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Precise Large Deviations for Partial Sums of a Class of Negatively Associated Random Arrays *

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Abstract

In this paper, under some mild conditions, precise large deviations for partial sums of negatively associated random arrays in multi-risk models are investigated. The obtained results extend some known ones, and we find out the asymptotic behavior of precise large deviations is also insensitive to negatively associated structures in multi-risk models.

Keywords: Negatively associated random arrays, large deviation, consistently varying tails.

AMS Subject Classification: 62F10, 60F05, 60G50.

§1. Introduction

More recently, precise large deviations for heavy-tailed sums have been investigated by many authors, since large deviation probabilities of the loss process can be used to characterize the ruin probability asymptotically, which is a very important objective in risk management. For some latest works of large deviations with heavy tails, we refer the reader to Ng et al. (2004), Tang (2006), Wang et al. (2006), Liu (2007), Chen and Zhang (2007), Yang et al. (2009), Liu (2009), among many others. However, all the works mentioned above are restricted to one type of risk. That is to say they always assume the insurer provides only one kind of insurance contract. In reality this assumption is not right, so large deviation problem of multi-risk models is more valuable. Motivated by this consideration, Wang and Wang (2007) firstly extended precise large deviation results to multi-risk models with independent claims. Obviously, independence assumption in Wang and Wang (2007) is much strong and out of place in reality. A weaker structure is negatively associated ones, which are introduced by Alam and Saxena (1981) and Joag-Dev and Proschan (1983).

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Definition 1.1 Let d be a positive integer, and let $\{X_n; n \in \mathcal{N}^d\}$ be a field of random variables. The field is called negatively associated (NA), if every pair of disjoint subsets S, T of $\mathcal{N}^d$ and any pair of coordinatewise increasing functions $f(X_i; i \in S)$, $g(X_j; j \in T)$, it holds that

$$\mathsf{Cov}\,(f(X_i; i \in S), g(X_j; j \in T)) \leq 0,$$

whenever the covariance exists.

Throughout this paper, we assume $\{X_{ij}, j \geq 1\}_{i=1}^k$ are NA random arrays, where for any $i = 1, \ldots, k$, $\{X_{ij}, j \geq 1\}$ denotes the ith related loss amounts with common distribution function $F_i(x)$, satisfying $\mathsf{E}X_{ij} = \mu_i < \infty$, $\overline{F}_i(x) = 1 - F_i(x) > 0$ for all $x \in (-\infty, \infty)$. We also assume $F_i \in \mathcal{C}$, for any $i = 1, \ldots, k$, where we say a distribution function F belongs to heavy-tailed subclass $\mathcal{C}$, if

$$\lim_{y \searrow 1} \liminf_{x \to \infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1 \qquad \text{or equivalently} \qquad \lim_{y \nearrow 1} \limsup_{x \to \infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1.$$

Such a distribution function F is usually said to have a consistently varying tail. The heavy-tailed subclass $\mathcal{C}$ was also studied by Cline et al. (1994) who calls it 'intermediate regular variation'. Another well-known heavy-tailed subclass is called the dominated variation class (denoted by $\mathcal{D}$). A distribution function F supported on $(-\infty, \infty)$ is in $\mathcal{D}$ if and only if

$$\limsup_{x \to \infty} \frac{\overline{F}(xy)}{\overline{F}(x)} < \infty$$

for any $0 < y < 1$ (or equivalently for some $0 < y < 1$). For more details of other heavy-tailed subclasses (eg. $\mathcal{R}, \mathcal{S}, \mathcal{L}$, and so on) and their relations, we refer the reader to Ng et al. (2004) or Wang and Wang (2007). Set

