{\overline{F}(\varepsilon x)}{\overline{F}(x)}\\ &= \frac{1-(\mathsf{E}Y^\alpha)^n}{1-\mathsf{E}Y^\alpha}(1-\varepsilon)^{-\alpha}+\frac{c(\mathsf{E}Y^\alpha)^n}{\mathsf{P}(Z>x_0)}\varepsilon^{-\alpha}.\end{aligned}$$

Since n and ε are arbitrary and $\mathsf{E}Y^\alpha<1$, first letting $n\to\infty$ and then letting $\varepsilon\to 0$ lead to the desired result that

$$\psi_S(x)\lesssim\frac{1}{1-\mathsf{E}Y^\alpha}\overline{F}(x).\tag{4.8}$$

Combining (4.1) and (4.8) we obtain (2.7). This ends the proof. #

4.2 Proof of Theorem 2.2

Introduce a survival d.f. R_S by $R_S(x) = (1 - \psi_S(x))1_{(x\geq 0)}$, which is a standard d.f. concentrated on $[0, \infty)$ with a mass $R_S(\{0\}) = 1 - \psi_S(0)$ at 0. Theorem 2.1 has proved that $R_S \in \mathcal{R}_{-\alpha}$. Hence, applying Lemma 3.4 to relation (3.2) yields that

$$\psi_T(x) \sim \mathsf{E}Y^\alpha \psi_S(x) \sim \frac{\mathsf{E}Y^\alpha}{1 - \mathsf{E}Y^\alpha}\overline{F}(x).$$

This ends the proof. #

References

[1] Bingham, N.H., Goldie, C.M., Teugels, J.L., *Regular Variation*, Cambridge University Press, Cambridge, 1987.

[2] Breiman, L., On some limit theorems similar to the arc-sin law, *Theor. Probability Appl.*, **10**(1965), 323–331.

[3] Chen, Y., Xie, X., The finite time ruin probability with the same heavy-tailed insurance and financial risks, *Acta Math. Appl. Sin. Engl. Ser.*, **21(1)**(2005), 153–156.

[4] Embrechts, P., Klüppelberg, C., Mikosch, T., *Modelling Extremal Events for Insurance and Finance*, Springer-Verlag, Berlin, 1997.

[5] Feller, W., *An Introduction to Probability Theory and Its Applications*, Vol. II, Second edition John Wiley & Sons, Inc., New York-London-Sydney, 1971.

[6] Norberg, R., Ruin problems with assets and liabilities of diffusion type, *Stochastic Process. Appl.*, **81(2)**(1999), 255–269.

[7] Nyrhinen, H., On the ruin probabilities in a general economic environment, *Stochastic Process. Appl.*, **83(2)**(1999), 319–330.

[8] Tang, Q., Tsitsiashvili, G., Precise estimates for the ruin probability in finite horizon in a discrete-time model with heavy-tailed insurance and financial risks, *Stochastic Process. Appl.*, **108(2)**(2003), 299–325.

[9] Vervaat, W., On a stochastic difference equation and a representation of nonnegative infinitely divisible random variables, *Adv. in Appl. Probab.*, **11(4)**(1979), 750–783.

在风险投资下破产概率的一个渐近显式

陈宜清 董效菊 谢湘生

(香港大学统计与精算学系，香港) (广东工业大学经济管理学院，广州， 510090)

在本文中，我们研究了一个离散时间风险模型的破产概率．在此风险模型中，保险公司的剩余资本被用于进行风险投资．我们运用纯概率的手法建立了无限时间破产概率的渐近显式，从而将 Tang 和 Tsitsiashvili (2003) 近期的一个结果推广到了无限时间的场合．

关 键 词: 渐近式，正则变化，破产概率，随机方程．

学 科 分 类 号: O211.3.