

Revuz Measures under Girsanov Transform

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Abstract

In this paper, we shall study how Revuz measures, energy functional, capacity and Lévy system change under Girsanov transform of Hunt processes.

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§1. Introduction and Preliminaries

Girsanov transform is an important transformation in the theory of Markov processes. It is always interesting and important to explore various kinds of relationships between X and the transformed process of X under Girsanov transform.

We start from a Hunt process $X = (\Omega, \mathcal{F}, \mathcal{F}_t, \theta_t, \mathbf{P}^x)_{t \in (0, +\infty)}$ on the state space (E, \mathcal{E}) with transition semigroup $(P_t)_{t \geq 0}$ and resolvent $(U^q)_{q \geq 0}$. E is, at least, a separable Radon space and \mathcal{E} is the Borel σ -field of E . A cemetery point Δ is adjoined to E as an isolated point of E and $E_\Delta := E \cup \{\Delta\}$, $\mathcal{E}_\Delta := \sigma(\mathcal{E} \cup \{\Delta\})$ (The symbol ‘:=’ is always read as ‘is defined to be’). Let $\zeta := \inf\{t : X_t = \Delta\}$ be the lifetime of X . The filtration $(\mathcal{F}, \mathcal{F}_t)$ is the augmented natural filtration of X . Denote by $\text{Exc}^q(X)$ and $S^q(X)$ ($q \geq 0$) the cones of q -excessive measures and functions, respectively. As usual we write $S := S^0$, $\text{Exc} := \text{Exc}^0$.

A raw additive functional (RAF) is a right continuous process $A = (A_t)$ satisfying $A_0 = 0$ and $A_{t+s} = A_t + A_s \circ \theta_t$ a.s. for all $s, t \geq 0$. An adapted raw AF is simply called an AF and a continuous AF is abbreviated as CAF. Refer to [1] for the theory of AF’s.

Given a excessive function h of X and suppose $0 < h < \infty$, so $h(X_t)$ is a right continuous nonnegative (\mathcal{F}_t) -supermartingale. By Doob-Meyer composition, we have

$$h(X_t) - h(X_0) = M_t^h - N_t^h,$$

where M_t^h is a local martingale, N_t^h is a continuous increasing additive functional.

Let

$$Z_t := \int_0^t \frac{dM_s^h}{h(X_{s-})},$$

which is a martingale additive functional of X . Note that for $t < \zeta$, we have

$$\Delta Z_t = \frac{1}{h(X_{t-})}(M_t^h - M_{t-}^h) = \frac{1}{h(X_{t-})}(h(X_t) - h(X_{t-})) = \frac{h(X_t)}{h(X_{t-})} - 1.$$

Let $Z_t = Z_t^c + Z_t^d$ be the decomposition as continuous and purely discontinuous parts. Denote by L_t the Doléan-Dade exponential martingale of Z_t . Then it admits a representation as the following

$$L_t = \exp\left(Z_t^c - \frac{1}{2}\langle Z^c \rangle_t\right) e^{Z_t^d} \prod_{s \leq t} \frac{h(X_s)}{h(X_{s-})} e^{(h(X_s)/h(X_{s-})-1)} 1_{\{t < \zeta\}},$$

which is a supermartingale of X . Consequently, the formula

$$\frac{dQ^x}{dP^x} \Big|_{\mathcal{F}_t} = L_t$$

uniquely determines a family of probability measure on $(\Omega, \mathcal{F}_\infty)$. It is known that X is a right Markov process on E under these new measures (see [8], Section 62). We will use $(Y, \mathcal{F}, \mathcal{F}_t, Q^x, x \in E)$ to denote the transformed process. Here $Y_t(\omega) = X_t(\omega)$ but we use Y_t for emphasis when working with Q^x . By Itô's formula, we can get

$$L_t = \frac{h(X_t)}{h(X_0)} \exp\left(\int_0^t \frac{dN_s^h}{h(X_s)}\right).$$

Let Q_t, V_t be the semigroup and resolvent of Y , respectively, that is, for any $f \in \mathcal{E}$, we have

$$Q_t f(x) = P^x(f(X_t) L_t) = P^x\left(f(X_t) \frac{h(X_t)}{h(X_0)} \exp\left(\int_0^t \frac{dN_s^h}{h(X_s)}\right)\right), \quad (1.1)$$

$$V^q f(x) = \int_0^\infty e^{-qt} Q_t f(x) dt = P^x\left(\int_0^\infty e^{-qt} f(X_t) \frac{h(X_t)}{h(X_0)} \exp\left(\int_0^t \frac{dN_s^h}{h(X_s)}\right) dt\right).$$

