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Asymptotically Optimal Investment for Risk Model with Random Income and Diffusions *

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Abstract

In this paper, an insurer with perturbed classical risk process and random premium income has the possibility of investment into a risky market. The price process of the risky market is assumed to follow a geometric Brownian motion. The aim of this paper is to obtain the asymptotical behavior of the ruin probability under the optimal strategy in the small claims. The constant (denoted by A^*) maximizing the Lundburg exponent is derived. It turns out that the optimal investment level convergence to A^* when the initial surplus tends to infinity. That is to say, the constant we found is the asymptotically optimal strategy.

Keywords: Lundburg inequality, optimal investment, ruin probability, Lundburg exponent.

AMS Subject Classification: 60K05, 62P05, 90A46.

§1. Introduction

In recent years there has bee an increasing attention in the utilization of stochastic control theory to the problems related to insurance affairs. See for example, Browne[2], Hipp and Plum[10][11], Schmidli[18], Yang and Zhang[19] and references therein. This due to the facts that insurance companies can invest in the stock market, can pay dividend to maximize a certain objective function and can purchase reinsurance et al.. The surplus process is assumed to be a compound Poisson process or a perturbed compound Poisson process or a Brownian motion with drift, where the variables, such as, reinsurance, new business, investment and dividend are adjusted dynamically.

The estimation of ruin probabilities has been a central topic in risk theory. It is known that if the claim sizes have exponential moments (i.e. so-called small claim case), the ruin probability decrease exponentially with the initial surplus, see, for instance, Asmussen[1]. However, when there is a stochastic return on investment, the situation may be different. Kalashnikov and Norberg[13] have investigated the problem under the additional

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assumption that all the surplus is invested in the risky market, likewise did by Paulsen and Gjessing[14], Frovola et al.[15]. In all of these cases it was shown that, even if the claim sizes are small claim size, the ruin probability decreases only with some negative power of the initial surplus. Thus, for large capital, investing more than the surplus into the risky market can not be optimal. Then, one interesting problem is: if an insurer has the opportunity to invest in a risky market, what is the minimal ruin probability she can obtain? In particularly, can she do better than keeping the funds in the bond? And if yes, how much can she do better?

Browne[2] considered this problem under the assumptions that the aggregate claims are modeled by a Brownian motion with drift, and the risky asset is modeled by a geometric Brownian motion and the corresponding ruin probability there is given by an exponential function. The compound Poisson process is one of the most popular models in risk theory, Hipp and Taksar[12] used the compound Poisson process to model the insurance business and considered the problem of optimal choice of new business to minimize the ultimate ruin probability. In the case of exponential distributed claim-size, explicit solutions can be obtained. However, in most cases, it is not so easy to obtain the explicit solution of optimal strategy. Consequently, there are some papers fall back on finding an asymptotically optimal strategy, see Gaier et al.[6] and Hipp and Schmidli[9].

In classical risk model and its some generalizations, the total premium income up to time t, denote by Π_t, is usually a linear function of time t. It is not so realistic in fact, thus, a natural generalization on this aspect is to assume that the premium income process $\Pi = \{\Pi_t, t \geq 0\}$ has more complex structure than the one in classical risk models. In this paper, we suppose that the premium income process can be stochastic and the logarithm of the asset price process of stock market be a drifted Brownian motion. It is assumed that the premiums and the claims have exponential moments. We investigate whether there are constants R and constant C such that the minimum ruin probability $\Psi(x)$ (i.e. the so-called value function in control theory), obtained by starting form an initial surplus x, satisfies

$$\Psi(x) \leq C\mathrm{e}^{-Rx}. \tag{1.1}$$

Of course, there always is the possibility not to invest at all, resulting in an exponential bound for the ruin probability $\Psi(x)$, which is the so-called Lundburg upper bound for classical risk model with stochastic premium income, under the assumption of a safety loading, see Melnikov[16]. In this paper, we want to find the optimal (i.e. the largest) coefficient R such that (1.1) holds, where $\Psi(x)$ is the minimum ruin probability under optimal strategy. It turn out that R is determined by an equation similar to the one that determines the Lundburg adjustment coefficient in classical risk model. The trading strategy that correspond to this optimal R is to invest constant amount of surplus in the risky market, independent of the current level of the surplus. We will later show that this constant strategy is asymptotically optimal, respectively asymptotically unique, in the sense that every "asymptotically different" Markovian strategy yields an "exponentially worse" decay of the ruin probability.

