

## 基于状态反馈的Markov切换随机时滞系统的 均方指数稳定性 \*

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### 摘 要

考察一类Markov切换时变时滞随机系统的均方指数稳定性. 利用基于Liapunov函数和线性矩阵不等式的方法, 给出了使状态反馈控制系统能克服不确定性和随机干扰, 在均方意义下达到指数稳定的充分条件. 当Markov链遍历所有模态时, 给出了一个独立于Markov链模态集的增益矩阵, 使得状态反馈控制系统均方指数稳定.

**关键词:** 随机系统, Markov切换, 时变时滞, 独立于模态集的状态反馈, 均方指数稳定.

**学科分类号:** O211.6.

### §1. 引 言

近年来, Markov切换随机时滞系统成为随机控制领域的研究热点问题之一<sup>[1-8]</sup>. 这类系统可以抽象为时间演化和事件驱动两类动态机制, 即由一系列基于Markov链有限模态集的随机系统通过模态的随机转移切换而成. 利用基于Liapunov函数和线性矩阵不等式的方法, Yue考察了一类Markov切换线性随机时滞系统的鲁棒稳定性问题<sup>[4]</sup>, Mao研究了一类Markov切换非线性时滞随机微分方程的稳定性<sup>[5]</sup>.

对于由Markov链切换而成的随机时滞系统, 其本身所固有的时滞效应, 参数的不确定性及双重的随机干扰源, 使其稳定性很难达到. 近年来的研究表明可以通过构造容许的反馈控制策略使得这类系统达到某种稳定性<sup>[1-3]</sup>. 基于状态反馈, Sathananthan等研究了一类Markov切换线性随机时滞系统的随机稳定性(stochastically stable)<sup>[1]</sup>. 基于状态观测器的输出反馈, Chen等也研究了一类Markov切换线性随机时滞系统的稳定性(stochastically stable)<sup>[2]</sup>. 基于状态反馈, Wang等研究了一类非线性随机时滞系统的均方指数稳定性(exponentially stable in mean square)<sup>[9]</sup>. 舒慧生等研究了时滞不确定性Markov切换线性随机微分系统的指数鲁棒稳定性<sup>[10]</sup>.

本文将进一步研究一类Markov切换状态反馈非线性随机时滞系统的均方指数稳定性, 并设计容许的独立于Markov链模态集的增益矩阵使相应的状态反馈控制系统达到均方指数稳定.

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## §2. 系统描述及假设

### 2.1 系统描述

设 $(\Omega, F, \{F_t\}_{t \geq 0}, P)$ 为一个带有自然流的完备概率空间,  $w(t)$ 是定义在 $(\Omega, F, \{F_t\}_{t \geq 0}, P)$ 上的一维标准布朗运动,  $\{\eta_t, t \geq 0\}$ 是定义在 $(\Omega, F, \{F_t\}_{t \geq 0}, P)$ 上取值于有限维状态空间 $S = \{1, 2, \dots, N\}$ 且右连续的Markov链, 其密度矩阵为 $\Gamma = \{\gamma_{ij}\} \in R^{n \times n}$ , ( $\gamma_{ij} \geq 0, i \neq j$ ,  $\sum_{j \in S} \gamma_{ij} = 0$ ), 即

$$P(\eta_{t+\delta} = j | \eta_t = i) = \begin{cases} \gamma_{ij}\delta + o(\delta), & i \neq j; \\ 1 + \gamma_{ii}\delta + o(\delta), & i = j, \end{cases}$$

这里 $\delta > 0$ ,  $|\cdot|$ 为Euclidean范数.  $\delta_k(t) : R_+ \rightarrow [0, \tau]$ ,  $k = 1, \dots, n$ ,  $\tau = \max_k \sup_t \delta_k(t) \geq 0$ ,  $\delta_k(t) \geq 0$ , 且 $\delta'_k(t) \leq \xi_k < 1$  ( $k = 1, \dots, n$ ), 并记 $\xi = \min(\xi_k)$ . 随机过程 $\varphi$ 可看作一个二元连续函数 $\varphi(\theta, \omega) : [-\tau, 0] \times \Omega \rightarrow R^n$ , 且具有范数 $\|\varphi\|^2 = \sup_{-\tau \leq t \leq 0} E|\varphi(t)|^2 < \infty$ . 并假定 $\eta_t, w(t), \varphi(t)$ 相互独立.

