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Markov-Modulated风险模型的极大值、极小值及零点数的联合分布

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摘 要

本文基于保险公司在首次破产后仍能继续运转的情形, 讨论并得到了Markov-modulated风险模型中关于末离零点前盈余过程极大值、极小值及零点数的联合分布.

关键词: Markov-modulated风险模型, 逐段决定马尔可夫过程, 补充变量.

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§1. 引 言

Markov-modulated风险模型是针对处于随机环境中的风险事业而建立的. 在此模型中, 定义$\{r(t), t \geq 0\}$为一环境过程, 影响着风险过程中的索赔到达次数和索赔额分布, 其满足:

(1) 具有有限状态空间$I = \{1, 2, \ldots, N\}$、轨道右连续且齐次不可约常返的马氏链;

(2) 密度阵为$Q = (q_{ij}), q_i := -q_{ii}$, π是其唯一平稳分布且此嵌入马氏链的转移阵为

$$p_{ij} = ((1 - \delta_{ij})q_{ij}/q_i), \qquad i, j \in I, \tag{1.1}$$

其中, $\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$;

(3) 当$r(t) = i$时索赔到达为一Poisson过程, 其参数为$\lambda_i \in R^+$, $i \in I$; 索赔额分布函数为G_i, 密度函数为g_i, 均值为μ_i, $i \in I$.

考虑盈余过程$\{U(t), t \geq 0\}$:

$$U(t) = u + ct - \sum_{n=1}^{N(t)} Y_n, \tag{1.2}$$

其中, 保费收入率c为一常数, $u \in R$为初始盈余; Y_n为第n次索赔额, 且$Y_1, Y_2, \ldots$相互独立; 令σ_n为第$n-1$次与第n次索赔的间隔时间, 则$N(t) := \sup\left\{n \in N \,\middle|\, \sum_{v=1}^{n} \sigma_v \leq t\right\}$为时刻$t$之

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前的索赔次数; 索赔到达的计数过程$\{N(t), t \geq 0\}$称为Markov-modulated Poisson过程, 其为Cox过程的一种特殊情况.

在初始环境为i的情况下, 初始盈余为u的破产概率为$\Psi_i(u) = \mathsf{P}_{u,i}(T < \infty)$, 生存概率为$\Phi_i(u) = 1 - \Psi_i(u)$, 净收益条件为$\rho = c\big(\sum\limits_{i\in I} \pi_i \lambda_i \mu_i\big)^{-1} - 1 > 0$.

由逐段决定马尔可夫过程理论[1], 补充变量可知$\{(U(t), r(t))\}_{t\geq 0}$为齐次逐段决定强马尔可夫过程, 其三元特征为

$$\varphi((u,i),t) = (u + ct, i), \tag{1.3}$$

$$F((u,i),t) = e^{-(\lambda_i + q_i)t}, \tag{1.4}$$

$$\begin{cases} Q((x,i),(x,j)) = \dfrac{q_{ij}}{\lambda_i + q_i}, & i \neq j; \\ \\ Q((x,i),(dy,j)) = \dfrac{\lambda_i}{\lambda_i + q_i} G_i(x - dy), & i = j, \end{cases} \tag{1.5}$$

其中, $x \in R^+$为索赔前瞬时的盈余额, $t \in R^+$, $i \in I$.

对于Markov-modulated风险模型, Reinhard(1984)、Asmussen(1995)、Bäuerle(1996)、Snoussi (1998)等诸多学者针对不同问题进行过考察. 近年来也有不少文章讨论了关于经典风险模型的极值分布, 如文献[6]和[7]. 本文利用过程$\{(U(t), r(t))\}_{t\geq 0}$的强马氏性及[1]中关于"首达时间分布"的讨论, 基于保险公司在首次破产后仍能继续运转的情形, 给出了Markov-modulated风险模型的极大值、极小值及零点数的一个联合分布.

§2. 主要结果

2.1 符号说明

据本文所需, 首先给出相关符号的定义, 以下各随机变量均定义在同一概率空间$(\Omega, \mathcal{F}, \mathsf{P})$上, 且$\mathcal{F}_t = \sigma\{(U(s), r(s)), s \leq t\}$.

