应用概率统计　第二十七卷
第六期　2011年12月

Chinese Journal of Applied Probability
and Statistics Vol.27 No.6 Dec. 2011

随机环境中马氏链的强大数定律 *

吴艳蕾　　吴小太

(安徽工程大学数学系, 芜湖, 241000)

摘　要

本文在随机环境的条件下, 借助鞅差序列的收敛定理, 给出随机环境中的马氏链的强极限定理, 并由此导出了随机环境中马氏链的强大数定律, 拓宽了已有文献中部分定理的适用范围.

关键词: 随机环境, 马氏链, 强大数定律.

学科分类号: O211.6.

§1. 引　　言

设$(S,\mathcal{A})$与$(T,\mathcal{B})$均为任意可测空间, $\overrightarrow{X}=\{X_n, n=0,1,\ldots\}$与$\overrightarrow{\xi}=\{\xi_n, n=0,1,\ldots\}$是概率空间$(\Omega,\mathcal{F},\mathsf{P})$上分别取值于$S$与$T$的随机变量序列. 对$\forall A\in\mathcal{A}$, $n\geq 0$, 若有

$$\mathsf{P}(X_0\in A|\overrightarrow{\xi}\,)=\mathsf{P}(X_0\in A|\xi_0), \tag{1.1}$$

$$\mathsf{P}(X_{n+1}\in A|\overrightarrow{X}{}_0^n,\overrightarrow{\xi}\,)=\mathsf{P}(\xi_n;X_n,A), \tag{1.2}$$

则称$\overrightarrow{X}$为随机环境$\overrightarrow{\xi}$中的马氏链, $\overrightarrow{\xi}$为环境序列. 若$\overrightarrow{\xi}$为马氏序列, 则称$\overrightarrow{X}$为马氏环境$\overrightarrow{\xi}$中的马氏链, 其中$\overrightarrow{X}{}_k^r=\{X_n, k\leq n\leq r\}$.

自上世纪80年代R. Cogburn等[1–3]人开始随机环境中的马氏链一般理论的研究以来, 随机环境中的马氏链就引起了国内学者的广泛关注, 戴永隆, 李应求, 王汉兴等对这一方向进行了深入的研究, 并取得了一系列深刻而丰富的成果[4–7]. 近年来在马氏环境中马氏链的强大数定律方面的研究也取得了较大的进展[8–11]. 本文在随机环境的条件下, 借助刘文与杨卫国在文献[12]中研究任意随机变量序列的方法, 给出随机环境下的马氏链的强大数定律. 文献[8]在马氏环境中给出了马氏链的强大数定律, 本文仅在随机环境的条件下得到了类似的结论, 从而拓宽了该文献中的部分定理的适用范围. 最后, 在假定双链序列$\{(X_n,\xi_n), n\geq 0\}$为$*$-混合序列的条件下, 讨论了随机环境中马氏链的强大数定律.

*国家自然科学基金项目(71171003)、安徽省自然科学青年基金项目(10040606Q03)和安徽工程大学校青年基金项目(2007YQ025)资助.

本文2009年4月9日收到.

§2. 主要结果

引理 2.1 $\overrightarrow{X}$为随机环境$\overrightarrow{\xi}$中的马氏链, $f(x)$为S上有界实可测函数, $\mathcal{F}_n=\sigma\{\overrightarrow{X}_0^n, \overrightarrow{\xi}_0^n\}$, 则

$$\mathsf{E}[f(X_n)|\mathcal{F}_{n-1}]=\mathsf{E}[f(X_n)|X_{n-1},\xi_{n-1}] \qquad \text{a.s..} \tag{2.1}$$

证明: 由(1.2)知, 对$\forall A\in\mathcal{A}$, 有

$$\begin{aligned}\mathsf{E}[I_A(X_n)|\mathcal{F}_{n-1}] &= \mathsf{E}[\mathsf{E}[I_A(X_n)|\mathcal{F}_{n-1}]|\vec{X}_0^{n-1},\vec{\xi}]\\ &= \mathsf{E}[\mathsf{E}[I_A(X_n)|X_{n-1},\xi_{n-1}]|\mathcal{F}_{n-1}]\\ &= \mathsf{P}(\xi_n;X_n,A),\end{aligned} \tag{2.2}$$