考察随机系统

$$\begin{aligned} dx(t) &= \{[A + \Delta A(t, \eta_t)]x(t) + B(\eta_t)f(x(t)) + C(\eta_t)g(z(t)) + Du(t)\}dt \\ &\quad + h(x(t), g(z(t)), \eta_t)dw(t), \end{aligned} \quad (2.1)$$

$$x(t) = \varphi(t), \quad \eta_t = \eta_0, \quad t \in [-\tau, 0]. \quad (2.2)$$

其中, 状态 $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in R^{n \times 1}$ , 输入 $u(t) = (u_1(t), u_2(t), \dots, u_m(t))^T \in R^{m \times 1}$ , 时滞状态 $z(t) = (x_1(t - \delta_1(t)), x_2(t - \delta_2(t)), \dots, x_n(t - \delta_n(t)))^T$ ,

$$f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T \in R^{n \times 1},$$

$$g(z(t)) = (g_1(x_1(t - \delta_1(t))), g_2(x_1(t - \delta_2(t))), \dots, g_n(x_n(t - \delta_n(t))))^T \in R^{n \times 1},$$

$$h(x(t), g(z(t)), \eta_t) \in R^{n \times 1}.$$

系数矩阵 $A, \Delta A(t, \eta_t), B(\eta_t), C(\eta_t), D \in R^{n \times n}$ , 任取 $\eta_t = i \in S$ .

随机系统(2.1)转化为

$$\begin{aligned} dx(t) &= \{[A + \Delta A_i(t)]x(t) + B_i f(x(t)) + C_i g(z(t)) + Du(t)\}dt \\ &\quad + h(x(t), g(z(t)), i)dw(t). \end{aligned} \quad (2.3)$$

记 $\Delta A_i(t) = \Delta A(t, \eta_t = i)$ , 满足 $\Delta A_i(t) = M_i H_i(t) N_i$ ,  $H_i^T(t) H_i(t) \leq I$ .

**定义 2.1** 如果存在常数 $c_i > 0$ 及 $\alpha_i > 0$ , 使得对任意 $t \geq 0$ , 都有

$$E|x(t)|^2 \leq c_i e^{-\alpha_i t} \|\varphi\|^2,$$

称随机系统(2.3)是均方指数稳定的.

## 2.2 假设

1)  $f(x(t)), g(z(t))$  的各分量  $f_k(\cdot), g_k(\cdot)$  满足  $f_k(0) = 0, g_k(0) = 0$ , 且存在非负实数族  $\{\mu_k, \beta_k, 1 \leq k \leq n\}$ , 使得对任意  $\xi_1, \xi_2 \in R$ , 都有

$$|f_k(\xi_1) - f_k(\xi_2)| \leq \mu_k |\xi_1 - \xi_2|, \quad |g_k(\xi_1) - g_k(\xi_2)| \leq \beta_k |\xi_1 - \xi_2|.$$

2) 对于  $h(x(t), g(z(t)), i)$ , 存在非负实数族  $\{\rho_{i1}, \rho_{i2}, i \in S\}$ , 使得

$$|h(x(t), g(z(t)), i)|^2 \leq \rho_{i1}^2 |x(t)|^2 + \rho_{i2}^2 |g(z(t))|^2.$$

记  $\beta = \max_k (\beta_k)$ ,

$$F = \text{diag}\{\beta_1^2, \beta_2^2, \dots, \beta_n^2\}, \quad J = \text{diag}\left\{\frac{\beta_1^2}{1 - \xi_1}, \frac{\beta_2^2}{1 - \xi_2}, \dots, \frac{\beta_n^2}{1 - \xi_n}\right\}.$$