$$J_F^* := \inf\Big\{-\frac{\log \overline{F}_*(y)}{\log y}, y > 1\Big\},$$

where $\overline{F}_*(y) = \liminf\limits_{x \to \infty} \overline{F}(xy)/\overline{F}(x)$. In the terminology of Tang (2006), J_F^* is called the (upper) Matuszewska index of F. Let $\{n_i, i = 1, \ldots, k\}$ be k positive integer sequences. For simplicity, we write $S_{n_i} = \sum\limits_{j=1}^{n_i} X_{ij}$, $i = 1 \ldots, k$; $S(k; n_1, \ldots, n_k) = \sum\limits_{i=1}^{k}\sum\limits_{j=1}^{n_i} X_{ij}$. Let $\{N_i(t), i = 1, \ldots, k\}$ be independent nonnegative integer valued counting processes for the claim numbers. We assume that $\{X_{ij}, j \geq 1\}_{i=1}^k$ and $\{N_i(t), i = 1, \ldots, k\}$ are mutually independent and that $\mathsf{E}N_i(t) = \lambda_i(t) \to \infty$ as $t \to \infty$ $(i = 1, \ldots, k)$. Let $S(k; t) = \sum\limits_{i=1}^{k}\sum\limits_{j=1}^{N_i(t)} X_{ij}$, $t \geq 0$. Tang (2006) studied precise large deviations for the sums of negatively dependent random variables with consistently varying tails. Chen et al. (2007) and Liu (2007) extended Tang (2006)'s results to random sums of negatively associated random variables with consistently varying tails respectively. In the present paper, we study precise large deviations for partial sums of negatively associated random arrays in multi-risk models. The obtained results extend some known ones, and we find out the asymptotic behavior of precise large deviations is also insensitive to negatively associated structures in multi-risk models. The rest of this paper is organized as follows. In Section 2, we present some preliminaries. Main results and their proofs are presented in Section 3. An application of main results is stated in Section 4.

§2. Preliminaries

In this section, by convention, we use the notations $S_n = \sum\limits_{i=1}^{n} X_i$, $S_{N(t)} = \sum\limits_{i=1}^{N(t)} X_i$ and $F \asymp G$ in the sense that

$$0 < \liminf \frac{F}{G} \le \limsup \frac{F}{G} < \infty.$$

Clearly, if $F \in \mathcal{D}$, then, for any $c > 0$, $\overline{F}(cx) \asymp \overline{F}(x)$. See also in Tang and Yan (2002). In the following we give some lemmas for the proofs of theorems. Lemma 2.1 is a slight adjustment of Joag-Dev and Proschan (1983).

Lemma 2.1 Let $\{X_k, 1 \le k \le n\}$ be NA. $A_1, \ldots, A_m$ are pairwise disjoint subsets of $\{1, \ldots, n\}$. If f_i, $i = 1, \ldots, m$ be coordinatedwise non-decreasing (or non-increasing) functions. Then $f_1(X_j, j \in A_1), \ldots, f_m(X_j, j \in A_m)$ also be NA and

$$\mathsf{P}\Big(\bigcap_{i=1}^{n}\{X_i > x_i\}\Big) \le \prod_{k=1}^{n} \mathsf{P}(X_k > x_k), \qquad \mathsf{P}\Big(\bigcap_{i=1}^{n}\{X_i \le x_i\}\Big) \le \prod_{k=1}^{n} \mathsf{P}(X_k \le x_k)$$

for any $n = 1, 2, \ldots$ and all $x_1, x_2, \ldots, x_n$.

Lemma 2.2 Let $\{X_k, k = 1, 2, \ldots\}$ be NA with common distribution F and mean μ, satisfying $\mathsf{E}(X_1^-)^r < \infty$ for some $r > 1$, where $X_1^- = \max\{0, -X_1\}$. If $F(-x) = o(\overline{F}(x))$ as $x \to \infty$ and $F \in \mathcal{D}$. Then for any $\gamma > 0$, as $n \to \infty$

$$\mathsf{P}(S_n - n\mu \le -x) = o(n\overline{F}(x))$$

holds uniformly for $x \ge \gamma n$.

Proof Note that $\{X_k, k = 1, 2, \ldots\}$ be NA, thus, by definition, $\{-X_k, k = 1, 2, \ldots\}$ be also NA. Lemma 2.3 of Tang (2006) implies, for any $x > \gamma n$, $\delta > 0$ and $p > J_F^*$, there exists positive numbers ν_0 and C such that

$$\begin{aligned}\mathsf{P}(S_n - n\mu \le -x) &= \mathsf{P}(-S_n + n\mu \ge x) \le n\mathsf{P}(-X_1 + \mu > \nu_0 x) + Cx^{-p} \\ &\le nF(-\nu_0 x + \mu) + Cx^{-p}.\end{aligned} \tag{2.1}$$

It is well known, for any fixed $p > J_F^*$, $x^{-p} = o(\overline{F}(x))$ $(x \to \infty)$ and for large x, $\overline{F}(\nu_0 x) \asymp \overline{F}(x)$. In (2.1), using $F(-x) = o(\overline{F}(x))$, we get

$$\begin{aligned}\frac{\mathsf{P}(S_n - n\mu \le -x)}{n\overline{F}(x)} &\le \frac{nF(-\nu_0 x + \mu) + Cx^{-p}}{n\overline{F}(x)} \\ &= \frac{no(\overline{F}(\nu_0 x)) + Cx^{-p}}{n\overline{F}(x)} \asymp \Big(1 + \frac{C}{n}\Big)\frac{o(\overline{F}(x))}{\overline{F}(x)}.\end{aligned}$$