The process Y is called the Girsanov transform of X by $(L_t)_{\{t>0\}}$. When X is a symmetric Markov process and $0 \leq h \in \mathcal{F}$ (the Dirichlet space associated with X), the above transform had been studied in [2], and the Dirichlet form associated with the process Y was given in [2].

In this article, we will get some potential objects under the transform, such as Revuz measures, energy functionals, capacity etc. Many authors have studied the same questions about other transforms, for instance, killing transform (by an decreasing multiplicative functional (MF)), refer to [11]; h -transform, refer to [5]; time-change transform, refer to [6].

This paper is organized as the following. In Section 2, we get the relationships of some potential objects under the Girsanov transform, including Revuz measures, energy functionals and capacity. In Section 3, we get the relationship of Lévy system under the Girsanov transform.

§2. Revuz Measures under Girsanov Transform

In this section, we will get the relationships of some potential objects between the process X and the transformed process Y which is defined in Section 1.

Lemma 2.1 For any $q \geq 0$, if $u \in S^q(Y)$, then $hu \in S^q(X)$.

Proof By the definition of q -excessive function, we have

$$u \in S^q(Y) \iff e^{-qt}Q_t u(x) \leq u(x), \quad \lim_{t \rightarrow 0} e^{-qt}Q_t u(x) = u(x).$$

From (1.1), we have

$$e^{-qt}P_t f(x) = Q^x \left(e^{-qt}f(X_t) \frac{h(X_0)}{h(X_t)} \exp \left(- \int_0^t \frac{dN_s^h}{h(X_s)} \right) \right).$$

Thus it follows that

$$\begin{aligned} e^{-qt}P_t(hu)(x) &= P^x(e^{-qt}h(X_t)u(X_t)) \\ &= Q^x \left(e^{-qt}h(X_t)u(X_t) \frac{h(X_0)}{h(X_t)} \exp \left(- \int_0^t \frac{dN_s^h}{h(X_s)} \right) \right) \\ &= h(x)Q^x \left(e^{-qt}u(X_t) \exp \left(- \int_0^t \frac{dN_s^h}{h(X_s)} \right) \right) \\ &\leq h(x)e^{-qt}Q_t u(x) \\ &\leq h(x)u(x) \end{aligned}$$

and

$$\lim_{t \rightarrow 0} e^{-qt}P_t(hu)(x) = h(x) \lim_{t \rightarrow 0} e^{-qt}Q^x(u(X_t) e^{-\int_0^t dN_s^h/h(X_s)}) = h(x)u(x).$$

Then also by the definition of q -excessive function, we obtain that $hu \in S^q(X)$, which is the desired result. \square

Lemma 2.2 The followings are the relationships of excessive measures between the process X and the process Y .

- (1) If $\eta \in \text{Exc}^q(Y)$, then $(1/h) \cdot \eta \in \text{Exc}^q(X)$, for any $q \geq 0$;
- (2) If $\eta \in \text{Pur}(Y)$, then $(1/h) \cdot \eta \in \text{Pur}(X)$;
- (3) If $\eta = \mu V \in \text{Pot}(Y)$, then $(1/h) \cdot \mu U \in \text{Pot}(X)$;
- (4) If $\eta \in \text{Con}(Y)$, then $(1/h) \cdot \eta \in \text{Con}(X)$.

Refer to [4] for the above notations.

Proof In the following we always assume that $f \in p\mathcal{E}$.

- (1) By the definition of q -excessive measure, we have $\eta \in \text{Exc}^q(Y)$ if and only if $\eta e^{-qt}Q_t \leq \eta$.

