应用概率统计　第二十六卷
第一期　2010年2月

Chinese Journal of Applied Probability
and Statistics Vol.26 No.1 Feb. 2010

The Asymptotic of Finite Time Ruin Probabilities for Risk Model with Variable Interest Rates *

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Abstract

Consider a discrete time risk model

$$U_n = (U_{n-1} + Y_n)(1 + r_n) - X_n, \qquad n = 1, 2, \cdots,$$

where $U_0 = x > 0$ is the initial reserve of an insurance company, r_n the interest rates, Y_n the total amount of premiums, X_n the total amount of claims and U_n the reserve at time n. Under some mild conditions on Y_n and r_n, we obtain the uniform asymptotics relation for the finite time ruin probabilities $\psi(x, N) \sim \sum\limits_{k=1}^{N} \overline{F}_X((1+r_1)\cdots(1+r_n)x)$ as $x \to \infty$, where $\psi(x, N) = \mathsf{P}\big(\min\limits_{0 \le n \le N} U_n < 0 \,|U_0 = x\big)$, $N \ge 1$, $\overline{F}_X(x)$ is the tail distribution of X_1, and the uniformity is with respect to $N \ge 1$.

Keywords: Discrete time risk model, heavy-tailed, interest rate, finite time ruin probability, asymptotics.

AMS Subject Classification: 60K10.

§1. Introduction

Consider a discrete time risk model, in which the surplus at time n, U_n, is expressed by a recursion:

$$U_n = (U_{n-1} + Y_n)(1 + r_n) - X_n, \qquad n = 1, 2, \cdots, \tag{1.1}$$

where $U_0 = x > 0$ is the initial surplus, $\{r_n; n \ge 1\}$ is a sequence of non-negative real numbers, $\{X_n; n \ge 1\}$ and $\{Y_n; n \ge 1\}$ are two sequences of independent identically distributed (i.i.d.) random variables with common distribution function F_X and F_Y, respectively. r_n denotes the interest rate during the nth period, i.e., from time $n-1$ to time n; Y_n is the amount of premium income during the nth period, and is received at the beginning of the nth period, i.e. at time $n-1$; X_n represents the claim amount of the

*Supported by the National Natural Science Foundation of China (10671149, 10801139) and Key Project of Philosophy and Social Sciences Research of the Ministry of Education (07JZD0010).

Received August 18, 2007. Revised December 12, 2008.

nth period, and is paid at the end of the nth period, i.e. at time n. $\{X_n\}$ and $\{Y_n\}$ are assumed to be independent.

The ruin probabilities of finite time horizon for a risk model with surplus process $\{U_n; n \geq 1\}$are given by

$$\psi(x, N) = \mathsf{P}\Big(\min_{0 \leq n \leq N} U_n < 0 \Big| U_0 = x\Big) \qquad \text{for } x > 0 \text{ and } N = 1, 2, \cdots. \tag{1.2}$$

The asymptotics of $\psi(x, N)$ is a classical topic in risk theory. For the discrete time risk model (1.1), Ng et al. (2002) discussed the asymptotics of $\psi(x, N)$ for a special case where $r_n \equiv r$ for $n = 1, 2, \cdots$, and $\triangle_n := X_n - Y_n(1 + r)$ follows the Pareto law with tail $\mathsf{P}(\triangle_n > x) \sim x^{-\alpha} L(x)$. They proved that for each $N \geq 1$,

$$\psi(x, N) \sim \frac{(1 + r)^{(N+1)\alpha} - (1 + r^{\alpha})}{(1 + r^{\alpha}) - 1} x^{-\alpha} L(x).$$

The uniformity for asymptotics of $\psi(x, N)$ is another interesting topic. Up to now, the uniformly result for the risk model (1.1) is remain undiscussed. This paper is intend to derive the uniformly asymptotics of $\psi(x, N)$ for this model, following the method used for a discrete time risk model which is slightly different from risk model (1.1) and is detailed as follows. Assume that the premiums are received and the claims are paid at the end of each period of time, then the claim amount minus the premium amount can be taken as one term – a gloss loss sequence $\{W_n; n \geq 1\}$ which has heavy-tailed distribution function (d.f.) $F(x)$. In this case, the surplus of this model is given by a recursion