故当$f(x)=I_A(x)$时(2.1)成立. 参照马氏链的等价定理[13, P.20]的证明方法, 即可得(2.1). □

定理 2.1 设$\overrightarrow{X}$为随机环境$\overrightarrow{\xi}$中的马氏链, $f_n(x)$为S上实可测函数列. $\{a_n,n\geq 0\}$是递增的正数序列且$a_n\uparrow\infty$. 设$\{\varphi_n(x),n\geq 0\}$是一列定义在R上的非负偶函数, 当$|x|\uparrow$时,

$$\frac{\varphi_n(x)}{|x|}\uparrow, \qquad \frac{\varphi_n(x)}{x^2}\downarrow. \tag{2.3}$$

设

$$\sum_{n=1}^{\infty}\mathsf{E}[\varphi_n(f_n(X_n))]/\varphi_n(a_n)<\infty, \tag{2.4}$$

则

$$\lim_{n\to\infty} a_n^{-1}\sum_{k=1}^{n}\{f_k(X_k)-\mathsf{E}[f_k(X_k)|X_{k-1},\xi_{k-1}]\}=0 \qquad \text{a.s..} \tag{2.5}$$

证明: 设$f_n^*(X_n)=f_n(X_n)I(|f_n(X_n)|\leq a_n)$, 由(2.3), (2.4)有

$$\begin{aligned}\sum_{n=1}^{\infty}\mathsf{P}(f_n(X_n)\neq f_n^*(X_n)) &= \sum_{n=1}^{\infty}\mathsf{E}I(|f_n(X_n)|>a_n)\\ &\leq \sum_{n=1}^{\infty}\mathsf{E}[\varphi_n(f_n(X_n))]/\varphi_n(a_n)<\infty \qquad \text{a.s..}\end{aligned} \tag{2.6}$$

由(2.6)可得

$$\mathsf{P}(|f_n(X_n)|>a_n,\ \text{i.o})=0,$$

从而有

$$\sum_{n=1}^{\infty}\frac{1}{a_n}[f_n(X_n)-f_n^*(X_n)]<\infty \qquad \text{a.s..} \tag{2.7}$$

设

$$Y_n=\frac{\{f_n^*(X_n)-\mathsf{E}[f_n^*(X_n)|X_{n-1},\xi_{n-1}]\}}{a_n},$$

由$f_n^*(X_n)/a_n \le 1$有界, 由引理2.1有$\{Y_n, \mathcal{F}_n, n \ge 1\}$为鞅差序列, 其中$\mathcal{F}_n = \sigma\{\overrightarrow{X}_0^n, \overrightarrow{\xi}_0^n\}$. 注意到

$$\begin{aligned} \mathsf{E}[Y_n^2|\mathcal{F}_{n-1}] &= \{\mathsf{E}[(f_n^*(X_n))^2|\mathcal{F}_{n-1}] - (\mathsf{E}[f_n^*(X_n)|\mathcal{F}_{n-1}])^2\}/a_n^2 \\ &\le \mathsf{E}[(f_n^*(X_n))^2|\mathcal{F}_{n-1}]/a_n^2 \\ &= \mathsf{E}\Big[\Big(\frac{f_n^*(X_n)}{a_n}\Big)^2\Big|X_{n-1}, \xi_{n-1}\Big] \qquad \text{a.s..} \end{aligned} \tag{2.8}$$

由(2.3)有

$$\begin{aligned} \mathsf{E}\Big[\Big(\frac{f_n^*(X_n)}{a_n}\Big)^2\Big|X_{n-1}, \xi_{n-1}\Big] &\le \mathsf{E}\Big[\frac{\varphi_n(f_n^*(X_n))}{\varphi_n(a_n)}\Big|X_{n-1}, \xi_{n-1}\Big] \\ &\le \mathsf{E}\Big[\frac{\varphi_n(f_n(X_n))}{\varphi_n(a_n)}\Big|X_{n-1}, \xi_{n-1}\Big]. \end{aligned} \tag{2.9}$$