破产时刻为$T = \inf\{t > 0; U(t) < 0\}$ (其中, $\inf\emptyset = \infty$); 盈余过程最后一次离开零点的时刻, 即末离时为$L = \sup\{t > 0; U(t) < 0\}$ (其中, $\sup\emptyset = 0$); 盈余过程首次达到a的时刻为$T_a = \inf\{t > 0; U(t) = a\}$ (其中, $\inf\emptyset = \infty$).

定义首次破产时刻为$T_1 = \inf\{t > 0; U(t) < 0\}$ (其中, $\inf\emptyset = \infty$), 即$T_1 = T$, 盈余过程首次到达零点的时刻为$T_1^0 = \inf\{t > 0; U(t) = 0\}$ (其中, $\inf\emptyset = \infty$); 一般地, 第k次破产时刻为$T_k = \inf\{t > T_{k-1}^0; U(t) < 0\}$ (其中, $\inf\emptyset = \infty$), 盈余过程第k次到达零点的时刻为$T_k^0 = \inf\{t > T_{k-1}^0; U(t) = 0\}$ (其中, $\inf\emptyset = \infty$), $k = 2, 3, \ldots$; 盈余过程到达零点数的总次数为$H = \sup\{k; T_k^0 < \infty\}$ (其中, $\sup\emptyset = 0$).

由于$\mathsf{P}_u\big(\lim\limits_{t\to\infty}U(t)=\infty\big)=1$, 则有$\mathsf{P}_u(H<\infty)=1$. 当$H>0$时, $T_H^0<\infty$且$T_{H+1}^0=\infty$; 当$L>0$时, $L=T_H^0$; 当$u\geq 0$时, $\mathsf{P}_u(T_k<T_k^0|T_k<\infty)=1$, $\forall\, k\geq 1$.

对于$t\geq 0$, 定义推移算子$\theta_t:\Omega\to\Omega$, $U_s(\theta_t\omega)=U_{s+t}(\omega)$; 对于有限停时$T$, 定义映射$\theta_t:\Omega\to\Omega$, 若$T(\omega)=t$, 则$\theta_T(\omega)=\theta_t(\omega)$, 此时$U_t\circ\theta_T=U_{t+T}$.

定义过程$\{U(t),r(t)\}_{t\geq 0}$的第n次随机跳时刻为τ_n, 而τ_1'为盈余过程$\{U(t)\}_{t\geq 0}$的第1次索赔到达时刻.

2.2　定理

引理 2.1　当$b>y>0$时, $f_1(-y,k;b;j_1)=\mathsf{P}_{-y,k}\big(\inf\limits_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1\big)$, $k,j_1\in I$是初始盈余为$-y$, 初始环境状态为k, 盈余过程在首次达到零点前极小值小于b且首次达到零点发生在状态j_1的概率, 则$f_1(-y,k;b;j_1)$为方程

$$\begin{aligned} f_1(-y,k;b;j_1) &= \int_0^{y/c}\lambda_k e^{-(\lambda_k+q_k)s}\int_0^{b+cs-y}f_1(cs-y-z,k;b;j_1)\mathrm{d}G_k(z)\mathrm{d}s \\ &\quad+\sum_{j\neq k}\int_0^{y/c}q_{kj}e^{-(\lambda_k+q_k)s}f_1(cs-y,j;b;j_1)\mathrm{d}s+\delta_{kj_1}e^{-(\lambda_k+q_k)y/c} \end{aligned} \tag{2.1}$$

的最小非负解.

证明:　由于

$$\begin{aligned} f_1(-y,k;b;j_1) &= \mathsf{P}_{-y,k}\Big(\inf_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1\Big) \\ &= \mathsf{P}_{-y,k}\Big(\inf_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1,0<T_1^0<\tau_1\Big) \\ &\quad+\mathsf{P}_{-y,k}\Big(\inf_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1,\tau_1\leq T_1^0\Big), \end{aligned} \tag{2.2}$$