## §3. 主要结果

对于随机系统(2.3), 引入状态反馈

$$u(t) = -K_i x(t). \quad (3.1)$$

将(3.1)代入(2.3)式, 得

$$\begin{aligned} dx(t) = & \{[A + \Delta A_i(t) - DK_i]x(t) + B_i f(x(t)) + C_i g(z(t))\}dt \\ & + h(x(t), g(z(t)), i)dw(t), \end{aligned} \quad (3.2)$$

记

$$F(x(t), g(z(t)), i) = [A - DK_i + \Delta A_i(t)]x(t) + B_i f(x(t)) + C_i g(z(t)), \quad (3.3)$$

则(3.2)式可简写为

$$dx(t) = F(x(t), g(z(t)), i)dt + h(x(t), g(z(t)), i)dw(t). \quad (3.4)$$

**引理 3.1** (Schur) 给定相应维数的矩阵  $\Omega_1, \Omega_2, \Omega_3$ , 其中  $\Omega_1 = \Omega_1^T, \Omega_2 > 0$ , 则  $\Omega_1 + \Omega_3^T \Omega_1^{-1} \Omega_3 < 0$  当且仅当  $\begin{pmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{pmatrix} < 0$ .

**引理 3.2** [8] 给定相应维数的矩阵  $X, Y$ , 则对任意  $\mu > 0$ , 都有

$$X^T Y + Y^T X \leq \mu X^T X + \mu^{-1} Y^T Y.$$

**引理 3.3** [9] 给定相应维数的矩阵  $M, N, H(t)$ , 其中  $M, N$  为常阵,  $H(t)^T H(t) \leq I$ , 则对任意  $\mu > 0$ , 都有

$$MH(t)N + N^T H(t)^T M^T \leq \mu MM^T + \mu^{-1} N^T N.$$

**定理 3.1** 对于状态反馈控制系统(2.3), 给定增益矩阵 $\{K_i, i \in S\}$ , 若存在正定矩阵族 $\{P_i, i \in S\}$ , 及正实数组 $\{\varepsilon_i, \theta_i, v_i, i \in S\}$ , 使得

$$\begin{pmatrix} \Pi_i & \Gamma_i^T \\ \Gamma_i & -I_n \end{pmatrix} < 0, \quad \rho_{i2}^2 \sigma_{in} + v_i^{-1} \geq 1,$$

则状态反馈控制系统(2.3)均方指数稳定, 其中

$$\begin{aligned} \Pi_i &= P_i[A - DK_i] + [A - DK_i]^T P_i + \varepsilon_i^{-1} N_i^T N_i + \theta_i^{-1} F + \rho_{i1}^2 \sigma_{in} I_n + \sum_{j \in S} \gamma_{ij} P_j, \\ \Gamma_i &= \left( \varepsilon_i^{1/2} P_i M_i \quad \theta_i^{1/2} P_i B_i \quad v_i^{1/2} P_i C_i \right)^T, \quad \sigma_{in} = \lambda_{\max}(P_i), \quad \sigma_{i1} = \lambda_{\min}(P_i). \end{aligned}$$

**证明:** 构造Liapunov函数

$$V(t, \eta_t, x(t))|_{\eta_t=i} = V_1(t, i, x(t)) + V_2(t, x(t)), \quad (3.5)$$

$$V_1(t, i, x(t)) = x(t)^T P_i x(t), \quad (3.6)$$

$$V_2(t, x(t)) = \sum_{k=1}^n \int_{t-\delta_k(t)}^t g_k^2(x_k(s)) ds, \quad (3.7)$$