This paper is organized as follows. In Section 2, an introduction to our problem are presented and in Section 3, the expression for the asymptotically optimal strategy and the bounds for the value function,say by $\Psi(x)$, are obtained. In Section 4, we prove that that the optimal strategy convergence to the constant strategy as the initial surplus tends to infinity.

§2. Model and Assumptions

Let $(\Omega, \mathcal{F}, \mathsf{P})$ be a complete probability space containing all the variables defined in this paper and a risk process with stochastic premium income is given by

$$S_t = u + pt + \Pi_t - X_t, \tag{2.1}$$

where S_t is the surplus of an insurer at time t, p is the fixed premium income rate and p is a positive constant, Π_t is the extra stochastic premium income up to time t, so $pt + \Pi_t$ is the total premium income up to time t. X_t is the total claims up to time t. In this paper, it is assumed that

$$\Pi_t = \sum_{i=1}^{N_S^1(t)} P_i, \qquad X_t = \sum_{i=1}^{N_S^2(t)} C_i,$$

where $N_S^1(t)$, $N_S^2(t)$ are two Poisson processes with parameters λ_1, λ_2. $\{P_i, i \geq 1\}$, $\{C_i, i \geq 1\}$ are two sequences of i.i.d. positive random variables. Let $F_P(x)$ and $F_C(x)$ denote the common distribution functions of $\{P_i, i \geq 1\}$ and $\{C_i, i \geq 1\}$, and similarly, denote the moment generating function of them by $M_P(r) := \mathsf{E}e^{rP}$ and $M_C(r) := \mathsf{E}e^{rC}$. For simplicity, $\{\Pi_t, t \geq 0\}$ and $\{X_t, t \geq 0\}$ are assumed to be independent. One maybe think that the independent assumption of N_S^1 and N_S^2 is quite bold. In fact, there are also some papers considering the dependent structure of N_S^1 and N_S^2 by term of "common shock", see Cossette and Marceau[3] for instance. By some modification, our model can also be used for coping with such correlated risk model, for exposition ease, we just assume that the independent structure hold here.

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If we take into account the competition of insurance market and assume that the number of insurance company is large enough and each company has only limit influence on the insurance market, then it is natural to use Gaussian diffusion for modeling the capital of insurance company, so the surplus of insurance company can be revived as

$$S_t = u + pt + \Pi_t - X_t + \sigma W_t, \tag{2.2}$$

where $\{W_t, t \geq 0\}$ is a standard Brownian Motion and is independent with $\{\Pi_t, t \geq 0\}$ and $\{X_t, t \geq 0\}$, σ is a positive constant. The diffusion term σW_t represents an additional uncertainty of aggregate claims. An alternative interpretation is that it adds an uncertainty to the premium income, see Dufresne and Gerber[4]. In this paper, we choose the latter interpretation. Hereafter, if we refer to S_t, it is the one has the form of (2.2). In this paper, we do not assume the safety loading hold, i.e. we do not assume that $\mathsf{E}[pt + \Pi_t - X_t] > 0$, i.e. $p + \lambda_1 \mathsf{E} P_1 - \lambda_2 \mathsf{E} C_1 > 0$ and reason we do not need to make such assumption can be found later.

We assume that the standard assumptions of continuous-time financial models hold, that is

1. continuous trading is permitted;
2. no transaction cost or tax is involved in trading;
3. all assets are infinitely divisible.