其中

$$\begin{aligned} &\mathsf{P}_{-y,k}\Big(\inf_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1,0<T_1^0<\tau_1\Big) \\ =\;& \mathsf{P}_{-y,k}(0<T_1^0<\tau_1,r(T_1^0)=j_1)=\delta_{kj_1}\mathsf{P}_{-y,k}\Big(\tau_1>\frac{y}{c}\Big) \\ =\;& \delta_{kj_1}e^{-(\lambda_k+q_k)y/c}, \end{aligned} \tag{2.3}$$

$$\begin{aligned} &\mathsf{P}_{-y,k}\Big(\inf_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1,\tau_1\leq T_1^0\Big) \\ =\;& \mathsf{E}_{-y,k}\Big[\mathrm{I}_{\{\tau_1\leq y/c,U(\tau_1)>-b\}}\mathsf{P}_{U(\tau_1),r(\tau_1)}\Big(\inf_{0\leq t<T_1^0}U(t)>-b,r(T_1^0)=j_1\Big)\Big] \\ =\;& \int_0^{y/c}\lambda_k e^{-(\lambda_k+q_k)s}\int_0^{b+cs-y}f_1(cs-y-z,k;b;j_1)\mathrm{d}G_k(z)\mathrm{d}s \\ &+\sum_{j\neq k}\int_0^{y/c}q_{kj}e^{-(\lambda_k+q_k)s}f_1(cs-y,j;b;j_1)\mathrm{d}s, \end{aligned} \tag{2.4}$$

将(2.3)、(2.4)式代入(2.2)式即得结论. □

引理 2.2 当$a > u \geq 0, y > 0$时, $f_2(u,i;a;y,k) = \mathsf{P}_{u,i}(T_1 < T_a, r(T_1) = k, U(T_1) > -y)$, $i,k \in I$是初始盈余额为u, 初始环境状态为i, 在首次破产前的盈余额小于a与首次破产赤字小于y且首次破产发生在状态k的联合分布, 则$f_2(u,i;a;y,k)$是方程

$$\begin{aligned} f_2(u,i;a;y,k) &= \int_0^{(a-u)/c} \lambda_i e^{-(\lambda_i+q_i)s} \int_0^{u+cs} f_2(u+cs-z,i;a;y,k)\mathrm{d}G_i(z)\mathrm{d}s \\ &\quad + \sum_{j\neq i}\int_0^{(a-u)/c} q_{ij}e^{-(\lambda_i+q_i)s} f_2(u+cs,j;a;y,k)\mathrm{d}s \\ &\quad + \delta_{ik}\int_0^{(a-u)/c} \lambda_i e^{-(\lambda_i+q_i)s}(G_i(u+cs+y-)-G_i(u+cs))\mathrm{d}s \end{aligned} \tag{2.5}$$

的最小非负解.

证明: 因为

$$\begin{aligned} f_2(u,i;a;y,k) &= \mathsf{P}_{u,i}(T_1 < T_a, r(T_1) = k, U(T_1) > -y) \\ &= \mathsf{P}_{u,i}(0 < T_1 \leq \tau_1 < T_a, r(T_1) = k, U(T_1) > -y) \\ &\quad + \mathsf{P}_{u,i}(\tau_1 < T_1 < T_a, r(T_1) = k, U(T_1) > -y), \end{aligned} \tag{2.6}$$

而

$$\begin{aligned} &\mathsf{P}_{u,i}(0 < T_1 \leq \tau_1 < T_a, r(T_1) = k, U(T_1) > -y) \\ = \;& \mathsf{P}_{u,i}\Big(T_1 = \tau_1', \tau_1' < \frac{a-u}{c}, r(\tau_1') = k, U(\tau_1') > -y\Big) \\ = \;& \delta_{ik}\int_0^{(a-u)/c} \lambda_i e^{-(\lambda_i+q_i)s}(G_i(u+cs+y-)-G_i(u+cs))\mathrm{d}s, \end{aligned} \tag{2.7}$$