由(2.4)有

$$\mathsf{E}\Big\{\sum_{n=1}^{\infty} \mathsf{E}\Big[\frac{\varphi_n(f_n(X_n))}{\varphi_n(a_n)}\Big|X_{n-1}, \xi_{n-1}\Big]\Big\} = \sum_{n=1}^{\infty} \mathsf{E}\Big[\frac{\varphi_n(f_n(X_n))}{\varphi_n(a_n)}\Big] < \infty,$$

可得

$$\sum_{n=1}^{\infty} \mathsf{E}\Big[\frac{\varphi_n(f_n(X_n))}{\varphi_n(a_n)}\Big|X_{n-1}, \xi_{n-1}\Big] < \infty \qquad \text{a.s..} \tag{2.10}$$

由(2.8), (2.9)与(2.10)有

$$\sum_{i=1}^{\infty} \mathsf{E}[Y_n^2|\mathcal{F}_{n-1}] < \infty \qquad \text{a.s..} \tag{2.11}$$

由(2.11)与鞅差序列的收敛定理有

$$\sum_{n=1}^{\infty} Y_n = \sum_{n=1}^{\infty} \frac{\{f_n^*(X_n) - \mathsf{E}[f_n^*(X_n)|X_{n-1}, \xi_{n-1}]\}}{a_n} < \infty \qquad \text{a.s..} \tag{2.12}$$

事实上

$$\begin{aligned} &|\{\mathsf{E}[f_n(X_n)|X_{n-1}, \xi_{n-1}] - \mathsf{E}[f_n^*(X_n)|X_{n-1}, \xi_{n-1}]\}/a_n| \\ \le\ & \mathsf{E}[|f_n(X_n) - f_n^*(X_n)|/a_n|X_{n-1}, \xi_{n-1}] \\ =\ & \mathsf{E}\Big[\frac{|f_n(X_n)|}{a_n} I(|f_n(X_n)| > a_n)\Big|X_{n-1}, \xi_{n-1}\Big] \\ \le\ & \mathsf{E}[(\varphi_n(f_n(X_n))|X_{n-1}, \xi_{n-1}]/\varphi_n(a_n), \end{aligned} \tag{2.13}$$

故由(2.10)与(2.13)有

$$\sum_{n=1}^{\infty} \{\mathsf{E}[f_n(X_n)|X_{n-1}, \xi_{n-1}] - \mathsf{E}[f_n^*(X_n)|X_{n-1}, \xi_{n-1}]\}/a_n < \infty \qquad \text{a.s..} \tag{2.14}$$

由(2.7), (2.12)与(2.14), 有

$$\sum_{n=1}^{\infty} \{f_n(X_n) - \mathsf{E}[f_n(X_n)|X_{k-1}, \xi_{k-1}]\}/a_n < \infty \qquad \text{a.s..} \tag{2.15}$$

由kronecker引理, 即得(2.5). □

推论 2.1 设$\overrightarrow{X}$为随机环境$\overrightarrow{\xi}$中的马氏链, $f_n(x)$为S上实可测函数列. $\varphi_n(x)$与a_n如定理2.1中定义, 设

$$\sum_{n=1}^{\infty} \mathsf{E}[\varphi_n(f_n(X_n))]/\varphi_n(a_n) < \infty, \tag{2.16}$$

则对$\forall m \ge 1$, 有

$$\lim_{n\to\infty} a_n^{-1} \sum_{k=1}^{n} \{f_k(X_k) - \mathsf{E}[f_k(X_k)|X_{k-m}, \xi_{k-m}]\} = 0 \qquad \text{a.s..} \tag{2.17}$$

这里约定X_{-m}, ξ_{-m}, $m \ge 1$恒为常数.