The price of the risky asset is assumed to follow the stochastic differential equation

$$\frac{\mathrm{d}P(t)}{P(t)} = \mathrm{d}Z_t = \alpha \mathrm{d}t + \beta \mathrm{d}B_t, \qquad \alpha > 0,\ \beta > 0,\ t \geq 0. \tag{2.3}$$

Here $\{B_t, t \geq 0\}$ is a standard Brownian Motion, Process $S = \{S_t, t \geq 0\}$ and Process $Z = \{Z_t, t \geq 0\}$ are assumed to be independent. Denote by $\boldsymbol{F} = \{\mathcal{F}_t\}_{t \geq 0}$ the smallest filtration satisfying the usual condition such that the process $S = \{S_t, t \geq 0\}$ and $Z = \{Z_t, t \geq 0\}$ are measurable.

The interpretation of (2.3) is that αt is the non-risky part of investments so that $Z_t = \alpha t$ implies that one unit invested at time zero will be worth $\mathrm{e}^{\alpha t}$ at time t. In mathematical finance it is very common to use (2.3) with $Z_t = \alpha t + \beta B_t$. This is in the case of the famous Black-Scholes option pricing formula, where the price of a stock is assumed to follow the stochastic differential equation $P_t = v + \int_0^t P_s \mathrm{d}Z_s$ with $P = P_0$ being the value of the stock at time zero. It is well known that this implies that $P = \{P_t, t \geq 0\}$ is a geometric Brownian motion so that the value of the stock at time t is $P_t = \exp\{(\alpha - \beta^2/2)t + \beta B_t\}$.

Let $\{A_t\}$ denote the amount invested into risky market at time t, we allow that the company invests more than its current surplus into risk market. In this case, money has to

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be borrowed for such strategies. However, we should note that for large capital, investing more than the surplus into the stock market can not be optimal, see Kalashinikov and Norberg[13]. The strategies $\{A_t, t \geq 0\}$ have to be predictable w.r.t. $\mathcal{F}_t$. This means in particular that the value of an admissible strategy at time t may depend on the history of the process S_t and Z_t up to time t, but it may not depend on the size of a claim occurring at time t. That is to say, the admissible set is

$$\mathcal{A} = \Big\{A = (A_t)_{t\geq 0} : A \text{ is predictable and } \mathsf{P}\Big[\int_0^t A^2(s)\mathrm{d}s < \infty\Big] \text{ for all } t \in [0,\infty)\Big\}.$$

In this paper, we focus on the Markov control, i.e.

$$A_t = A(Y_{t-}^{A,b}), \tag{2.4}$$

where $A(\cdot)$ is called the *defining function* of the Markov strategy A_t. The dynamic of wealth of the insurer with such investment strategy is given by

$$\begin{aligned} \mathrm{d}Y_t^{x,A} &= p\mathrm{d}t + \mathrm{d}\Pi_t + \sigma\mathrm{d}W_t + A_{t-}\mathrm{d}Z_t - \mathrm{d}X_t \\ &= [\alpha A_t + p]\mathrm{d}t + \sigma\mathrm{d}W_t + \mathrm{d}\Pi_t + A_t\beta\mathrm{d}B_t - \mathrm{d}X_t, \\ Y_t^{x,A} &= x. \end{aligned} \tag{2.5}$$

Denote the time of ruin with initial surplus x and strategy A by

$$\tau(x,A) = \inf\{t \geq 0 : Y_t^{x,A} < 0\} \tag{2.6}$$

and corresponding ruin probability by $\Psi(x,A) = \mathsf{P}(\tau(x,A) < \infty)$. The value function is

$$\Psi(x) = \inf_{A\in\mathcal{A}} \Psi(x,A). \tag{2.7}$$

Under aforementioned assumptions and Theorem (32) of Protter (2004, P294), $Y_t^{x,A}$ is a strong Markov process and its infinitesimal operator (with control process A_t) is

$$\begin{aligned} \mathcal{A}^A g(x) &= [\alpha A + p]g_x + \frac{1}{2}(\sigma^2 + \beta^2 A^2)g_{xx} \\ &\quad + \lambda_1\mathsf{E}[g(x+P_1) - g(x)] + \lambda_2\mathsf{E}[g(x-C_1) - g(x)], \end{aligned} \tag{2.8}$$