$$\begin{aligned} &\mathsf{P}_{u,i}(\tau_1 < T_1 < T_a, r(T_1) = k, U(T_1) > -y) \\ = \;& \mathsf{P}_{u,i}\Big(\tau_1 < \frac{a-u}{c}, U(\tau_1) \geq 0, \tau_1 < T_1 < T_a, r(T_1) = k, U(T_1) > -y\Big) \\ = \;& \mathsf{E}_{u,i}\big[\mathrm{I}_{\{\tau_1 < (a-u)/c, U(\tau_1)\geq 0\}}\mathsf{P}_{U(\tau_1),r(\tau_1)}(T_1 < T_a, r(T_1) = k, U(T_1) > -y)\big] \\ = \;& \int_0^{(a-u)/c} \lambda_i e^{-(\lambda_i+q_i)s}\int_0^{u+cs} f_2(u+cs-z,i;a;y,k)\mathrm{d}G_i(z)\mathrm{d}s \\ & + \sum_{j\neq i}\int_0^{(a-u)/c} q_{ij}e^{-(\lambda_i+q_i)s} f_2(u+cs,j;a;y,k)\mathrm{d}s, \end{aligned} \tag{2.8}$$

将(2.7)、(2.8)式代入(2.6)式即得结论. □

定理 2.1 当$a > u \geq 0$, $b > y > 0$时, 若$f_3(u,i;a,b;j_1)$是初始盈余为u, 初始环境状态为i, 在T_1^0之前极大盈余额小于a与极小盈余额小于b且首次达到零点发生在状态j_1的联合

分布$\mathsf{P}_{u,i}\big(\sup_{0\le t<T_1^0} U(t)<a, \inf_{0\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1, T_1^0<\infty\big)$, 则有

$$f_3(u,i;a,b;j_1)=\sum_{k\in I}\int_0^b f_1(-y,k;b;j_1)f_2(u,i;a;dy,k). \tag{2.9}$$

证明: 由过程的轨道性质可知, 当$T_1^0<\infty$时, $\sup_{0\le t<T_1^0} U(t)=\sup_{0\le t<T_1} U(t)$, $\inf_{0\le t<T_1^0} U(t)=\inf_{T_1\le t<T_1^0} U(t)$并且$(T_1<\infty)\triangleq(T_1^0<\infty)$, 这里$\triangleq$表示两个集合的对称差为0, 即若集合$A,B$有$A\triangle B=\{\omega:\omega\in A$且$\omega\notin B$; 或者$\omega\in B$且$\omega\notin A\}$, 则称$A\triangle B$为集合$A,B$的对称差. 对于$a>u$, 有$\mathsf{P}_{u,i}(T_a<\infty)=1$; 当$T_1<\infty$时, $T_1<T_1^0$. 而

$$\begin{aligned}
f_3(u,i;a,b;j_1) &= \mathsf{P}_{u,i}\Big(\sup_{0\le t<T_1^0} U(t)<a, \inf_{0\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1, T_1^0<\infty\Big)\\
&= \mathsf{P}_{u,i}\Big(\sup_{0\le t<T_1} U(t)<a, \inf_{T_1\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1, T_1<\infty\Big)\\
&= \mathsf{P}_{u,i}\Big(T_1<T_a, \inf_{T_1\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1\Big)\\
&= \sum_{k\in I}\mathsf{P}_{u,i}\Big(T_1<T_a, r(T_1)=k, \inf_{T_1\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1\Big),
\end{aligned} \tag{2.10}$$

利用过程$\{(U(t),r(t))\}_{t\ge0}$的强马氏性, 可得

$$\begin{aligned}
&\mathsf{P}_{u,i}\Big(T_1<T_a, r(T_1)=k, \inf_{T_1\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1\Big)\\
=\ &\mathsf{E}_{u,i}\Big[\mathsf{P}_{u,i}\Big(T_1<T_a, r(T_1)=k, \theta_{T_1}\Big(\inf_{0\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1\Big)\Big|\mathcal{F}_{T_1}\Big)\Big]\\
=\ &\mathsf{E}_{u,i}\Big[\mathrm{I}_{\{T_1<T_a, r(T_1)=k\}}\mathsf{P}_{(U(T_1),k)}\Big(\inf_{0\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1\Big)\Big]\\
=\ &\int_0^b \mathsf{P}_{-y,k}\Big(\inf_{0\le t<T_1^0} U(t)>-b, r(T_1^0)=j_1\Big)\\
&\cdot\mathsf{P}_{u,i}(T_1<T_a, r(T_1)=k, U(T_1)\in(-y,-y+dy)),
\end{aligned} \tag{2.11}$$