Thus it follows that

$$\begin{aligned}
 \frac{1}{h} \eta e^{-qt} P_t f(x) &= \int_E e^{-qt} \mathbf{P}^x(f(X_t)) \frac{1}{h(x)} \eta(\mathrm{d}x) \\
 &= \int_E e^{-qt} \frac{1}{h(x)} \mathbf{Q}^x\left(f(X_t) \frac{h(X_0)}{h(X_t)} \exp\left(-\int_0^t \frac{\mathrm{d}N_s^h}{h(X_s)}\right)\right) \eta(\mathrm{d}x) \\
 &\leq \int_E e^{-qt} \mathbf{Q}^x\left(\frac{f(X_t)}{h(X_t)}\right) \eta(\mathrm{d}x) \leq \eta\left(\frac{f}{h}\right) \\
 &= \frac{1}{h} \eta(f).
 \end{aligned}$$

Thus we get the desired result.

(2) By the definition of pure excessive measure, we have $\eta \in \text{Pur}(Y)$ if and only if $\eta Q_t \rightarrow 0$ as $t \rightarrow \infty$. So

$$\frac{1}{h} \eta P_t f \leq \eta Q_t \left(\frac{f}{h}\right) \rightarrow 0, \quad \text{as } t \rightarrow \infty.$$

Thus we get the desired result.

(3) Since

$$\begin{aligned}
 \frac{1}{h} \mu U f(x) &= \int_E \frac{1}{h(x)} \mathbf{P}^x\left(\int_0^\infty f(X_t) \mathrm{d}t\right) \mu(\mathrm{d}x) \\
 &= \int_E \frac{1}{h(x)} \left(\int_0^\infty \mathbf{Q}^x\left(f(X_t) \frac{h(X_0)}{h(X_t)} \exp\left(-\int_0^t \frac{\mathrm{d}N_s^h}{h(X_s)}\right)\right) \mathrm{d}t\right) \mu(\mathrm{d}x) \\
 &\leq \mu V\left(\frac{f}{h}\right)(x).
 \end{aligned}$$

So $(1/h) \cdot \mu U$ is σ -finite, that is, $(1/h) \cdot \mu U \in \text{Pot}(X)$.

(4) If $f > 0$ and $\eta(f) < \infty$. Since

$$Uf(x) \leq \frac{1}{h(x)} V(hf)(x),$$

thus

$$\{V(hf) < \infty\} \supset \{Uf < \infty\}.$$

Since $\eta \in \text{Con}(Y)$, thus $\eta\{V(hf) < \infty\} = 0$. Then $(1/h) \cdot \eta\{Uf < \infty\} = 0$.

Thus it follows that

$$\frac{1}{h} \eta \in \text{Con}(X). \quad \square$$

The above two lemmas give the relationships of excessive functions and excessive measures. We see that

$$\exp\left(-\int_0^t \frac{\mathrm{d}N_s^h}{h(X_s)}\right) \leq 1$$

plays an important role in the above proofs. And when $\eta \in \text{Inv}(\text{Dis}, \text{Har})$, we can not get $(1/h) \cdot \eta \in \text{Inv}(\text{Dis}, \text{Har})$, which is different with the h-transform, see [5].

Revuz measures were first introduced for ordinary additive functionals in [7]. Let $m \in \text{Exc}(X)$ and $A \in \text{RAF}$. Then the Revuz measure of A relative to m is defined by

$$v_A^m(f) := \uparrow \lim_{t \downarrow 0} \frac{1}{t} \mathbf{P}^m \left(\int_0^t f(X_s) dA_s \right),$$

where $\uparrow (\downarrow)$ means increasing (decreasing) and the following is the same, we shall not repeat it. We know that if $A \in \text{RAF}$ and $m \in \text{Dis}(X)$, we have the following representation

$$v_A^m(f) = L(m, U_A f),$$

where

$$U_A f(x) := \mathbf{P}^x \left(\int_0^\infty f(X_t) dA_t \right).$$

Let ν, μ denote the Revuz measures of X and Y , respectively.

Theorem 2.1 Let $\eta \in \text{Exc}(Y)$ and A be an increasing CAF which is finite on $[0, \zeta]$. Then

$$\mu_A^\eta = h \cdot \nu_A^{(1/h) \cdot \eta}.$$

Proof If

$$A_t = \int_0^t a(X_s) ds,$$

where a is a bounded positive Borel function, then for any $m \in \text{Exc}(X)$, $f \in p\mathcal{E}$, we have

$$\begin{aligned} \nu_A^m(f) &= \lim_{t \rightarrow 0} \frac{1}{t} \mathbf{P}^m \int_0^t f(X_s) a(X_s) ds = \lim_{t \rightarrow 0} \frac{1}{t} \int_0^t m P_s(fa) ds \\ &= \lim_{t \rightarrow 0} m P_t(fa) = m(fa) \\ &= a \cdot m(f). \end{aligned}$$

That is $\nu_A^m = a \cdot m$.