依广义Itô公式, 并结合(3.4)式, 有

$$\begin{aligned} LV_1(t, i, x(t)) &= x(t)^T P_i F(x(t), g(z(t)), i) + F(x(t), g(z(t)), i)^T P_i x(t) \\ &\quad + h(x(t), g(z(t)), i)^T P_i h(x(t), g(z(t)), i) + \sum_{j \in S} \gamma_{ij} V_1(t, j, x(t)), \end{aligned} \quad (3.8)$$

$$LV_2(t, x(t)) = \sum_{k=1}^n g_k^2(x_k(t)) - \sum_{k=1}^n g_k^2(x_k(t - \delta_k(t))), \quad (3.9)$$

$$LV(t, i, x(t)) = LV_1(t, i, x(t)) + LV_2(t, x(t)). \quad (3.10)$$

令

$$\begin{aligned} \Xi_i &= P_i[A - DK_i] + [A - DK_i]^T P_i + \varepsilon_i P_i M_i M_i^T P_i + \varepsilon_i^{-1} N_i^T N_i \\ &\quad + \theta_i P_i B_i B_i^T P_i + \theta_i^{-1} F + v_i P_i C_i C_i^T P_i + \rho_{i1}^2 \sigma_{in} I_n + \sum_{j \in S} \gamma_{ij} P_j, \end{aligned}$$

由引理3.1及定理条件, 得

$$\Xi_i < 0. \quad (3.11)$$

记 $\lambda_i = \lambda_{\min}(-\Xi_i)$ , 令 $W(t, i, x(t)) = e^{\alpha_i t} V(t, i, x(t))$ , 其中,  $\alpha_i$ 满足

$$\max_{1 \leq k \leq n} \left\{ \alpha_i \sigma_{in} + (\alpha_i \tau e^{\alpha_i \tau} + 1) \beta_k^2 + (\rho_{i2}^2 \sigma_{in} + v_i^{-1} - 1) e^{\alpha_i \tau} \frac{\beta_k^2}{1 - \xi_k} \right\} = \lambda_i. \quad (3.12)$$

则

$$LW(t, i, x(t)) = e^{\alpha_i t} [\alpha_i V(t, i, x(t)) + LV(t, i, x(t))], \quad (3.13)$$

且对任意  $T \geq 0$ , 都有

$$\begin{aligned} \mathbb{E}\{W(t, i, x(t))|_0^T\} &= \mathbb{E}\left\{\int_0^T LW(t, i, x(t))dt\right\} \\ &= \mathbb{E}\left\{\int_0^T e^{\alpha_i t} [\alpha_i V(t, i, x(t)) + LV(t, i, x(t))]\right\}, \end{aligned} \quad (3.14)$$

$$\begin{aligned} \mathbb{E}\{W(0, i, x(0))\} &= \mathbb{E}\{V(0, i, x(0))\} \\ &= \mathbb{E}\left\{x(0)^T P_i x(0) + \sum_{k=1}^n \int_{-\delta_k(0)}^0 g_k^2(x_k(s))ds\right\} \leq c_{i1} \|\varphi\|^2, \end{aligned} \quad (3.15)$$

其中,  $c_{i1} = \sigma_{in} + \tau\beta^2$ .

由(3.5)–(3.12)式, 得

$$\begin{aligned} &\alpha_i V(t, i, x(t)) + LV(t, i, x(t)) \\ &= \alpha_i x(t)^T P_i x(t) + \alpha_i \sum_{k=1}^n \int_{t-\delta_k(t)}^t g_k^2(x_k(s))ds \\ &\quad + x(t)^T P_i [A - DK_i + \Delta A_i(t)]x(t) + x(t)^T [A - DK_i + \Delta A_i(t)]^T P_i x(t) \\ &\quad + x(t)^T P_i B_i f(x(t)) + f(x(t))^T B_i^T P_i x(t) + x(t)^T P_i C_i g(z(t)) \\ &\quad + g(z(t))^T C_i^T P_i x(t) + h(x(t), g(z(t)), i)^T P_i h(x(t), g(z(t)), i) \\ &\quad + \sum_{j \in S} x(t)^T P_j x(t) + \sum_{k=1}^n g_k^2(x_k(t)) - \sum_{k=1}^n g_k^2(x_k(t - \delta_k(t))). \end{aligned} \quad (3.16)$$