证明: 由定理2.1知(2.17)在$m=1$时成立. 由(2.16)有

$$\sum_{n=1}^{\infty} \frac{\mathsf{E}[\varphi_{nm+t}(f_{nm+t}(X_{nm+t}))]}{\varphi_{nm+t}(a_{nm+t})} < \infty, \qquad t = 0, 1, \ldots, m-1.$$

仿照定理2.1的证明, 有

$$\sum_{n=1}^{\infty} \frac{1}{a_{nm+t}} \{f_{nm+t}(X_{nm+t}) - \mathsf{E}[f_{nm+t}(X_{nm+t})|X_{(n-1)m+t}, \xi_{(n-1)m+t}]\} < \infty \qquad \text{a.s..} \tag{2.18}$$

由(2.18)有

$$\begin{aligned}
& \sum_{n=m}^{\infty} a_n^{-1}[f_n(X_n) - \mathsf{E}(f_n(X_n)|X_{n-m}, \xi_{n-m})] \\
\le\ & \sum_{t=0}^{m-1} \sum_{n=1}^{\infty} \frac{1}{a_{nm+t}} \{f_{nm+t}(X_{nm+t}) - \mathsf{E}[f_{nm+t}(X_{nm+t})|X_{(n-1)m+t}, \xi_{(n-1)m+t}]\} \\
<\ & \infty \qquad \text{a.s..}
\end{aligned} \tag{2.19}$$

故由kronecker引理, 即得(2.17). $\square$

定理 2.2 设$\overrightarrow{X}$为随机环境$\overrightarrow{\xi}$中的马氏链, $f_n(x)$为S上一致有界的实可测函数列, 即$\exists M$对$\forall n \ge 0$, 有$|f_n(x)| \le M$. 若对$\forall k \ge 1$,

$$\lim_{m\to\infty} \sup_{k,(x_1,\theta_1),(x_2,\theta_2)} \|P^{(k,k+m)}(x_1;\theta_1,.) - P^{(k,k+m)}(x_2;\theta_2,.)\|, \tag{2.20}$$

则

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} [f_k(X_k) - \mathsf{E} f_k(X_k)] = 0 \qquad \text{a.s..} \tag{2.21}$$

证明: 在推论2.1中, 令$\varphi_n(x) = x^2$, $a_n = n$, 由

$$\sum_{n=1}^{\infty} \frac{\mathsf{E}[(f_n(X_n))^2]}{n^2} \le M^2 \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty,$$

对$\forall m \ge 1$, 由推论2.1有

$$\lim_{n\to\infty} n^{-1} \sum_{k=1}^{n} \{f_k(X_k) - \mathsf{E}[f_k(X_k)|X_{k-m}, \xi_{k-m}]\} = 0 \qquad \text{a.s..} \tag{2.22}$$

记$\mathsf{P}(X_{n+m}\in A|\xi_n;X_n)=P^{(n,n+m)}(\xi_n;X_n,A)$, $\overline{P}_n(A)=\mathsf{P}(X_n\in A)$, $P_n(A;B)=\mathsf{P}(X_n\in A,\xi_n\in B)$. 注意到

$$\begin{aligned}
&|\mathsf{E}[f_{k+m}(X_{k+m})|X_k,\xi_k]-\mathsf{E}f_{k+m}(X_{k+m})|\\
=\ &\Big|\int P^{(k,k+m)}(X_k;\xi_k,x)f_{k+m}(x)\mathrm{d}x-\int \overline{P}_{k+m}(x)f_{k+m}(x)\mathrm{d}x\Big|\\
\le\ &\sup_{(x_1,\theta_1)}\Big|\int [P^{(k,k+m)}(x_1;\theta_1,x)-\overline{P}_{k+m}(x)]f_{k+m}(x)\mathrm{d}x\Big|\\
\le\ &M\sup_{(x_1,\theta_1)}\|P^{(k,k+m)}(x_1;\theta_1,\cdot)-\overline{P}_{k+m}(\cdot)\|, \qquad (2.23)
\end{aligned}$$