Thus

$$\mu_A^\eta = a \cdot \eta, \quad \nu_A^{(1/h) \cdot \eta} = a \cdot \frac{1}{h} \eta.$$

Then

$$\mu_A^\eta = h \cdot \nu_A^{(1/h) \cdot \eta}.$$

For a general A which is an increasing CAF, define a strictly increasing CAF H by $H_t := A_t + t \wedge \zeta$, then

$$\mu_H^\eta = \mu_A^\eta + \eta := \tilde{\eta}, \quad \nu_H^{(1/h) \cdot \eta} = \nu_A^{(1/h) \cdot \eta} + \frac{1}{h} \eta := \widetilde{\frac{1}{h} \eta}.$$

By time-change, let

$$\tilde{X}_t := X_{H_t^{-1}}, \quad \tilde{Y}_t := Y_{H_t^{-1}}, \quad \tilde{A}_t := A_{H_t^{-1}}.$$

Since $A_t \ll H_t$, then $\tilde{A}_t \ll dt$. By Motoo's theorem, there is a bounded positive Borel function \tilde{a} , such that

$$\tilde{A}_t = \int_0^t \tilde{a}(X_s) ds$$

for all $t \geq 0$ a.s. $\mathbf{P}^{(1/h) \cdot \eta}$. By the discussion in the first part of the proof, we have

$$\tilde{\nu}_{\tilde{A}_t}^{(\widetilde{(1/h) \cdot \eta})} = \tilde{a} \frac{1}{h} \eta = \tilde{a} \left(\nu_A^{(1/h) \cdot \eta} + \frac{1}{h} \eta \right), \quad \tilde{\mu}_{\tilde{A}_t}^{\tilde{\eta}} = \tilde{a} \tilde{\eta} = \tilde{a} (\mu_A^\eta + \eta),$$

where $\tilde{\nu}$ and $\tilde{\mu}$ denote the Revuz measures of \tilde{X} and \tilde{Y} , respectively.

And by time-change (refer to [6]), we have

$$\nu_A^{(1/h) \cdot \eta} = \tilde{\nu}_{\tilde{A}_t}^{(\widetilde{(1/h) \cdot \eta})}, \quad \mu_A^\eta = \tilde{\mu}_{\tilde{A}_t}^{\tilde{\eta}}.$$

Thus it follows that

$$\mu_A^\eta = h \cdot \nu_A^{(1/h) \cdot \eta}. \quad \square$$

This theorem gives the relationship of Revuz measures between X and Y . In the proof, we use the method in [3]. We see that time-change plays an important role in the above proof.

The energy functional L (of X) is defined on $\text{Exc} \times S$ by

$$L(m, u) = \sup\{\mu(u) : \mu \geq 0 \text{ and } \mu U \leq m\}.$$

Refer to [4] for the basic properties of L , among which are $L(\mu U, u) = \mu(u)$ and $L(m, Uf) = m_d(f)$ where m_d is the dissipative part of m .

Let L^X, L^Y denote the energy functional of X and Y , respectively.

Before we get our results, we will explain our ideas as followings. Let X^h denote the h -transform of the process X . Since $0 < h < \infty$, let $P_t^{(h)}$ denote the semigroup of X^h , that is, for $t > 0$,

$$P_t^{(h)} f(x) = \mathbf{P}^x \left(f(X_t) \frac{h(X_t)}{h(X_0)} \right).$$

From (1.1), we see that the process X^h can be seen as the killing process of Y by the decreasing MF \overline{M} , where

$$\overline{M} = \exp \left(- \int_0^t \frac{dN_s^h}{h(X_s)} \right).$$

Since the relationships of energy functional and capacity between the process X and the h -transform process X^h have been known (refer to [5] and [10]), so we can use the process

X^h as a bridge to get the relationships of energy functional and capacity between the process X and the process Y . We will get the relationship between L^X and L^Y in the following Theorem 2.2.