将(2.11)式代入(2.10)式, 再由引理2.1和引理2.2可知结论得证. $\square$

定理 2.2 当$a>u\ge0$, $b>y>0$时, 若$f(u,i;a,b,n)$是初始盈余为u, 初始环境状态为i, 在末离零点前盈余过程极大值、极小值及零点数的联合分布$\mathsf{P}_{u,i}\big(\sup_{0\le t<L} U(t)<a, \inf_{0\le t<L} U(t)>-b, H=n\big)$, 则有

$$f(u,i;a,b,n)=\sum_{j_1\cdots j_n\in I} f_3(u,i;a,b;j_1)\prod_{m=1}^{n-1} f_3(0,j_m;a,b;j_{m+1})\Phi_{j_n}(0). \tag{2.12}$$

证明: 在$L>0$上, 有$L=T_H^0$, $T_H^0<\infty$, $T_{H+1}^0=\infty$, 则$H=n\Longleftrightarrow T_n^0<\infty$, $T_{n+1}^0=\infty$. 因为

$$\begin{aligned} f(u,i;a,b,n) &= \mathsf{P}_{u,i}\Big(\sup_{0\le t<L}U(t)<a,\ \inf_{0\le t<L}U(t)>-b,H=n\Big)\\ &= \mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,T_{n+1}^0=\infty\Big)\\ &= \sum_{j_n\in I}\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,\\ &\qquad r(T_n^0)=j_n,T_{n+1}^0=\infty\Big), \end{aligned} \tag{2.13}$$

当$T_n^0<\infty$时, $U(T_n^0)=0$, 由过程$\{(U(t),r(t))\}_{t\ge0}$的强马氏性, 有下列等式

$$\begin{aligned} &\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n,T_{n+1}^0=\infty\Big)\\ =\ &\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n,T_{n+1}=\infty\Big)\\ =\ &\mathsf{E}_{u,i}\Big[\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n,\theta_{T_n^0}T_1=\infty\Big|\mathcal{F}_{T_n^0}\Big)\Big]\\ =\ &\mathsf{E}_{u,i}\Big[\mathrm{I}_{\{\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n\}}\mathsf{P}_{0,j_n}(T_1=\infty)\Big]\\ =\ &\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n\Big)\Phi_{j_n}(0), \end{aligned} \tag{2.14}$$

其中,

$$\begin{aligned} &\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_n^0}U(t)<a,\ \inf_{0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n\Big)\\ =\ &\sum_{j_1\cdots j_{n-1}\in I}\mathsf{P}_{u,i}\Big\{\Big(\sup_{0\le t<T_1^0}U(t)<a,\ \inf_{0\le t<T_1^0}U(t)>-b,T_1^0<\infty,r(T_1^0)=j_1\Big)\bigcap\cdots\\ &\bigcap\Big(\sup_{T_{n-2}^0\le t<T_{n-1}^0}U(t)<a,\ \inf_{T_{n-2}^0\le t<T_{n-1}^0}U(t)>-b,T_{n-1}^0<\infty,r(T_{n-1}^0)=j_{n-1}\Big)\\ &\bigcap\Big(\sup_{T_{n-1}^0\le t<T_n^0}U(t)<a,\ \inf_{T_{n-1}^0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n\Big)\Big\}. \end{aligned} \tag{2.15}$$

这里

$$\begin{aligned} &\mathsf{P}_{u,i}\Big\{\Big(\sup_{0\le t<T_1^0}U(t)<a,\ \inf_{0\le t<T_1^0}U(t)>-b,T_1^0<\infty,r(T_1^0)=j_1\Big)\bigcap\cdots\\ &\bigcap\Big(\sup_{T_{n-2}^0\le t<T_{n-1}^0}U(t)<a,\ \inf_{T_{n-2}^0\le t<T_{n-1}^0}U(t)>-b,T_{n-1}^0<\infty,r(T_{n-1}^0)=j_{n-1}\Big)\\ &\bigcap\Big(\sup_{T_{n-1}^0\le t<T_n^0}U(t)<a,\ \inf_{T_{n-1}^0\le t<T_n^0}U(t)>-b,T_n^0<\infty,r(T_n^0)=j_n\Big)\Big\} \end{aligned}$$