where g_x, g_{xx} denote, respectively, the first order and the second order partial derivatives w.r.t. x. From Hipp and Plum[10][11], we know that $\Psi(x)$ is twice continuously differentiable on $[0,\infty)$, and $\Psi(x)$ solves the Hamilton-Jacob-Bellman equation

$$\begin{aligned} &\inf_{A\in\mathcal{A}} \Big\{[\alpha + p]\Psi_x + \frac{1}{2}(\sigma^2 + \beta^2 A^2)\Psi_{xx} \\ &+ \lambda_1\mathsf{E}[\Psi(x+P_1) - \Psi(x)] + \lambda_2\mathsf{E}[\Psi(x-C_1) - \Psi(x)]\Big\} = 0, \end{aligned} \tag{2.9}$$

where $\Psi(x) = 1$ for all $x < 0$. The optimal strategy $\widetilde{A}(u)$ is the value of A in the Hamilton-Jacob-Bellman equation for which the minimum is taken:

$$\widetilde{A}(x) = -\frac{\alpha}{\beta^2}\frac{\Psi_x(x)}{\Psi_{xx}(x)}. \tag{2.10}$$

§3. Lundburg Bounds

In order to find the Lundburg bounds, we assume that the tail distributions of the premium income sizes and the claim sizes are decreasing exponentially fast, i.e. that $M_C(r)$ and $M_P(r)$ exist for $r \in (-\infty, r_\infty)$, $r_\infty = \min\{r_C^\infty, r_P^\infty\}$, where r_C^∞ and r_P^∞ are the corresponding positive constants such that

$$\lim_{r\uparrow r_C^\infty} M_C(r) = \infty, \qquad \lim_{r\uparrow r_P^\infty} M_P(r) = \infty.$$

When $\lambda_1 = 0$, these conditions are exactly the conditions implying $\Psi(x) \sim C\mathrm{e}^{-Rx}$ in the classical risk model.

To obtain the upper bound for the ruin probability (value function) under optimal strategy, we start by defining the Lundburg exponent R. Let $R(A)$ be the solution to equation

$$\frac{1}{2}(\sigma^2 + A^2\beta^2)r^2 + \lambda_1[M_P(-r) - 1] + \lambda_2[M_C(r) - 1] - (p + \alpha A)r = 0. \tag{3.1}$$

This is the Lundburg exponent of process (2.2) with constant investment strategy A. The Lundburg exponent for our problem is $R = \sup R(A)$. This means we maximize the Lundburg exponent in order to obtain an asymptotically optimal constant strategy. Note that the function on the left hand side of (3.1) (denoted by $f(A, r)$) is nonnegative at R, therefore, R is the solution to

$$\inf_{A\geq 0}\Big\{\frac{1}{2}(\sigma^2 + A^2\beta^2)r^2 + \lambda_1[M_P(-r) - 1] + \lambda_2[M_C(r) - 1] - (p + \alpha A)r\Big\} = 0. \tag{3.2}$$

Remark 1 Note that for any given $A \geq 0$, $f(A, r)$ is a convex function w.r.t. r. Since $R \geq R(A)$, it follows that $f(A, R) \geq f(A, R(A)) = 0$. Thus the constant optimal strategy A^*, which corresponds to the largest Lundburg coefficient R, should be the one satisfying

$$f(A^*, R) = 0 \leq f(A, R), \tag{3.3}$$

i.e. R is the solution of equation $\inf\limits_{A}\{f(A, r)\} = 0$. The idea of finding such R can also be seen in Hipp and Schmidli[9].

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The following step is to find R and A^*. Suppose that R has already been determined, then

$$f(A,R)=\frac{1}{2}(\sigma^2+A^2\beta^2)R^2+\lambda_1[M_P(-R)-1]+\lambda_2[M_C(R)-1]-(p+\alpha A)R. \tag{3.4}$$

Note that only $A^*=\alpha/(\beta^2 R)$ minimizing $f(A,R)$, that is to say our Lundburg exponent R is the solution to

$$\frac{1}{2}\sigma^2r^2+\lambda_1M_P(-r)+\lambda_2M_C(r)=\lambda_1+\lambda_2+cr+\frac{1}{2}\frac{\alpha^2}{\beta^2}. \tag{3.5}$$

This equation has a solution R, which is greater than the Lundburg exponent corresponding to 0 investment policy, say $R_0=R(0)$. Even in the case that the Lundburg exponent for classical risk model does not exist (e.g. positive safety loading does not hold), we can still assure Equation (3.5) has a positive solution. That is why do not need to assume the positive safety loading hold in this paper.