由引理3.3, 得

$$P_i \Delta A_i(t) + \Delta A_i(t)^T P_i \leq \varepsilon_i P_i M_i M_i^T P_i + \varepsilon_i^{-1} N_i^T N_i. \quad (3.17)$$

由假设1), 2)及引理3.2, 得

$$\begin{aligned} &x(t)^T P_i B_i f(x(t)) + f(x(t))^T B_i^T P_i x(t) \\ &\leq \theta_i x(t)^T P_i B_i B_i^T P_i x(t) + \theta_i^{-1} f(x(t))^T f(x(t)) \\ &\leq x(t)^T [\theta_i P_i B_i B_i^T P_i + \theta_i^{-1} F]x(t), \end{aligned} \quad (3.18)$$

$$\begin{aligned} &x(t)^T P_i C_i g(z(t)) + g(z(t))^T C_i^T P_i x(t) \\ &\leq v_i x(t)^T P_i C_i C_i^T P_i x(t) + v_i^{-1} g(z(t))^T g(z(t)), \end{aligned} \quad (3.19)$$

$$\begin{aligned} &h(x(t), g(z(t)), i)^T P_i h(x(t), g(z(t)), i) \\ &\leq \sigma_{in} |h(x(t), g(z(t)), i)|^2 \\ &\leq x(t)^T [\rho_{i1}^2 \sigma_{in} I_n]x(t) + g(z(t))^T [\rho_{i2}^2 \sigma_{in} I_n]g(z(t)). \end{aligned} \quad (3.20)$$

记

$$\begin{aligned} \Theta_i &= \alpha_i P_i + P_i [A - DK_i] + [A - DK_i]^T P_i + \varepsilon_i P_i M_i M_i^T P_i + \varepsilon_i^{-1} N_i^T N_i \\ &\quad + \theta_i P_i B_i B_i^T P_i + \theta_i^{-1} F + v_i P_i C_i C_i^T P_i + \rho_{i1}^2 \sigma_{in} I_n + \sum_{j \in S} \gamma_{ij} P_j, \end{aligned}$$

则由(3.16)–(3.20)式, 得

$$\begin{aligned} & \alpha_i V(t, i, x(t)) + LV(t, i, x(t)) \\ & \leq x(t)^T \Theta_i x(t) + \alpha_i \sum_{k=1}^n \int_{t-\delta_k(t)}^t g_k^2(x_k(s)) ds + \sum_{k=1}^n g_k^2(x_k(t)) \\ & \quad + (\rho_{i2}^2 \sigma_{in} + v_i^{-1} - 1) \sum_{k=1}^n g_k^2(x_k(t - \delta_k(t))). \end{aligned} \quad (3.21)$$

结合(3.14), (3.21)式, 有

$$\begin{aligned} & \mathbb{E} \left\{ \int_0^T LW(t, i, x(t)) dt \right\} \\ & \leq \mathbb{E} \left\{ \int_0^T e^{\alpha_i t} \left[ x(t)^T \Theta_i x(t) + \alpha_i \sum_{k=1}^n \int_{t-\delta_k(t)}^t g_k^2(x_k(s)) ds \right. \right. \\ & \quad \left. \left. + \sum_{k=1}^n g_k^2(x_k(t)) + (\rho_{i2}^2 \sigma_{in} + v_i^{-1} - 1) \sum_{k=1}^n g_k^2(x_k(t - \delta_k(t))) \right] dt \right\}. \end{aligned} \quad (3.22)$$