与

$$\begin{aligned}
&\sup_{(x_1,\theta_1)}\|P^{(k,k+m)}(x_1;\theta_1,\cdot)-\overline{P}_{k+m}(\cdot)\|\\
=\ &\sup_{(x_1,\theta_1)}\Big\|P^{(k,k+m)}(x_1;\theta_1,\cdot)-\int P^{(k,k+m)}(x;\theta,\cdot)P_k(\mathrm{d}x;\mathrm{d}\theta)\Big\|\\
=\ &\sup_{(x_1,\theta_1)}\Big\|\int [P^{(k,k+m)}(x_1;\theta_1,\cdot)-P^{(k,k+m)}(x;\theta,\cdot)]P_k(\mathrm{d}x;\mathrm{d}\theta)\Big\|\\
\le\ &\sup_{(x_1,\theta_1),(x_2,\theta_2)}\|P^{(k,k+m)}(x_1;\theta_1,\cdot)-P^{(k,k+m)}(x_2;\theta_2,\cdot)\|. \qquad (2.24)
\end{aligned}$$

由(2.23), (2.24), 故有

$$\begin{aligned}
&\limsup_m\limsup_n\Big|\frac{1}{n}\sum_{k=1}^n\{\mathsf{E}[f_k(X_k)|X_{k-m},\xi_{k-m}]-\mathsf{E}f_k(X_k)\}\Big|\\
\le\ &M\limsup_m\limsup_n\sup_{(x_1,\theta_1),(x_2,\theta_2)}\frac{1}{n}\sum_{k=1}^n\sup_k\|(P^{(k,k+m)}(x_1;\theta_1,\cdot)-P^{(k,k+m)}(x_2;\theta_2,\cdot)\|\\
\le\ &M\limsup_m\sup_k\sup_{(x_1,\theta_1),(x_2,\theta_2)}\|(P^{(k,k+m)}(x_1;\theta_1,\cdot)-P^{(k,k+m)}(x_2;\theta_2,\cdot)\|=0. \qquad (2.25)
\end{aligned}$$

当$k<m$时, 由推论2.1中的约定知, (2.25)式成立. 由于

$$\begin{aligned}
\Big|\frac{1}{n}\sum_{k=1}^n[f_k(X_k)-\mathsf{E}f_k(X_k)]\Big| \le\ &\Big|\frac{1}{n}\sum_{k=1}^n[f_k(X_k)-\mathsf{E}[f_k(X_k)|X_{k-m},\xi_{k-m}]\Big|\\
&+\Big|\frac{1}{n}\sum_{k=1}^n[\mathsf{E}[f_k(X_k)|X_{k-m},\xi_{k-m}]-\mathsf{E}f_k(X_k)]\Big|, \qquad (2.26)
\end{aligned}$$

故由(2.22), (2.25)与(2.26), 即得(2.21). □

定理 2.3 设$\vec{X}$为随机环境$\vec{\xi}$中的马氏链, $f(x)$为S上有界实可测函数(不妨设$|f(x)|\le M$). 若

$$\lim_{N\to\infty}\sup_{k,(x_1,\theta_1)}\Big\|\frac{1}{N}\sum_{m=1}^N P^{(k,k+m)}(x_1;\theta_1,\cdot)-\overline{\mathsf{P}}(\cdot)\Big\|=0, \qquad (2.27)$$

其中$\overline{\mathsf{P}}$为$\mathcal{A}$上的概率测度, 则

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}f(X_k)=\int\overline{\mathsf{P}}(x)f(x)\mathrm{d}x\qquad\text{a.s.}.\tag{2.28}$$

证明: 对$\forall m\geq 1$, 由$|f(x)|\leq M$有

$$\mathsf{E}\Big[\sum_{n=1}^{\infty}\frac{\mathsf{E}[f_{n+m}(X_{n+m})|X_n,\xi_n]^2}{n^2}\Big]\leq\sum_{n=1}^{\infty}\frac{\mathsf{E}[f_{n+m}(X_{n+m})]^2}{n^2}<\infty\qquad\text{a.s.},\tag{2.29}$$