$$U_n = U_{n-1}(1 + r_n) - W_n, \qquad n = 1, 2, \cdots. \tag{1.3}$$

The uniform asymptotics of $\psi(x, N)$ defined in (1.2) for the risk model (1.3) have been discussed in several references. When $r_n \equiv 0$ for all $n \geq 1$, Korschunov (2002) obtained that, under certain conditions on the heavy tail of the gloss loss, it holds uniformly for $N = 1, 2, \cdots$ that

$$\psi(x, N) \sim \frac{1}{\mu} \int_x^{x+N\mu} \overline{F}(u) \mathrm{d}u,$$

as $x \to +\infty$, where $\mu := \displaystyle\int_0^{+\infty} \overline{F}(x) \mathrm{d}x$. When $r_n \equiv r$ for all $n \geq 1$, Jiang and Tang (2003) gave an asymptotic relation for $\psi(x, N)$, generalizing the corresponding result of Ng et al. (2002). They proved that under certain conditions,

$$\psi(x, N) \sim \sum_{k=1}^{N} \overline{F}((1 + r)^k x)$$

holds uniformly for $N = 1, 2, \cdots$, that is

$$\lim_{x \to \infty} \sup_{N \geq 1} \left| \frac{\psi(x, N)}{\sum\limits_{k=1}^{N} \overline{F}((1 + r)^k x)} - 1 \right| = 0.$$

In this paper, we consider the risk model (1.1), in which the premiums are received and the claims are paid at different time and the interest rates are variable. We shall prove that under some reasonable conditions,

$$\psi(x,N) \sim \sum_{k=1}^{N} \overline{F}_X((1+r_1)\cdots(1+r_k)x)$$

holds uniformly for $N = 1, 2, \cdots$.

The rest of the paper is organized as follows. In Section 2, some notations and preliminaries are provided. The main results are stated in Section 3. In Section 4, the proofs of the main results are provided.

§2. Notations and Preliminaries

First we give some notations. For two positive infinitesimals $A(x)$ and $B(x)$, $A(x) \lesssim B(x)$ if $\limsup\limits_{x\to\infty} A(x)/B(x) \le 1$, $A(x) \gtrsim B(x)$ if $\liminf\limits_{x\to\infty} A(x)/B(x) \ge 1$, and $A(x) \sim B(x)$ if $A(x) \lesssim B(x)$ and $A(x) \gtrsim B(x)$.

Like many researches in insurance mathematics, we restrict our interest to the case of heavy tails. An important subclass of heavy-tailed distribution functions is $R_{-\alpha}$, the regularly varying class. An extended version of $R_{-\alpha}$ is the so-called extended regularly varying (ERV) class.

Definition 2.1 Let X be a non-negative random variable with d.f. F. X (or its d.f.) has a regularly varying tail if $\overline{F}(x) = x^{-\alpha}L(x)$ for some $\alpha > 0$, where $L(x)$ is a slowly varying function as $x \to \infty$. We write $\overline{F} \in R_{-\alpha}$.

Definition 2.2 A non-negative random variable X with d.f. F is said to be in the ERV class, if there exist constants $1 < \alpha \le \beta < \infty$, such that for $y \ge 1$,

$$y^{-\beta} \le \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \le \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \le y^{-\alpha},$$

or equivalently, for $v \le 1$,

$$v^{\beta} \le \liminf_{x\to\infty} \frac{\overline{F}(x/v)}{\overline{F}(x)} \le \limsup_{x\to\infty} \frac{\overline{F}(x/v)}{\overline{F}(x)} \le v^{\alpha}.$$

We write $\overline{F} \in \mathrm{ERV}(-\alpha, -\beta)$. If $\alpha = \beta$, then $\overline{F} \in R_{-\alpha}$.

Remark 1 For a d.f. F with $\overline{F} \in \mathrm{ERV}(-\alpha, -\beta)$, it holds obviously that

$$\lim_{x\to\infty} \frac{\overline{F}(x+L)}{\overline{F}(x)} = 1 \qquad \text{for any fixed } L > 0. \tag{2.1}$$

Relation (2.1) characterizes the class, L, of long-tailed distributions. We write $\overline{F} \in L$.

We provide here a result given by Ng et al. (2002), which will be needed in our proofs.