由假设1)得

$$\begin{aligned} & \int_0^T \left[ \alpha_i \sum_{k=1}^n \int_{t-\delta_k(t)}^t g_k^2(x_k(s)) ds \right] dt \\ & = \alpha_i \sum_{k=1}^n \int_{-\delta_k(0)}^T \left[ \int_{s \vee 0}^{s+\delta_k(\cdot) \wedge T} e^{\alpha_i t} dt \right] g_k^2(x_k(s)) ds \\ & \leq \alpha_i \sum_{k=1}^n \int_{-\delta_k(0)}^T [\delta_k(\cdot) e^{\alpha_i \delta_k(\cdot)} e^{\alpha_i s} g_k^2(x_k(s))] ds \\ & \leq \alpha_i \tau e^{\alpha_i \tau} \sum_{k=1}^n \int_{-\tau}^T [e^{\alpha_i s} g_k^2(x_k(s))] ds \leq \sum_{k=1}^n \int_{-\tau}^T e^{\alpha_i s} x(s)^T [(\alpha_i \tau e^{\alpha_i \tau}) G] x(s) ds \\ & = \sum_{k=1}^n \int_{-\tau}^0 e^{\alpha_i s} x(s)^T [(\alpha_i \tau e^{\alpha_i \tau}) G] x(s) ds + \sum_{k=1}^n \int_0^T e^{\alpha_i s} x(s)^T [(\alpha_i \tau e^{\alpha_i \tau}) G] x(s) ds. \end{aligned} \quad (3.23)$$

记  $y_k(t) = t - \delta_k(t)$ , 有  $y'_k(t) \geq 1 - \xi_k > 0$ , 于是

$$\begin{aligned} \int_0^T e^{\alpha_i t} \left[ \sum_{k=1}^n g_k^2(x_k(t - \delta_k(t))) \right] dt & = \sum_{k=1}^n \int_0^T [e^{\alpha_i t} g_k^2(x_k(t - \delta_k(t)))] dt \\ & \leq \sum_{k=1}^n \int_{-\delta_k(0)}^{T-\delta_k(T)} \left[ e^{\alpha_i(s+\delta_k(\cdot))} g_k^2(x_k(s)) \frac{1}{1-\xi_k} \right] ds \\ & \leq \sum_{k=1}^n \int_{-\tau}^T \left[ e^{\alpha_i(s+\delta_k(\cdot))} \beta_k^2 x_k^2(s) \frac{1}{1-\xi_k} \right] ds \\ & = \int_{-\tau}^T e^{\alpha_i s} x(s)^T [e^{\alpha_i \tau J}] x(s) ds \\ & = \int_{-\tau}^0 e^{\alpha_i s} x(s)^T [e^{\alpha_i \tau J}] x(s) ds \\ & = \int_0^T e^{\alpha_i s} x(s)^T [e^{\alpha_i \tau J}] x(s) ds, \end{aligned} \quad (3.24)$$

$$\begin{aligned} \int_0^T e^{\alpha_i t} \left[ \sum_{k=1}^n g_k^2(x_k(t)) \right] dt &= \sum_{k=1}^n \int_0^T [e^{\alpha_i t} g_k^2(x_k(t))] dt \\ &\leq \int_0^T e^{\alpha_i t} x(t)^T G x(t) dt. \end{aligned} \quad (3.25)$$

记  $\Omega_i = \Theta_i + (\alpha_i \tau e^{\alpha_i \tau})G + (\rho_{i2}^2 \sigma_{in} + v_i^{-1} - 1)e^{\alpha_i \tau} J$ , 结合(3.11), (3.12)式, 有

$$\Omega_i \leq 0. \quad (3.26)$$

由(3.22)–(3.26)式, 得

$$\mathbb{E} \left\{ \int_0^T LW(t, i, x(t)) dt \right\} \leq c_{i2} \|\varphi\|^2 + \mathbb{E} \left\{ \int_0^T x(t)^T \Omega_i x(t) dt \right\} \leq c_{i2} \|\varphi\|^2, \quad (3.27)$$

其中,

$$c_{i2} = (1 - e^{-\alpha_i \tau}) \left[ \alpha_i \tau e^{\alpha_i \tau} \beta^2 + (\rho_{i2}^2 \sigma_{in} + v_i^{-1} - 1)e^{\alpha_i \tau} \frac{\beta^2}{\alpha_i(1 - \xi)} \right].$$