Lemma 3.1 Let $x\geq 0$ and $\alpha\neq 0$, $\beta\neq 0$. There exists a unique $0<R<r_\infty$ satisfying the equation

$$\frac{1}{2}\sigma^2r^2+\lambda_1M_P(-r)+\lambda_2M_C(r)=\lambda_1+\lambda_2+cr+\frac{1}{2}\frac{\alpha^2}{\beta^2}. \tag{3.6}$$

Proof Denote by $h_1(r)$ and $h_2(r)$ the left hand side and the right hand side of (3.6). Noting that

$$h_1(0)=\lambda_1+\lambda_2<h_2(0)=\lambda_1+\lambda_2+\frac{1}{2}\frac{\alpha^2}{\beta^2}.$$

Obviously, $h_1'(r)>0$, $r\in(0,r_\infty)$ and $\lim\limits_{r\to r_\infty}h_1(r)=\infty$, so it is easy to find that there exist a R such that $h_1(R)=h_2(R)$. The proof is completed. $\square$

Lemma 3.2 Conditioning on the above mentioned R and A^*, $M_t:=\exp\{-RY_t^{x,A^*}\}$ is a martingale.

Proof Obviously, M_t is measurable w.r.t. $\mathcal{F}_t$. On the one side, $\forall\, 0<s\leq t<\infty$

$$\begin{aligned}\mathsf{E}[M_t|\mathcal{F}_s] &= \mathsf{E}[\exp\{-RY_t^{x,A^*}\}|\mathcal{F}_s]\\ &= \mathsf{E}[\exp\{-R[Y_t^{x,A^*}-Y_s^{x,A^*}\}\exp\{Y_s^{x,A^*}\}|\mathcal{F}_s]\\ &= M_s\exp\{f(A^*,R)(t-s)\}\\ &= M_s. \qquad \square\end{aligned} \tag{3.7}$$

Remark 2 The above argument also show that for each $r\in(0,R)$, there exists two constants process $A_{1,2}\in\mathcal{A}$ such that the process $\exp\{-RY_t^{x,A_{1,2}}\}$ are martingales.

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The value $A_{1,2}$ are given in the following way:

$$A_{1,2}(r) = \frac{\alpha}{\beta^2 r} \pm \sqrt{\Delta(r)}, \tag{3.8}$$

where

$$\begin{aligned} \Delta(r) &:= \frac{2}{\beta^2 r^2}\Big(\frac{1}{2}\sigma^2 r^2 + \lambda_1 M_P(-r) + \lambda_2 M_C(r) - \lambda_1 - \lambda_2 - cr - \frac{1}{2}\frac{\alpha^2}{\beta^2}\Big) \\ &\geq 0, \qquad \text{for } r \leq R. \end{aligned} \tag{3.9}$$

Note that for $r = R$ we obtain $\Delta(R) = 0$, and therefore $A_1(R) = A_2(R) = A^*$.

Theorem 3.1 Let $\alpha \neq 0$, $\beta \neq 0$, for the constant investment strategy $A^* = \alpha/(\beta^2 R)$, the ruin probability has the following upper bound,

$$\Psi(x, A^*) \leq \mathrm{e}^{-Rx}. \tag{3.10}$$

Proof Since $\{M_t\}_{t\geq 0}$ is a nonnegative martingale, thus the stopped process $M^{\tau(x,A^*)} := M_{t\wedge\tau(x,A^*)}$ is also a martingale. By using this we have