Theorem Ng Suppose that the d.f. $\overline{F}_i \in L$ for $i \geq 1$. Then we have that, for each $n \geq 1$,

$$\mathsf{P}\Big(\max_{1\leq k\leq n} S_k > x\Big) \sim \mathsf{P}(S_n > x), \tag{2.2}$$

where $S_n = \sum\limits_{i=1}^{n} X_i$ and X_i has d.f. $F_i(x)$.

§3. Main Results

Recall that the risk model (1.1). First we provide the asymptotic result of $\psi(x, N)$ for each $N \geq 1$.

Theorem 3.1 Suppose that $\overline{F}_X \in \mathrm{ERV}(-\alpha, -\beta)$ for some $1 < \alpha \leq \beta < \infty$. Assume that Y_1 has finite mean $\mu := \mathsf{E}(Y_1)$ and finite variance $D(Y_1)$. Then for each $N \geq 1$,

$$\psi(x, N) \sim \sum_{k=1}^{N} \overline{F}_X((1+r_1)\cdots(1+r_k)x) \tag{3.1}$$

holds, that is

$$\lim_{x\to\infty}\left|\frac{\psi(x,N)}{\sum\limits_{k=1}^{N} \overline{F}_X((1+r_1)\cdots(1+r_k)x)} - 1\right| = 0.$$

The uniform asymptotic result of $\psi(x, N)$ for the risk model (1.1) is as follows.

Theorem 3.2 Suppose that $\overline{F}_X \in \mathrm{ERV}(-\alpha, -\beta)$ for some $1 < \alpha \leq \beta < \infty$. Assume that Y_1 has finite mean $\mu := \mathsf{E}(Y_1)$, finite variance $D(Y_1)$ and

$$\lim_{n\to\infty} \frac{r_1 + \cdots + r_n}{n} = r \tag{3.2}$$

for some $r > 0$. Then

$$\psi(x, N) \sim \sum_{k=1}^{N} \overline{F}_X((1+r_1)\cdots(1+r_k)x) \tag{3.3}$$

holds uniformly for $N \geq 1$, that is

$$\lim_{x\to\infty}\sup_{N\geq 1}\left|\frac{\psi(x,N)}{\sum\limits_{k=1}^{N} \overline{F}_X((1+r_1)\cdots(1+r_k)x)} - 1\right| = 0.$$

§4. Proofs

To prove the main results, we need some lemmas. The last two lemmas came from Jiang and Tang (2003), for the proofs of these two lemmas, readers may refer to Jiang and Tang (2003).

Lemma 4.1 Let $F = F_X * F_{-Y}$, F_X is the d.f. of X concentrated on $(0, +\infty)$ and $\overline{F}_X \in \mathrm{ERV}(-\alpha, -\beta)$ for $1 < \alpha \le \beta < \infty$, F_{-Y} is the d.f. of $-Y$, while Y is a non-negative random variable with finite mean $\mu = \mathsf{E}(Y)$ and finite variance $D(Y)$. Then $\overline{F}(x) \sim \overline{F}_X(x)$ and $\overline{F}(x) \in \mathrm{ERV}(-\alpha, -\beta)$.

Proof of Lemma 4.1 For every $t > 0$, by the independence of X and Y and the law of total expectation,

$$\begin{aligned}
\overline{F}(x) = \mathsf{P}(X - Y > x) &= \int_{-\infty}^{+\infty} \mathsf{P}(X - Y > x | Y = y) \mathrm{d}\mathsf{P}(Y \le y) \\
&\ge \int_{|y - \mathsf{E}Y| \le t} \mathsf{P}(X - y > x) \mathrm{d}\mathsf{P}(Y \le y) \\
&\ge \mathsf{P}(X > x + \mu + t)\mathsf{P}(|Y - \mathsf{E}Y| \le t) \\
&\ge \mathsf{P}(X > x + \mu + t)\Big(1 - \frac{D(Y)}{t^2}\Big).
\end{aligned}$$

Letting $t = x/n$ for $x > 0$ and $n = 1, 2, \cdots$, by Remark 1 and Definition 2.2,

$$\begin{aligned}
\liminf_{x\to\infty} \frac{\overline{F}(x)}{\overline{F}_X(x)} &\ge \liminf_{x\to\infty} \Big\{\frac{\mathsf{P}(X > x + \mu + x/n)}{\mathsf{P}(X > x)}\Big(1 - \frac{n^2 D(Y)}{x^2}\Big)\Big\} \\
&\ge \liminf_{x\to\infty} \frac{\mathsf{P}(X > [(n+1)/n]x + \mu)}{\mathsf{P}(X > x)} \\
&\ge \Big(\frac{n+1}{n}\Big)^{-\beta}.
\end{aligned}$$