结合(3.14), (3.15)及(3.27)式, 有

$$\begin{aligned} \sigma_{i1} e^{\alpha_i T} \mathbb{E}\{|x(T)|^2\} &\leq \mathbb{E}\{W(T, i, x(T))\} \\ &= \mathbb{E}\left\{W(0, i, x(0)) + \mathbb{E}\left\{\int_0^T LW(t, i, x(t)) dt\right\}\right\} \\ &\leq (c_{i1} + c_{i2}) \|\varphi\|^2, \end{aligned} \quad (3.28)$$

即证  $\mathbb{E}\{|x(T)|^2\} \leq c_i e^{-\alpha_i T} \|\varphi\|^2$ , 其中,  $c_i = (c_{i1} + c_{i2})/\sigma_{i1}$ .  $\square$

为了设计能够使状态反馈控制系统(2.3)满足均方指数稳定性的容许增益矩阵  $\{K_i, i \in S\}$ , 下面给出定理3.2, 并给出一个独立于Markov链模态集的增益矩阵  $K$ , 使得状态反馈控制系统(2.3)均方指数稳定.

**定理 3.2** 对于随机系统(2.3), 若存在正定矩阵族  $\{P_i, i \in S\}$ , 及正实数组  $\{\rho_i, \varepsilon_i, \theta_i, v_i, i \in S\}$ , 使得

$$\begin{pmatrix} \Lambda_i & \Gamma_i^T \\ \Gamma_i & -I_n \end{pmatrix} < 0, \quad \rho_{i2}^2 \sigma_{in} + v_i^{-1} \geq 1, \quad K_i = \rho_i D^T P_i,$$

则状态反馈控制系统(2.3)是均方指数稳定的, 其中

$$\Lambda_i = P_i A + A^T P_i - 2\rho_i P_i D D^T P_i + \varepsilon_i^{-1} N_i^T N_i + \theta_i^{-1} F + \rho_{i1}^2 \sigma_{in} I_n + \sum_{j \in S} \gamma_{ij} P_j,$$

$$\Gamma_i = \begin{pmatrix} \varepsilon_i^{1/2} P_i M_i & \theta_i^{1/2} P_i B_i & v_i^{1/2} P_i C_i \end{pmatrix}^T.$$

证明: 令

$$\begin{aligned} \Psi_i &= P_i A + A^T P_i - 2\rho_i P_i D D^T P_i + \varepsilon_i P_i M_i M_i^T P_i + \varepsilon_i^{-1} N_i^T N_i \\ &\quad + \theta_i P_i B_i B_i^T P_i + \theta_i^{-1} F + v_i P_i C_i C_i^T P_i + \rho_{i1}^2 \sigma_{in} I_n + \sum_{j \in S} \gamma_{ij} P_j, \end{aligned}$$

由引理3.1及定理条件, 得

$$\Psi_i < 0. \quad (3.29)$$

取增益矩阵  $K_i = \rho_i D^T P_i$ , 得  $\Xi_i = \Psi_i < 0$ , 故依定理3.1的证明过程, 状态反馈控制系统(2.3)均方指数稳定.  $\square$

**推论 3.1** 对于随机系统(2.1), 若存在正定矩阵族  $\{P_i, i \in S\}$ , 及正实数组  $\{\rho_i, \varepsilon_i, \theta_i, v_i, i \in S\}$ , 使得

$$\begin{pmatrix} \Lambda_i & \Gamma_i^T \\ \Gamma_i & -I_n \end{pmatrix} < 0, \quad \rho_{i2}^2 \sigma_{in} + v_i^{-1} \geq 1, \quad K = \rho \sigma D^T,$$

则状态反馈控制系统(2.1)均方指数稳定, 其中  $\rho = \max_{i \in S}(\rho_i)$ ,  $\sigma = \max_{i \in S}\{\lambda_{\max}(P_i)\}$ .