$$\begin{aligned} \mathrm{e}^{-Rx} &= \mathsf{E}M^{\tau(x,A^*)} \\ &= \mathsf{E}[M^{\tau(x,A^*)} 1_{\{\tau(x,A^*)\leq t\}}] + \mathsf{E}[M^{\tau(x,A^*)} 1_{\{\tau(x,A^*)> t\}}], \end{aligned} \tag{3.11}$$

where 1_C is the indicate function of set C. By monotone convergence theorem, it follows that

$$\lim_{t\to\infty} \mathsf{E}[M^{\tau(x,A^*)} 1_{\{\tau(x,A^*)\leq t\}}] = \mathsf{E}[M^{\tau(x,A^*)} 1_{\{\tau(x,A^*)<\infty\}}]. \tag{3.12}$$

Hence,

$$\mathrm{e}^{-Rx} \geq \mathsf{E}[M^{\tau(x,A^*)} | \tau(x, A^*) < \infty] \mathsf{P}(\tau(x, A^*) < \infty). \tag{3.13}$$

Thus, we have

$$\begin{aligned} \Psi(x, A^*) &= \mathsf{P}(\tau(x, A^*) < \infty) \\ &\leq \frac{\mathrm{e}^{-Rx}}{\mathsf{E}[M^{\tau(x,A^*)} | \tau(x, A^*) < \infty]}. \end{aligned} \tag{3.14}$$

By the definition of $M^{\tau(x,A^*)}$ we know that the denominator of the right hand side of (3.14) is greater than 1. This completes the proof. □

The following corollary is a trivial result of Theorem 3.1 and the definition of $\Psi(x)$, but it would be very important in the proof for the asymptotic optimality of the constant strategy A^*.

Corollary 3.1

$$\Psi(x) = \inf_{A\in\mathcal{A}} \Psi(x, A) \leq \Psi(x, A^*) \leq \mathrm{e}^{-Rx}. \tag{3.15}$$

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§4. Asymptotic Optimality and Asymptotic Uniqueness of the Constant Investment Strategy

In this section we want to show an asymptotic optimality, respectively, asymptotic uniqueness result for the constant investment strategy A^* and the exponent R. We will need the following assumption on the exponential tail distribution of the claim sizes:

Definition 4.1 Let $0 < r < r_\infty$ be given. We say that C has a uniform exponential moment in the tail distribution for r, if the following condition holds:

$$\sup_{y\geq 0} \mathsf{E}[\mathrm{e}^{-r(y-C)}|C > y] < \infty. \tag{4.1}$$

Remark 3 From now on we shall assume that the random variable C which models the claims size, has a uniform exponential moment in the tail distribution for R. One condition to assure Definition 4.1 holds for R is to assume that C has a hazard rate

$$h(y) := \frac{g_C(y)}{1 - G_C(y)} > 0, \tag{4.2}$$

satisfying

$$\liminf_{y\to\infty} h(y) > R, \tag{4.3}$$

here $G_C(y)$ and $g_C(y)$ are the distribution function and density function of C. Partly we make such assumption for the ease of exposition, partly because we need the assumption to go from a local martingale to a true submartingale in the proof of the following Lemmas.

Lemma 4.1 Assume that C has a uniform exponential moment in the tail distribution for R. Then for each $A \in \mathcal{A}$, the process $(\exp\{-RY_t^{x,A}\})_{t\geq 0}$ is a uniformly integrable submartingale.

Lemma 4.2 If C has a uniform exponential moment in the tail distribution for R, then for arbitrary $A \in \mathcal{A}$ and $x \in (0, +\infty)$, the wealth process $Y_t^{x,A}$ converges almost surely on $\{\tau(x, A) = \infty\}$ to ∞ for $t \to \infty$.

The proofs for Lemma 4.1 and Lemma 4.2 are similar to the proofs for Theorem 4 and Lemma 5 in Gaier et al.[6] and are omitted here. By the Lemma 4.1 and Lemma 4.2 we have the following Theorem 4.1, which is a lower bound for the ruin probability under the strategy A^*.