Therefore, proof of Lemma 2.2 is now completed. $\Box$

Remark 1 (1) In the proof of Lemma 2.2, for any $\varepsilon > 0$, replacing x with εx, as $n \to \infty$, one can easily get

$$\mathsf{P}(S_n - n\mu \le -\varepsilon x) = o(n\overline{F}(x)) \tag{2.2}$$

holds uniformly for $x \ge \gamma n$.

(2) If $\{X_{ij}, j \ge 1\}_{i=1}^k$ are NA and $F_i(x)$ $(i = 1, \ldots, k)$ satisfy the conditions of Lemma 2.2, for any $\varepsilon > 0$, as $n_i \to \infty$, by induction one can prove

$$\mathsf{P}\Big(\sum_{i=1}^k S_{n_i} - \sum_{i=1}^k n_i\mu_i \le -\varepsilon x\Big) = o\Big(\sum_{i=1}^k n_i\overline{F}_i(x)\Big) \tag{2.3}$$

holds uniformly for all $x \ge \max\{\gamma n_i, i = 1, \ldots, k\}$. In fact, for $k = 2$ and any $\delta \in (0, 1/2)$, Lemma 2.1, Lemma 2.2 and NA property yield

$$\begin{aligned}&\mathsf{P}\Big(\sum_{i=1}^2 S_{n_i} - \sum_{i=1}^2 n_i\mu_i \le -\varepsilon x\Big) \\ \le\ &\mathsf{P}(S_{n_1} - n_1\mu_1 \le -(1-\delta)\varepsilon x) + \mathsf{P}(S_{n_2} - n_2\mu_2 \le -(1-\delta)\varepsilon x) \\ &+\mathsf{P}(S_{n_1} - n_1\mu_1 \le -\delta\varepsilon x)\mathsf{P}(S_{n_2} - n_2\mu_2 \le -\delta\varepsilon x) \\ =\ &o(n_1\overline{F}_1(x)) + o(n_2\overline{F}_2(x)) + o(n_1\overline{F}_1(x))o(n_2\overline{F}_2(x)) \\ =\ &o(n_1\overline{F}_1(x)) + n_2\overline{F}_2(x)).\end{aligned} \tag{2.4}$$

Therefore, (2.3) directly derives from (2.4) and induction hypothesis.

§3. Main Results and Their Proofs

Theorem 3.1 Let $\{X_{ij}, j \ge 1\}_{i=1}^k$ be NA random arrays with common distribution function $F_i(x)$ satisfying $\mathsf{E}X_{ij} = \mu_i < \infty$ and $xF_i(-x) = o(\overline{F}_i(x))$, $x \to \infty$. Let $\{n_i\}$ be a positive integer sequence. If $\mathsf{E}|X_{ij}|^r < \infty$ for some $r > 1$ and $F_i \in \mathcal{C}$ for all $i = 1, \ldots, k$, then, for any fixed $\gamma > 0$, we have that as $n_i \to \infty$, all $i = 1, \ldots, k$,

$$\mathsf{P}\Big(S(k; n_1, \ldots, n_k) - \sum_{i=1}^{k} n_i\mu_i > x\Big) \sim \sum_{i=1}^{k} n_i\overline{F}_i(x) \tag{3.1}$$

holds uniformly for all $x \geq \max\{\gamma n_i, i = 1, \ldots, k\} := \Delta(k)$.

Remark 2 If all $F_i(x)$ $(i = 1, \ldots, k)$ are the same distribution function, then (3.1) implies Theorem 1.1 of Tang (2006). Particularly, if we also assume $\{X_{ij}, j \geq 1\}_{i=1}^{k}$ are nonnegative random variables, one can easily check the conditions of Theorem 3.1 naturally satisfy. Therefore, (3.1) implies Theorem 2.1 of Liu (2007). If $\{X_{ij}, j \geq 1, i = 1, \ldots, k\}$ are assumed to be mutually independent, (3.1) yields Theorem 3.1 of Wang and Wang (2007).