$$
\begin{aligned}
= \ & \mathsf{E}_{u,i}\Big[\mathrm{I}_{\{\sup\limits_{0\le t<T_1^0} U(t)<a,\ \inf\limits_{0\le t<T_1^0} U(t)>-b, T_1^0<\infty, r(T_1^0)=j_1\}} \\
& \cdot \mathsf{P}_{0,j_1}\Big\{\Big(\sup_{0\le t<T_1^0} U(t)<a,\ \inf_{0\le t<T_1^0} U(t)>-b, T_1^0<\infty, r(T_1^0)=j_2\Big)\bigcap\cdots \\
& \bigcap\Big(\sup_{T_{n-2}^0\le t<T_{n-1}^0} U(t)<a,\ \inf_{T_{n-1}^0\le t<T_1^0} U(t)>-b, T_{n-1}^0<\infty, r(T_{n-1}^0)=j_n\Big)\Big\}\Big] \\
= \ & \mathsf{P}_{u,i}\Big(\sup_{0\le t<T_1^0} U(t)<a,\ \inf_{0\le t<T_1^0} U(t)>-b, T_1^0<\infty, r(T_1^0)=j_1\Big) \\
& \cdot \mathsf{P}_{0,j_1}\Big\{\Big(\sup_{0\le t<T_1^0} U(t)<a,\ \inf_{0\le t<T_1^0} U(t)>-b, T_1^0<\infty, r(T_1^0)=j_2\Big)\bigcap\cdots \\
& \bigcap\Big(\sup_{T_{n-2}^0\le t<T_{n-1}^0} U(t)<a,\ \inf_{T_{n-2}^0\le t<T_{n-1}^0} U(t)>-b, T_{n-1}^0<\infty, r(T_{n-1}^0)=j_n\Big)\Big\}. \qquad (2.16)
\end{aligned}
$$

由定理2.1知,

$$
\mathsf{P}_{u,i}\Big(\sup_{0\le t<T_1^0} U(t)<a,\ \inf_{0\le t<T_1^0} U(t)>-b, T_1^0<\infty, r(T_1^0)=j_1\Big) = f_3(u,i;a,b;j_1), \qquad (2.17)
$$

则

$$
\begin{aligned}
& \mathsf{P}_{0,j_1}\Big\{\Big(\sup_{0\le t<T_1^0} U(t)<a,\ \inf_{0\le t<T_1^0} U(t)>-b, T_1^0<\infty, r(T_1^0)=j_2\Big)\bigcap\cdots \\
& \bigcap\Big(\sup_{T_{n-2}^0\le t<T_{n-1}^0} U(t)<a,\ \inf_{T_{n-2}^0\le t<T_{n-1}^0} U(t)>-b, T_{n-1}^0<\infty, r(T_{n-1}^0)=j_n\Big)\Big\} \\
= \ & f_3(0,j_1;a,b;j_2) f_3(0,j_2;a,b;j_3)\ldots f_3(0,j_{n-1};a,b;j_n) \\
= \ & \prod_{m=1}^{n-1} f_3(0,j_m;a,b;j_{m+1}). \qquad (2.18)
\end{aligned}
$$

综上(2.13)-(2.18)式, 结论得证. □

特别地, 若$a > u \ge 0$, $b > 0$, 当$N = 1$时, 初始盈余为u, 盈余过程在末离零点前极大值、极小值及零点数的联合分布为经典风险模型中的结果, 即

$$
f(u,a,b,n) = \Phi(0) f_3(u,a,b)(f_3(0,a,b))^{n-1}, \qquad n \ge 1. \qquad (2.19)
$$

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The Joint Distribution of the Supremum, the Infimum and the Number of Zero in the Markov-Modulated Risk Model

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In this paper, when the surplus has negative value, we allow the surplus process to continue. We consider, in the Markov-modulated risk model, the joint distribution of the supremum, the infimum and the number of zero of the surplus process before it leaves zero ultimately.

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