Letting $n \to \infty$ in the above inequality, then

$$\liminf_{x\to\infty} \frac{\overline{F}(x)}{\overline{F}_X(x)} \ge 1.$$

On the other hand,

$$\limsup_{x\to\infty} \frac{\overline{F}(x)}{\overline{F}_X(x)} \le \limsup_{x\to\infty} \frac{\mathsf{P}(X - Y > x)}{\mathsf{P}(X > x)} \le \limsup_{x\to\infty} \frac{\mathsf{P}(X > x)}{\mathsf{P}(X > x)} = 1.$$

Hence, $\overline{F}(x) \sim \overline{F}_X(x)$. Note that

$$\frac{\overline{F}(xy)}{\overline{F}(x)} = \frac{\overline{F}(xy)}{\overline{F}_X(xy)} \frac{\overline{F}_X(xy)}{\overline{F}_X(x)} \frac{\overline{F}_X(x)}{\overline{F}(x)}$$

for every $y \ge 1$, it is easy to known that $\overline{F} \in \mathrm{ERV}(-\alpha, -\beta)$. Lemma 4.1 is proved. □

Lemma 4.2 Let n be some positive integer, $F = F_1 * F_2 * \cdots * F_n$, where $\overline{F}_k$ is the d.f. of X_k and $\overline{F}_k \in \mathrm{ERV}(-\alpha, -\beta)$ for $1 < \alpha \le \beta < \infty$ for $k = 1, 2, \cdots, n$. Then $\overline{F} \in \mathrm{ERV}(-\alpha, -\beta)$ and

$$\overline{F}(x) \sim \overline{F}_1(x) + \overline{F}_2(x) + \cdots + \overline{F}_n(x). \tag{4.1}$$

Lemma 4.3 Let $F = F_1 * F_2$, where F_1 and F_2 are the d.f.'s of X_1 and X_2, respectively, which is concentrated on $(-\infty, \infty)$. If $\overline{F}_1 \in \mathrm{ERV}(-\alpha, -\beta)$ for $1 < \alpha \le \beta < \infty$ and $\overline{F}_2(x) \lesssim c\overline{F}_1(x)$ for some $c > 0$, then

$$\overline{F}(x) \lesssim (1+c)\overline{F}_1(x). \tag{4.2}$$

Proof of Theorem 3.1 Note that

$$\begin{aligned} U_n &= (U_{n-1}+Y_n)(1+r_n) - X_n \\ &= U_0(1+r_1)\cdots(1+r_n) + \sum_{k=1}^{n}(Y_k(1+r_k)\cdots(1+r_n)) - \sum_{k=1}^{n-1}(X_k(1+r_{k+1})\cdots(1+r_n)) - X_n, \end{aligned}$$

then

$$\begin{aligned} &\psi(x,N) \\ =\ & \mathsf{P}\Big(\min_{0\le n\le N}[(1+r_1)\cdots(1+r_n)]^{-1}U_n < 0\Big) \\ =\ & \mathsf{P}\Big(\max_{0\le n\le N}\Big\{\sum_{k=1}^{n} X_k[(1+r_1)\cdots(1+r_k)]^{-1} - \sum_{k=1}^{n} Y_k[(1+r_1)\cdots(1+r_{k-1})]^{-1}\Big\} > x\Big) \\ =\ & \mathsf{P}\Big(\max_{0\le n\le N}\sum_{k=1}^{n}[(1+r_1)\cdots(1+r_k)]^{-1}[X_k - Y_k(1+r_k)] > x\Big). \end{aligned}$$