Proof We use induction to prove (3.1). For the case of $k = 2$, we first show that

$$\liminf_{n_1, n_2 \to \infty} \inf_{x \geq \Delta(2)} \frac{\mathsf{P}\Big(S(2; n_1, n_2) - \sum_{i=1}^{2} n_i\mu_i > x\Big)}{n_1\overline{F}_1(x) + n_2\overline{F}_2(x)} \geq 1. \tag{3.2}$$

Notice that for any $0 < \varepsilon < 1$ and any $x > 0$,

$$\begin{aligned}
& \mathsf{P}(S(2; n_1, n_2) - n_1\mu_1 - n_2\mu_2 > x) \\
\geq\ & \mathsf{P}(S_{n_1} - n_1\mu_1 > (1+\varepsilon)x, S_{n_2} - n_2\mu_2 > -\varepsilon x) \\
& + \mathsf{P}(S_{n_2} - n_2\mu_2 > (1+\varepsilon)x, S_{n_1} - n_1\mu_1 > -\varepsilon x) \\
& - \mathsf{P}(S_{n_1} - n_1\mu_1 > (1+\varepsilon)x, S_{n_2} - n_2\mu_2 > (1+\varepsilon)x) \\
:=\ & I_1 + I_2 - I_3.
\end{aligned} \tag{3.3}$$

We first deal with I_1. Noting that

$$\begin{aligned}
I_1 &= \mathsf{P}(S_{n_1} - n_1\mu_1 > (1+\varepsilon)x, S_{n_2} - n_2\mu_2 > -\varepsilon x) \\
&\geq \mathsf{P}(S_{n_1} - n_1\mu_1 > (1+\varepsilon)x) - \mathsf{P}(S_{n_2} - n_2\mu_2 \leq -\varepsilon x),
\end{aligned} \tag{3.4}$$

by Theorem 2.1 of Tang (2006), for any $0 < \delta < 1$, and sufficiently large n_1, we have

$$\sup_{x \geq \gamma n_1} \left| \frac{\mathsf{P}(S_{n_1} - n_1\mu_1 > (1+\varepsilon)x)}{n_1\overline{F}_1((1+\varepsilon)x)} - 1 \right| < \delta. \tag{3.5}$$

Since $xF_2(-x) = o(\overline{F}_2(x))$ implies $F_2(-x) = o(\overline{F}_2(x))$, Lemma 2.2 yields $\mathsf{P}(S_{n_2} - n_2\mu_2 \leq -\varepsilon x) = o(n_2\overline{F}_2(x))$ holds uniformly for $x > \gamma n_2$. Thus, for sufficiently large n_1, n_2, and uniformly for $x \geq \Delta(2)$,

$$I_1 \geq (1-\delta)n_1\overline{F}_1((1+\varepsilon)x) + o(n_2\overline{F}_2(x)). \tag{3.6}$$

By the same argument as above, we get $I_2 \geq (1-\delta)n_2\overline{F}_2((1+\varepsilon)x)+o(n_1\overline{F}_1(x))$. Finally we consider I_3. Similar to (3.15) in Wang and Wang (2007), we have

$$\lim_{\varepsilon\downarrow 0}\lim_{n_i\to\infty}\sup_{x\geq\gamma n_i}\left|\frac{\overline{F}_i((1\pm\varepsilon)x)}{\overline{F}_i(x)}-1\right|=0, \qquad i=1,2. \tag{3.7}$$

Noting $\{X_{11},\ldots,X_{1n_1},X_{21},\ldots,X_{2n_2}\}$ are NA, Lemma 2.1 imply S_{n_1} and S_{n_2} are also NA. Thus, by Theorem 2.1 of Tang (2006) and (3.11), we get

$$\begin{aligned} I_3 &\leq \mathsf{P}(S_{n_1}-n_1\mu_1>(1+\varepsilon)x)\mathsf{P}(S_{n_2}-n_2\mu_2>(1+\varepsilon)x)\\ &\leq (1+\delta)^2 n_1\overline{F}_1((1+\varepsilon)x)n_2\overline{F}_2((1+\varepsilon)x)\\ &\leq (1+\delta)^4 n_1\overline{F}_1(x)n_2\overline{F}_2(x)\\ &= o(n_1\overline{F}_1(x)+n_2\overline{F}_2(x)). \end{aligned} \tag{3.8}$$

(3.3)-(3.8) yield that

$$\begin{aligned} &\mathsf{P}(S(2;n_1,n_2)-n_1\mu_1-n_2\mu_2>x)\\ \geq\ &(1-\delta)^2(n_1\overline{F}_1(x)+n_2\overline{F}_2(x))+o(n_1\overline{F}_1(x)+n_2\overline{F}_2(x)). \end{aligned}$$

Therefore, letting $\delta\downarrow 0$, we obtain (3.2).