Theorem 4.1 Assume that C has a uniform exponential moment in the tail distribution for R. Then the ruin probability satisfies, for every admissible control process $A(t) \in \mathcal{A}$,

$$\Psi(x, A) \geq L\mathrm{e}^{-Rx}, \tag{4.4}$$

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where

$$L=\inf_{y\geq 0}\frac{\displaystyle\int_y^{\infty}\mathrm{d}G_C(u)}{\displaystyle\int_y^{\infty}\mathrm{e}^{-R(y-u)}\mathrm{d}G_C(u)}=\frac{1}{\sup\limits_{y\geq 0}\mathsf{E}[\mathrm{e}^{-r(y-C)}|C>y]}>0. \tag{4.5}$$

Proof Note that N_S^1 and N_S^2 are independent, by using Doob's optional sampling theorem and the method adopted in the proof for Theorem 6.3, Chapter 3 of Asmussen[1], one can completes the proof immediately. By integrating Lemma 4.1 and Lemma 4.2 and Theorem 4.1, with a similar discussion to the proof for the Theorem 7 of Gaier et al.[6], one can similarly obtained the following Lemma 4.3. Since the proof is analogue to the one for Theorem 7 of Gaier et al.[6], it is omitted here. □

Lemma 4.3 Under the conditions of Theorem 4.1, if there exists $\epsilon>0$ and $x_\epsilon\geq 0$ such that

$$|A(x)-A^*|\geq\epsilon \text{ for all } x\geq x_\epsilon, \tag{4.6}$$

then there exist r_ϵ and A_ϵ such that

$$\Psi(x,A)\geq A_\epsilon\mathrm{e}^{-r_\epsilon x}. \tag{4.7}$$

Theorem 4.2 Under the conditions of Theorem 4.1, Let $\widetilde{A}(\cdot)$ be the defining function of the optimal investment strategy $\widetilde{A}$. If this function has a limit for $x\to\infty$, then this limit is given by

$$\lim_{x\to\infty}A(x)=A^*. \tag{4.8}$$

Proof Assume that $\lim\limits_{x\to\widetilde{A}(x)}\neq A^*$. Then there exist ϵ, $x_\epsilon>0$ such that

$$|\widetilde{A}(x)-A^*|>\epsilon \text{ for } x\geq x_\epsilon. \tag{4.9}$$

Therefore, by Lemma 4.3, one obtains that

$$\Psi(x)\geq A_\epsilon\mathrm{e}^{-r_\epsilon x} \tag{4.10}$$

for some $r_\epsilon<R$, which together with the corollary 1 yields the apparent contradiction to the optimality of $\widetilde{A}$:

$$\lim_{x\to\infty}\frac{\Psi(x)}{\mathrm{e}^{-Rx}}=\infty. \qquad \square \tag{4.11}$$

Remark 4 The asymptotic investment for an insurer with classical surplus risk process and geometric Brownian motion asset price process had been investigated in Gaier et al.[6]. In this paper, we consider a more general surplus risk process: the classical risk process with stochastic premium income with diffusions and we obtain some results about the asymptotically optimal investment strategy for the insurer. We can view our result as an extension of Gaier et al.[6] in the perturbed model with random income.

Remark 5 What can we say from Theorem 4.2? One can find that when the surplus x tends to ∞, the optimal strategy tends to the constant strategy A^*. Obviously, when the surplus is very large, such strategy is very conservative. So from 4.2 we know that minimizing the ruin probability is an extremely conservative approach for insurers. As it was described in Gaier et al.[6], "a more proper way to deal with the probability of ruin in the presence of control variables (such as investment) apparently consist in imposing a certain threshold level on this probability while optimizing w.r.t. other criteria, for example, the expected value of discounted dividends, et al.".

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带扰动的随机保费模型的渐近最优投资

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本文研究了一类具有随机投资回报的随机保费模型的最小破产概率的渐近性质. 在假定常值投资策略的情形下, 通过最小化调节系数, 我们得到了与此调节系数相对应的最优的常值投资策略. 最后我们证明当初始盈余趋向于无穷的时候, 最优的投资策略趋向于这个常值策略.

关键词: 林德伯格不等式, 最优投资, 破产概率, 林德伯格指数.

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