Theorem 2.2 Let $\eta \in \text{Exc}(Y)$, $u \in S(Y)$, then

$$L^X\left(\frac{1}{h}\eta, hu\right) = L^Y(\eta, u) + \nu_{N^h}^{(1/h)\cdot\eta}(u).$$

Proof Due to [5] (4.8), we have

$$L^{X^h}(\eta, u) = L^X\left(\frac{1}{h}\eta, hu\right). \quad (2.1)$$

The process X^h can be seen as the \overline{M} -subprocess of Y , where

$$\overline{M} = \exp\left(-\int_0^t \frac{dN_s^h}{h(X_s)}\right).$$

Due to Theorem 3.3 in [11], we have

$$L^{X^h}(\eta, u) = L^Y(\eta, u) + \mu_{\overline{M}}^\eta(u). \quad (2.2)$$

Thus we have

$$L^X\left(\frac{1}{h}\eta, u\right) = L^Y(\eta, u) + \mu_{\overline{M}}^\eta(u). \quad (2.3)$$

By Theorem 2.1, we have

$$\mu_{\overline{M}}^\eta = h \cdot \nu_{\overline{M}}^{(1/h)\cdot\eta}. \quad (2.4)$$

The Stieltjes logarithm of \overline{M} is

$$[\overline{M}]_t := \int_0^t \frac{d(-\overline{M}_s)}{\overline{M}} = \int_0^t \frac{dN_s^h}{h(X_s)}.$$

Due to [11] Corollary 2.8, we have

$$\nu_{\overline{M}}^{(1/h)\cdot\eta} = \nu_{[\overline{M}]}^{(1/h)\cdot\eta} = \frac{1}{h} \cdot \nu_{N^h}^{(1/h)\cdot\eta}. \quad (2.5)$$

Using (2.2) (2.3) (2.4), it follows that

$$L^X\left(\frac{1}{h}\eta, hu\right) = L^Y(\eta, u) + \nu_{N^h}^{(1/h)\cdot\eta}(u). \quad \square$$

The Hunt's balayage operation R_T^q is defined on Exc^q ($q > 0$). That is, for T is an terminal time,

$$R_T^q m(f) := L^q(m, P_T^q U^q f), \quad \text{for } m \in \text{Exc}^q(X).$$

For $m \in \text{Exc}(X)$,

$$R_T m := \uparrow \lim_{q \downarrow 0} R_T^q m.$$

If $m \in \text{Dis}(X)$, $R_T m(f) = L(m, P_T U f)$ (refer to [4]).

Let $\mathcal{E}^e := \sigma\left(\bigcup_{q \geq 0} S^q\right)$. If $B \in \mathcal{E}^e$ and $m \in \text{Exc}^q$ ($q \geq 0$), we shall write $R_B^q m$ in place of $R_{T_B}^q m$.

For $B \in \mathcal{E}^e$ and $q \geq 0$, the q -capacity of B with $m \in \text{Exc}(X)$ is defined by

$$\begin{aligned}\Gamma_m^q(B) &:= L^q(m, P_B^q 1) = L^q(R_B^q m, 1), \\ \Gamma_m(B) &:= \Gamma^0(B).\end{aligned}$$

An exact terminal time T is called strict if $T \circ \theta_T = 0$ a.s.. In the case that $T = T_B$, the hitting time of $B \in \mathcal{E}^e$, T is strict if and only if $X_T \in B^r$, where B^r is the set of regular points of B . The set B is called strict if T_B is. Let Γ^X , Γ^Y denote the capacity of X and Y , respectively. We are now in a position to get the relationship of capacity between X and Y .

Theorem 2.3 Let $B \in \mathcal{E}$ and be strict, $\eta \in \text{Exc}(Y)$, then

$$\Gamma_\eta^Y(B) + \frac{1}{h} \nu_{N^h}^{(1/h) \cdot R_B \eta}(P_B h) = L^X\left(\frac{1}{h} \eta, P_B h\right) = L^X\left(R_B\left(\frac{1}{h} \eta\right), h\right).$$

Proof Due to X^h is the h -transform process of X , we have

$$\Gamma_\eta^{X^h}(B) = L^{X^h}(\eta, P_B^h 1) = L^X\left(\frac{1}{h} \eta, h P_B^h 1\right) = L^X\left(\frac{1}{h} \eta, P_B h\right) = L^X\left(R_B\left(\frac{1}{h} \eta\right), h\right). \quad (2.6)$$

In the above reasoning, we use (2.1) to get the second equality and use [5] (5.4)(i) to get the third equality.