Next we show that

$$\limsup_{n_1,n_2\to\infty}\sup_{x\geq\Delta(2)}\frac{\mathsf{P}(S(2;n_1,n_2)-n_1\mu_1-n_2\mu_2>x)}{n_1\overline{F}_1(x)+n_2\overline{F}_2(x)}\leq 1. \tag{3.9}$$

Notice that for any $\varepsilon\in(0,1/2)$ and any $x>0$, then, by NA property, Lemma 2.1 and Theorem 2.1 of Tang (2006), we arrive at

$$\begin{aligned} &\mathsf{P}(S(2;n_1,n_2)-n_1\mu_1-n_2\mu_2>x)\\ \leq\ &\mathsf{P}(S_{n_1}-n_1\mu_1>(1-\varepsilon)x)+\mathsf{P}(S_{n_2}-n_2\mu_2>(1-\varepsilon)x)\\ &+\mathsf{P}(S_{n_1}-n_1\mu_1>\varepsilon x)\mathsf{P}(S_{n_2}-n_2\mu_2>\varepsilon x)\\ \asymp\ &(n_1\overline{F}_1(x)+n_2\overline{F}_2(x))++o(n_1\overline{F}_1(x)+n_2\overline{F}_2(x)). \end{aligned} \tag{3.10}$$

(3.9) directly derives from (3.10). Thus (3.1) holds for $k=2$. Now suppose (3.1) holds for $k-1$, for the case of k, using the similar argument as (3.3), we have that,

$$\begin{aligned} &\mathsf{P}\Big(S(k;n_1,\ldots,n_k)-\sum_{i=1}^{k}n_i\mu_i>x\Big)\\ \geq\ &\mathsf{P}\Big(\sum_{i=1}^{k-1}S_{n_i}-\sum_{i=1}^{k-1}n_i\mu_i>(1+\varepsilon)x, S_{n_k}-n_k\mu_k>-\varepsilon x\Big)\\ &+\mathsf{P}\Big(S_{n_k}-n_k\mu_k>(1+\varepsilon)x, \sum_{i=1}^{k-1}S_{n_i}-\sum_{i=1}^{k-1}n_i\mu_i>-\varepsilon x\Big)\\ &-\mathsf{P}\Big(\sum_{i=1}^{k-1}S_{n_i}-\sum_{i=1}^{k-1}n_i\mu_i>(1+\varepsilon)x, S_{n_k}-n_k\mu_k>(1+\varepsilon)x\Big). \end{aligned} \tag{3.11}$$

NA property, Remark 1 and induction hypothesis yields that,

$$\liminf_{n_1,\dots,n_k\to\infty}\inf_{x\geq\Delta(k)}\frac{\mathsf{P}\Big(S(k;n_1,\dots,n_k)-\sum\limits_{i=1}^{k}n_i\mu_i>x\Big)}{\sum\limits_{i=1}^{k}n_i\overline{F}_i(x)}\geq 1. \tag{3.12}$$

For the reverse inequality, using the similar argument as (3.10) and induction hypothesis,

$$\limsup_{n_1,\dots,n_k\to\infty}\sup_{x\geq\Delta(k)}\frac{\mathsf{P}\Big(S(k;n_1,\dots,n_k)-\sum\limits_{i=1}^{k}n_i\mu_i>x\Big)}{\sum\limits_{i=1}^{k}n_i\overline{F}_i(x)}\leq 1. \tag{3.13}$$

Combining (3.12) and (3.13), the proof of Theorem 3.1 is now completed. □

Theorem 3.2 Let $\{X_{ij},j\geq 1\}_{i=1}^{k}$ be NA random arrays with common distribution function $F_i(x)$ that has finite expectation $\mu_i<0$, and $xF_i(-x)=o(\overline{F}_i(x))$, $x\to\infty$ and $\mathsf{E}|X_{ij}|^r<\infty$ for some $r>1$ and $F_i\in\mathcal{C}$ for all $i=1,\dots,k$. Let $\{N_i(t)\}_{i=1}^{k}$ be independent nonnegative integer-valued process independent of $\{X_{ij},j\geq 1\}_{i=1}^{k}$. If $\{N_i(t)\}_{i=1}^{k}$ satisfy, for any $i=1,\dots k,\delta>0$ and some $p_i>J_{F_i}^{*}$,

$$\mathsf{E}N_i^{p_i}(t)I_{(N_i(t)>(1+\delta)\lambda_i(t))}=O(\lambda_i(t)), \tag{3.14}$$

where $I_{\{\cdot\}}$ is the indicator function. Then for any fixed $\gamma>\max\{|\mu_i|,i=1,\dots,k\}$, as $t\to\infty$,

$$\mathsf{P}\Big(S(k;t)-\sum_{i=1}^{k}\mu_i\lambda_i(t)>x\Big)\sim\sum_{i=1}^{k}\lambda_i(t)\overline{F}_i(x), \tag{3.15}$$

uniformly for $x\geq\max\{\gamma\lambda_i(t),i=1,\dots,k\}:=\Gamma(k)$.