For every $0 < k \le N$, Y_k has finite mean μ and finite variance $D(Y_1)$, r_k is a non-negative real number, then $-Y_k(1+r_k)$ also has finite mean and finite variance. Note that $\overline{F}_{X_k} \in \mathrm{ERV}(-\alpha,-\beta)$ for some $1 < \alpha \le \beta < \infty$, then by Lemma 4.1, $\overline{F}_k(x) \sim \overline{F}_{X_k}(x)$ and $\overline{F}_k(x) \in \mathrm{ERV}(-\alpha,-\beta)$, where $\overline{F}_k(x) = \overline{F}_{X_k} * \overline{F}_{-Y_k(1+r_k)}(x)$. By Lemmas 4.1 and 4.2 and Theorem Ng,

$$\begin{aligned} \psi(x,N) &= \mathsf{P}\Big(\max_{0\le n\le N}\sum_{k=1}^{n}[(+r_1)\cdots(1+r_k)]^{-1}[X_k - Y_k(1+r_k)] > x\Big) \\ &\sim \mathsf{P}\Big(\sum_{k=1}^{N}[(1+r_1)\cdots(1+r_k)]^{-1}[X_k - Y_k(1+r_k)] > x\Big) \\ &\sim \sum_{k=1}^{N}\mathsf{P}([(1+r_1)\cdots(1+r_k)]^{-1}[X_k - Y_k(1+r_k)] > x) \\ &= \sum_{k=1}^{N}\overline{F}_k((1+r_1)\cdots(1+r_k)X_k > x) \\ &\sim \sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)). \end{aligned}$$

This ends the proof of Theorem 3.1. □

Proof of Theorem 3.2 From the assumption that

$$\frac{r_1+\cdots+r_n}{n} \to r, \qquad \text{as} \qquad n\to\infty,$$

one can easily show that $\sum_{k=1}^{\infty}((1+r_2)\cdots(1+r_k))^{-\alpha}$, $\alpha>1$ converges as $n\to\infty$, and thus for every $\epsilon>0$ there exists a large enough integer $m=m(\epsilon)\geq 1$ such that

$$\sum_{k=m+1}^{\infty}((1+r_2)\cdots(1+r_k))^{-\alpha}<\epsilon. \tag{4.3}$$

Applying successively Theorem 3.1, we know that there exists some $M=M(\epsilon)>0$ such that for all $1\leq N\leq m$ and $x\geq M$,

$$(1-\epsilon)\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k))\leq\psi(x,N)\leq(1+\epsilon)\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)). \tag{4.4}$$

Now we consider $N>m$. By (4.9),

$$\psi(x,N)\geq\psi(x,m)\geq(1-\epsilon)\Big(\sum_{k=1}^{N}-\sum_{k=m+1}^{N}\Big)\overline{F}_X(x(1+r_1)\cdots(1+r_k)).$$

Since $\overline{F}_X\in\mathrm{ERV}(-\alpha,-\beta)$ for some $1<\alpha\leq\beta<\infty$, there exists positive number x_0 such that

$$\frac{\overline{F}_X(xy)}{\overline{F}_X(x)}\leq y^{-\alpha} \tag{4.5}$$

holds for all $xy\geq x\geq x_0$ (See ref.[6], Proposition 2.2.1). Then by (4.9) and (4.11), it is clear that

$$\begin{aligned}\sum_{k=m+1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)) &\leq \sum_{k=m+1}^{\infty}\overline{F}_X(x(1+r_1)\cdots(1+r_k))\\ &\leq \sum_{k=m+1}^{\infty}((1+r_2)\cdots(1+r_k))^{-\alpha}\overline{F}_X(x(1+r_1))\\ &\leq \epsilon\overline{F}_X(x(1+r_1))\\ &\leq \epsilon\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)).\end{aligned}$$

Hence it holds for all $N>m$ that

$$\begin{aligned}\psi(x,N) &\geq (1-\epsilon)\Big(\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k))-\epsilon\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k))\Big)\\ &= (1-\epsilon)^2\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)).\end{aligned}$$

Next we will derive the upper bound for $\psi(x,N)$. Clearly,

$$\begin{aligned}&\psi(x,N)\\ =\ &\mathsf{P}\Big(\max_{0\leq n\leq N}\sum_{k=1}^{n}[(1+r_1)\cdots(1+r_k)]^{-1}[X_k-Y_k(1+r_k)]>x\Big)\\ \leq\ &\mathsf{P}\Big(\max_{0\leq n\leq N}\sum_{k=1}^{n}[(1+r_1)\cdots(1+r_k)]^{-1}X_k>x\Big)\\ \leq\ &\mathsf{P}\Big(\max_{0\leq n\leq m}\sum_{k=1}^{n}[(1+r_1)\cdots(1+r_k)]^{-1}X_k+\sum_{k=m+1}^{N}[(1+r_1)\cdots(1+r_k)]^{-1}X_k>x\Big),\end{aligned}$$