## §4. 数值例子

下面基于定理3.2, 给出状态反馈控制系统(2.1)的一个二维的数值例子. 取Markov链的状态空间  $S = \{1, 2\}$ , 密度矩阵

$$\Gamma = \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix},$$

各系数矩阵及参数如下

$$\begin{aligned} A &= \begin{pmatrix} -2 & 1 \\ 1 & -4 \end{pmatrix}, & D &= \begin{pmatrix} 0.8 & 0.5 \\ -0.3 & 0.4 \end{pmatrix}, \\ M_1 &= \begin{pmatrix} 0.3 & 0 \\ 0 & 0.3 \end{pmatrix}, & N_1 &= \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}, \\ M_2 &= N_2 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \\ B_1 &= \begin{pmatrix} 0.8 & 0.7 \\ -0.5 & -1 \end{pmatrix}, & B_2 &= \begin{pmatrix} 1.5 & 0.8 \\ -0.7 & 1 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} -0.1 & -0.5 \\ 0.2 & 0.6 \end{pmatrix}, & C_2 &= \begin{pmatrix} -0.7 & -1 \\ 0.3 & 1 \end{pmatrix}, \\ \xi_1 &= \xi_2 = 0, & \mu_1 &= 0.5, & \mu_2 &= 0.3, \\ \beta_1 &= 0.2, & \beta_2 &= 0.1, & \rho_{11} &= 0.15, & \rho_{21} &= 0.1, \\ \rho_{12} &= \rho_{22} = 0.1, & \tau &= 0.2. \end{aligned}$$

取  $\varepsilon_1 = 0.2$ ,  $\varepsilon_2 = 0.1$ ,  $\theta_1 = 0.25$ ,  $\theta_2 = 0.2$ ,  $v_1 = 1$ ,  $v_2 = 0.5$ ,  $\rho_1 = 10$ ,  $\rho_2 = 5$ ,

$$P_1 = \begin{pmatrix} 1.9026 & 0.3460 \\ 0.3460 & 1.5344 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 1.9606 & 0.3893 \\ 0.3893 & 1.5936 \end{pmatrix},$$



$$K_1 = \begin{pmatrix} 14.1825 & -1.3850 \\ 10.8969 & 7.8677 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 7.2583 & -0.8333 \\ 5.6800 & 4.1603 \end{pmatrix},$$

$$K = \begin{pmatrix} 17.6594 & -6.6223 \\ 11.0371 & 8.8297 \end{pmatrix},$$

$$\alpha_1 = 6.0075, \quad \alpha_2 = 3.5432, \quad c_1 = 1.6125, \quad c_2 = 1.6825.$$

由数值结果知: 对任意  $\eta_t \in S = \{1, 2\}$ , 若取独立于Markov链模态集的增益矩阵  $K$ , 则状态反馈控制系统(2.1)均方指数稳定.

## §5. 结 论

本文研究了一类Markov切换时滞状态反馈控制系统的均方指数稳定性. 利用基于Liapunov函数和线性矩阵不等式的方法, 证明了这类状态反馈控制系统是能够达到均方指数稳定的, 同时给出了容许的增益矩阵使相应的状态反馈控制系统满足均方指数稳定性. 并设计出了一个独立于Markov链模态集的增益矩阵, 使得状态反馈控制系统均方指数稳定, 即当Markov链遍历所有模态时, 通过这一反馈控制策略, 状态反馈控制系统都是均方指数稳定的. 数值例子表明, 本方法是可行的.

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## Exponential Stabilization in Mean Square of Stochastic Systems with Delay and Markovian Switching via State-Feedback

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Exponential stabilization in mean square is investigated for a class of stochastic systems, where Markovian switching and time-varying delay is introduced. In order to guarantee the exponential stability in mean square for the system, a sufficient condition is derived using the method of Liapunov function and LMI. Furthermore, a kind of desirable gain matrix is designed to make the system the exponential stability in mean square by the state feedback. Finally, when the Markov chain goes around all its modes, a mode-independent gain matrix is presented to guarantee the exponential stability in mean square for this system.

**Keywords:** Stochastic system, Markovian switching, time-varying delay, mode-independent state-feedback, exponential stability in mean square.

**AMS Subject Classification:** 93E15.