Remark 3 If all $F_i(x)\,(i=1,\dots,k)$ are the same distribution function, then (3.15) implies Theorem 1.2 of Chen and Zhang (2007). Particularly, if we also assume $\{X_{ij},j\geq 1,i=1,\dots,k\}$ are nonnegative random variables, one can easily check the conditions of Theorem 3.2 naturally satisfy. Therefore, (3.15) implies Theorem 2.2 of Liu (2007). If $\{X_{ij},j\geq 1,i=1,\dots,k\}$ are assumed to be mutually independent, (3.15) yields Theorem 4.1 of Wang and Wang (2007).

Proof Again by induction as Theorem 3.1, it is sufficient to show (3.15) for $k=2$. We first show that

$$\liminf_{t\to\infty}\inf_{x\geq\Gamma(2)}\frac{\mathsf{P}(S(2;t)-\lambda_1(t)\mu_1-\lambda_2(t)\mu_2>x)}{\lambda_1(t)\overline{F}_1(x)+\lambda_2(t)\overline{F}_2(x)}\geq 1. \tag{3.16}$$

The same argument yields that, for any $0<\varepsilon<1$ and any $x>0$,

$$
\begin{aligned}
& \mathsf{P}(S(2;t)-\lambda_1(t)\mu_1-\lambda_2(t)\mu_2>x) \\
\geq\ & \mathsf{P}(S_{N_1(t)}-\lambda_1(t)\mu_1>(1+\varepsilon)x, S_{N_2(t)}-\lambda_2(t)\mu_2>-\varepsilon x) \\
& +\mathsf{P}(S_{N_2(t)}-\lambda_2(t)\mu_2>(1+\varepsilon)x, S_{N_1(t)}-\lambda_1(t)\mu_1>-\varepsilon x) \\
& -\mathsf{P}(S_{N_1(t)}-\lambda_1(t)\mu_1>(1+\varepsilon)x, S_{N_2(t)}-\lambda_2(t)\mu_2>(1+\varepsilon)x) \\
:=\ & J_1+J_2-J_3. \qquad (3.17)
\end{aligned}
$$

We first deal with J_1. Note that

$$
J_1\geq \mathsf{P}(S_{N_1(t)}-\lambda_1(t)\mu_1>(1+\varepsilon)x)-\mathsf{P}(S_{N_2(t)}-\lambda_2(t)\mu_2\leq-\varepsilon x). \qquad (3.18)
$$

By Theorem 1.2 of Chen and Zhang (2007), we easily conclude

$$
\lim_{t\to\infty}\sup_{x\geq\gamma\lambda_1(t)}\left|\frac{\mathsf{P}(S_{N_1(t)}-\lambda_1(t)\mu_1>(1+\varepsilon)x)}{\lambda_1(t)\overline{F}_1((1+\varepsilon)x)}-1\right|=0. \qquad (3.19)
$$

Now for any $\beta\in(0,\varepsilon)$, note that $\mu_2<0$,

$$
\begin{aligned}
& \mathsf{P}(S_{N_2(t)}-\lambda_2(t)\mu_2\leq-\varepsilon x) \\
=\ & \sum_{n=1}^{\infty}\mathsf{P}(S_n-n\mu_2\leq-\varepsilon x+(n-\lambda_2(t))|\mu_2|)\mathsf{P}(N_2(t)=n) \\
=\ & \sum_{-\varepsilon x+(n-\lambda_2(t))|\mu_2|\leq-\beta x}+\sum_{-\varepsilon x+(n-\lambda_2(t))|\mu_2|>-\beta x} \\
:=\ & K_1+K_2. \qquad (3.20)
\end{aligned}
$$