From the assumption (3.2) we know that $\sum\limits_{k=1}^{\infty}((1+r_1)^{-1}(1+r_2)\cdots(1+r_k))^{-\alpha/2}$ converges as $n\to\infty$, and thus for every $\epsilon>0$ there exist $m=m(\epsilon)$ large enough such that

$$\sum_{k=m+1}^{\infty}((1+r_1)^{-1}(1+r_2)\cdots(1+r_k))^{-\alpha/2}<\epsilon. \tag{4.6}$$

Hence by (4.11) and (4.12),

$$\begin{aligned}
&\mathsf{P}\Big(\sum_{k=m+1}^{N}[(1+r_1)\cdots(1+r_k)]^{-1}X_k>x\Big)\\
\le\ &\mathsf{P}\Big(\sum_{k=m+1}^{\infty}[(1+r_1)\cdots(1+r_k)]^{-1}X_k>\sum_{k=m+1}^{\infty}((1+r_1)\cdots(1+r_k))^{-1/2}x\Big)\\
\le\ &\mathsf{P}\Big(\bigcup_{k=m+1}^{\infty}(X_k>(1+r_1)^{1/2}\cdots(1+r_k)^{1/2}x)\Big)\\
\le\ &\sum_{k=m+1}^{\infty}\mathsf{P}(X_k>(1+r_1)^{1/2}\cdots(1+r_k)^{1/2}x)\\
\le\ &\sum_{k=m+1}^{\infty}((1+r_1)^{-1}(1+r_2)\cdots(1+r_k))^{-\alpha/2}\mathsf{P}(X>(1+r_1)x)\\
\le\ &\epsilon\overline{F}_X(x(1+r_1))\\
\le\ &\epsilon\mathsf{P}\Big(\max_{0\le n\le m}\sum_{k=1}^{n}[(1+r_1)\cdots(1+r_k)]^{-1}X_k>x\Big).
\end{aligned}$$

Therefore, by Lemma 4.2 and the inequality (4.10) we obtain that

$$\begin{aligned}
\psi(x,N)\ &\le\ (1+\epsilon)\mathsf{P}\Big(\max_{0\le n\le m}\sum_{k=1}^{n}[(1+r_1)\cdots(1+r_k)]^{-1}X_k>x\Big)\\
&\le\ (1+\epsilon)^2\sum_{k=1}^{m}\overline{F}_X(x(1+r_1)\cdots(1+r_k))\\
&\le\ (1+\epsilon)^2\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)).
\end{aligned}$$

Consequently we get uniformly for $N>m$,

$$\begin{aligned}
&(1-\epsilon)^2\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k))\\
\lesssim\ &\psi(x,N)\lesssim(1+\epsilon)^2\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k)).
\end{aligned}\tag{4.7}$$

From (4.10) and (4.13), we get to know that the two-sided inequality

$$(1-\epsilon)^2\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k))\le\psi(x,N)\le(1+\epsilon)^2\sum_{k=1}^{N}\overline{F}_X(x(1+r_1)\cdots(1+r_k))$$

holds uniformly for all $N\ge1$. Hence, the result in (3.3) follows from the arbitrariness of $\epsilon>0$. This ends the proof of Theorem 3.2. $\square$

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变利率风险模型有限时间破产概率的渐近

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本文考虑离散时间风险模型$U_n = (U_{n-1} + Y_n)(1 + r_n) - X_n$, $n = 1, 2, \cdots$, 其中$U_0 = x > 0$为保险公司的初始准备金, r_n为在第n个时刻的利率, Y_n为到时刻n为止的总保费收入, X_n为到时刻n为止的所支付的全部索赔, U_n表示保险公司在时刻n的盈余. 当Y_n和r_n满足某些温和条件时, 我们得到了在$x \to \infty$时, 有限时间破产概率$\psi(x, N) = \mathsf{P}\big(\min\limits_{0\le n\le N} U_n < 0 | U_0 = x\big)$关于$N \geq 1$的一致渐近的关系式$\psi(x, N) \sim \sum\limits_{k=1}^{N} \overline{F}_X((1 + r_1)\cdots(1 + r_n)x)$, 其中$\overline{F}_X(x)$是$X_1$的尾分布.

关键词: 离散时间风险模型, 重尾, 利率, 有限时间破产概率, 渐近.

学科分类号: O211.4, O211.9.