Since the process X^h is the \overline{M} -subprocess of Y , and by using [11] (4.11), we have

$$\Gamma_\eta^{X^h}(B) = \Gamma_\eta^Y(B) + \mu_{\overline{M}}^{R_B \eta}(Q_{\overline{M}}^B 1). \quad (2.7)$$

By (2.4) and (2.5), we get

$$\mu_{\overline{M}}^\eta = \nu_{N^h}^{(1/h) \cdot \eta} \quad (2.8)$$

and

$$Q_{\overline{M}}^B 1 = \frac{1}{h} P_B h. \quad (2.9)$$

By (2.6), (2.7), (2.8), (2.9), we can easily get the result. \square

§3. Lévy System

A Lévy system for X is a pair (N, H) , where N is a kernel on (E, \mathcal{E}^u) (where \mathcal{E}^u is the σ -algebra of universally measurable subsets of E) with $N(x, \{x\}) = 0$ for any $x \in E$ and

H is a continuous AF of X having bounded 1-potential, such that for any $F \in p\mathcal{E}^u \times \mathcal{E}^u$ vanishing on the diagonal and any predictable process Z , we have

$$\mathbf{P}^x \sum_{0 < s \leq t} Z_s F(X_{s-}, X_s) = \mathbf{P}^x \int_0^t Z_s N F(X_s) dH_s,$$

where

$$N F(x) := \int F(x, y) N(x, dy).$$

Refer to [8] § 73 for the existence of Lévy systems. Now we state the relationship of Lévy system of X and Y .

Theorem 3.1 If (N, H) is a Lévy system of X , then (N', H) is a Lévy system of Y , where

$$N'(x, dy) := N(x, dy) \frac{h(y)}{h(x)}.$$

Proof By the definition of Lévy system, for any $F \in p\mathcal{E}^u$, we have

$$\begin{aligned} \mathbf{Q}^x \sum_{0 < s \leq t} Z_s F(X_{s-}, X_s) &= \mathbf{P}^x \sum_{0 < s \leq t} Z_s F(X_{s-}, X_s) L_s \\ &= \mathbf{P}^x \sum_{0 < s \leq t} Z_s F(X_{s-}, X_s) \frac{h(X_s)}{h(X_0)} e^{\int_0^s dN_r^h / h(X_r)} \\ &= \frac{1}{h(x)} \mathbf{P}^x \sum_{0 < s \leq t} Z_s h(X_{s-}) F(X_{s-}, X_s) \frac{h(X_s)}{h(X_{s-})} e^{\int_0^s dN_r^h / h(X_r)} \\ &= \frac{1}{h(x)} \mathbf{P}^x \int_0^t Z_s h(X_{s-}) e^{\int_0^s dN_r^h / h(X_r)} N' F(X_s) dH_s \\ &= \mathbf{P}^x \int_0^t Z_s \frac{h(X_s)}{h(X_0)} e^{\int_0^s dN_r^h / h(X_r)} N' F(X_s) dH_s \\ &= \mathbf{Q}^x \int_0^t Z_s N' F(X_s) dH_s. \end{aligned}$$

Then also by the definition of Lévy system, we have that (N', H) is a Lévy system of the process Y . \square

In the above proof, we use the method in [9] for proving Theorem 3.4.

By Theorem 3.1, we can see that the relationship of Lévy system between the process Y and the process X is the same as the relationship of Lévy system between the process X^h and the process X (refer to [10]). When X is a symmetric Markov process and $0 \leq h \in \mathcal{E}$ (the Dirichlet space of X), the relationship of Lévy system between the process Y and the process X is the same as the above theorem, see [2] Lemma 2.9.

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Girsanov变换下的Revuz测度

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在本文中, 我们将研究Hunt过程在Girsanov变换下的Revuz测度、能量泛函、容量以及Lévy系是如何变换的.

关键词: Girsanov变换, Revuz测度, 能量泛函, 容量, Lévy系.

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