Firstly, using Lemma 2.2, we get

$$
\begin{aligned}
K_1 &\leq \sum_{-\varepsilon x+(n-\lambda_2(t))|\mu_2|\leq-\beta x}\mathsf{P}(S_n-\lambda_2(t)\mu_2\leq-\beta x)\mathsf{P}(N_2(t)=n) \\
&= \sum_{-\varepsilon x+(n-\lambda_2(t))|\mu_2|\leq-\beta x} o(n\overline{F}_2(x))\mathsf{P}(N_2(t)=n) \\
&\leq o(\overline{F}_2(x))\sum_{n=1}^{\infty}n\mathsf{P}(N_2(t)=n)=o(\lambda_2(t)\overline{F}_2(x)). \qquad (3.21)
\end{aligned}
$$

Now we deal with K_2. For simplicity, we denote $(\varepsilon-\beta)/|\mu_2|$ by C in the following. In fact, for any $p>J^*_{F_2}$, noting that $\mu_2<0$, using Tchebychef inequality, we have

$$
\begin{aligned}
K_2 &= \sum_{-\varepsilon x+(n-\lambda_2(t))|\mu_2|>-\beta x}\mathsf{P}(S_n-\lambda_2(t)\mu_2\leq-\varepsilon x)\mathsf{P}(N_2(t)=n) \\
&\leq \mathsf{P}\Big(N_2(t)>\frac{\varepsilon-\beta}{|\mu_2|}x+\lambda_2(t)\Big)\leq\frac{\mathsf{E}N_2^p(t)I_{\{N_2(t)>Cx+\lambda_2(t)\}}}{(Cx+\lambda_2(t))^p} \\
&\leq \frac{\mathsf{E}N_2^p(t)I_{\{N_2(t)>(1+C\gamma)\lambda_2(t)\}}}{(Cx)^p}=C^{-p}x^{-p}O(\lambda_2(t)) \\
&= o(\lambda_2(t)\overline{F}_2(x)). \qquad (3.22)
\end{aligned}
$$

The last equality holds because $x^{-p} = o(\overline{F}_2(x))$ by Lemma 2.1 of Tang (2006). Combining (3.18)-(3.22), for any $\delta > 0$, we get

$$J_1 \geq (1-\delta)\lambda_1(t)\overline{F}_1((1+\varepsilon)x) + o(\lambda_2(t)\overline{F}_2(x)). \tag{3.23}$$

Next by the same argument as above, we also get $J_2 \geq (1-\delta)\lambda_2(t)\overline{F}_2((1+\varepsilon)x) + o(\lambda_1(t)\overline{F}_1(x))$. Finally we consider J_3. Similar to (3.7) we easily arrive at

$$\lim_{\varepsilon\downarrow 0}\lim_{t\to\infty}\sup_{x\geq\gamma\lambda_i(t)}\left|\frac{\overline{F}_i((1\pm\varepsilon)x)}{\overline{F}_i(x)} - 1\right| = 0, \qquad i = 1, 2. \tag{3.24}$$

Note that $\{N_i(t)\}_{i=1}^2$ be independent and $\{X_{ij}\}_{i=1}^2$ be NA, by Lemma 2.1, Theorem 2.1 of Chen and Zhang (2007) and (3.24), one get

$$\begin{aligned} J_3 &\leq \mathsf{P}(S_{N_1(t)} - \lambda_1(t)\mu_1 > (1+\varepsilon)x)\mathsf{P}(S_{N_2(t)} - \lambda_2(t)\mu_2 > (1+\varepsilon)x) \\ &\sim \lambda_1(t)\overline{F}_1((1+\varepsilon)x)\lambda_2(t)\overline{F}_2((1+\varepsilon)x) \\ &= o(\lambda_1(t)\overline{F}_1(x) + \lambda_2(t)\overline{F}_2(x)). \end{aligned} \tag{3.25}$$

Therefore, by (3.23)-(3.25) and letting $\delta \downarrow 0$, for any sufficiently large t, $x \geq \Gamma(2)$,

$$\liminf_{t\to\infty}\inf_{x\geq\Gamma(2)}\frac{\mathsf{P}(S(2;t) - \lambda_1(t)\mu_1 - \lambda_2(t)\mu_2 > x)}{\lambda_1(t)\overline{F}_1(x) + \lambda_2(t)\overline{F}_2(x)} \geq 1.$$

This proves (3.16).

We now show that

$$\limsup_{t\to\infty}\sup_{x\geq\Gamma(2)}\frac{\mathsf{P}(S(2;t) - \lambda_1(t)\mu_1 - \lambda_2(t)\mu_2 > x)}{\lambda_1(t)\overline{F}_1(x) + \lambda_2(t)\overline{F}_2(x)} \leq 1. \tag{3.26}$$

Notice that for any $\varepsilon \in (0, 1/2)$ and any $x > 0$, by NA property and the same argument as (3.10), we have, as $t \to \infty$, $x \geq \Gamma(2)$,

$$\begin{aligned} &\mathsf{P}(S(2;t) - \lambda_1(t)\mu_1 - \lambda_2(t)\mu_2 > x) \\ \leq\ &\mathsf{P}(S_{N_1(t)} - \lambda_1(t)\mu_1 > (1-\varepsilon)x) + \mathsf{P}(S_{N_2(t)} - \lambda_2(t)\mu_2 > (1-\varepsilon)x) \\ &+ \mathsf{P}(S_{N_1(t)} - \lambda_1(t)\mu_1 > \varepsilon x)\mathsf{P}(S_{N_2(t)} - \lambda_2(t)\mu_2 > \varepsilon x) \\ \sim\ &\lambda_1(t)\overline{F}_1((1-\varepsilon)x) + \lambda_2(t)\overline{F}_2((1-\varepsilon)x) + \lambda_1(t)\overline{F}_1(\varepsilon x)\lambda_2(t)\overline{F}_2(\varepsilon x) \\ \sim\ &\lambda_1(t)\overline{F}_1(x) + \lambda_2(t)\overline{F}_2(x) + o(\lambda_1(t)\overline{F}_1(x) + \lambda_2(t)\overline{F}_2(x)). \end{aligned} \tag{3.27}$$

Thus we get (3.26).

Combining (3.16) and (3.26), (3.15) holds for $k = 2$. The proof of Theorem 3.2 is now completed. $\square$

§4. Applications

In this section we give an application of the main results. Assume there are two types of contracts in an insurer. The first loss amounts $X=\{X_j, j\geq 1\}$ are NA nonnegative random variables with common distribution $F\in\mathcal{C}$ and finite expectation μ. An ordinary renewal counting process $\{N_1(t), t\geq 0\}$ denotes their related claim numbers, where $\lambda_1(t)=\mathsf{E}N_1(t)$. Let $\{I_j, j\geq 1\}$ be a sequence of Bernoulli random variables with $\mathsf{E}I_j=q$, $0<q\leq 1$. The second loss amounts $\{Y_j, j\geq 1\}$ are also NA nonnegative random variables with distribution $G(\neq F)\in\mathcal{C}$ and finite expectation ν, and related claim numbers $N_2(t)=N(\Lambda(t))$ be a Cox process, where $N(t)$ be an ordinary renewal process and $\{\Lambda(t), t\geq 0\}$ be another right-continuous nondecreasing process with $\lambda^*(t)=\mathsf{E}\Lambda(t)$, independent of $N(t)$. Suppose that sequences of $\{X_j, j\geq 1\}$, $\{I_j, j\geq 1\}$, $\{Y_j, j\geq 1\}$ are NA, independent of $\{N_1(t), t\geq 0\}$, $\{N_2(t), t\geq 0\}$. Then the total claim amount up to time t is

$$S(t)=\sum_{j=1}^{N_1(t)}X_jI_j+\sum_{j=1}^{N_2(t)}Y_j,\qquad t\geq 0. \tag{4.1}$$

We also assume that for some $p>\gamma_G$ and any $\theta>0$, $\mathsf{E}\Lambda^p(t)1_{(\Lambda(t))>(1+\theta)\lambda^*(t)}=O(\lambda^*(t))$. Write $N_1^*(t)=\sup\{\sigma_n\geq t, I_n=1\}$, $t\geq 0$. (4.1) can be rewritten as

$$S(t)=\sum_{j=1}^{N_1^*(t)}X_j+\sum_{j=1}^{N_2(t)}Y_j.$$

Using the same method in Section 5 of Wang et al. (2007) and by Theorem 3.2, we get, as $t\to\infty$, for any $\gamma>0$ and uniformly for $x\geq\max\{\gamma\lambda_1(t), \gamma\lambda^*(t)\}$,

$$\mathsf{P}(S(t)-q\lambda_1(t)\mu-\nu\lambda^*(t)>x)\sim q\lambda_1(t)\overline{F}(x)+\lambda^*(t)\overline{G}(x).$$

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一类负相伴随机阵列部分和的精致大偏差

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本文在一些适当的条件下得到了多风险模型中负相伴随机阵列的精致大偏差, 推广了一些已知的结果, 同时表明在多风险模型中负相伴结构对精致大偏差同样不具有敏感性.

关键词: 负相伴随机阵列, 大偏差, 一致变化尾.

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