可得

$$\lim_{n\to\infty}\frac{\mathsf{E}[f_{n+m}(X_{n+m})|X_n,\xi_n]}{n}=0\qquad\text{a.s.}.\tag{2.30}$$

由(2.22)与(2.30), 有

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\{f_k(X_k)-\mathsf{E}[f_{k+m}(X_{k+m})|X_k,\xi_k]\}=0\qquad\text{a.s.}.\tag{2.31}$$

对$\forall N\geq 1$, 由(2.31)有

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\Big\{f(X_k)-\frac{1}{N}\sum_{m=1}^{N}\mathsf{E}[f(X_{k+m})|X_k,\xi_k]\Big\}=0\qquad\text{a.s.}.\tag{2.32}$$

事实上

$$\begin{aligned}
&\Big|\frac{1}{N}\sum_{m=1}^{N}\mathsf{E}[f(X_{k+m})|X_k,\xi_k]-\int\overline{\mathsf{P}}(x)f(x)\mathrm{d}x\Big|\\
\leq\ &\Big|\frac{1}{N}\sum_{m=1}^{N}\int f(x)P^{(k,k+m)}(X_k;\xi_k,x)\mathrm{d}x-\int\overline{\mathsf{P}}(x)f(x)\mathrm{d}x\Big|\\
\leq\ &M\int\Big|\frac{1}{N}\sum_{m=1}^{N}P^{(k,k+m)}(X_k;\xi_k,x)-\overline{\mathsf{P}}(x)\Big|\mathrm{d}x\\
\leq\ &M\sup_{k,(x_1,\theta_1)}\int\Big|\frac{1}{N}\sum_{m=1}^{N}P^{(k,k+m)}(x_1;\theta_1,x)-\overline{\mathsf{P}}(x)\Big|\mathrm{d}x\\
\leq\ &M\sup_{k,(x_1,\theta_1)}\Big\|\frac{1}{N}\sum_{m=1}^{N}P^{(k,k+m)}(x_1;\theta_1,\cdot)-\overline{\mathsf{P}}(\cdot)\Big\|.
\end{aligned}\tag{2.33}$$

当$N\to\infty$时, 由(2.33)有

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\Big\{\frac{1}{N}\sum_{m=1}^{N}\mathsf{E}[f(X_{k+m})|X_k,\xi_k]-\int\overline{\mathsf{P}}(x)f(x)\mathrm{d}x\Big\}=0\qquad\text{a.s.}.\tag{2.34}$$

由(2.32)与(2.34), 即得(2.28). □

定义 2.1[12] 设$\{X_n,n\geq 0\}$为一随机变量序列, $\mathcal{F}_n^m=\sigma(X_n,X_{n+1},\ldots,X_m)$. 若存在正整数$N$, $\psi(n)$为定义在整数$n\geq N$上的函数且$\lim\limits_{n\to\infty}\psi(n)=0$. 设对$\forall n\geq N$, $m\geq 1$, $A\in\mathcal{F}_0^m$与$B\in\mathcal{F}_{m+n}^{\infty}$, 有

$$|\mathsf{P}(AB)-\mathsf{P}(A)\mathsf{P}(B)|\leq\psi(n)\mathsf{P}(A)\mathsf{P}(B),\tag{2.35}$$

则称$\{X_n, n \geq 0\}$为$*$-混合序列. 由(2.35)有

$$|\mathsf{E}[X_{n+m}|\mathcal{F}_0^m] - \mathsf{E}X_{n+m}| \leq \psi(n)\mathsf{E}|X_{n+m}| \qquad \text{a.s..} \tag{2.36}$$

设双链序列$\{(X_n, \xi_n), n \geq 0\}$为$*$-混合序列, $f_n(x)$为S上的实可测函数列, 由(2.1)与(2.36), 对$\forall m \geq 1$有

$$|\mathsf{E}[f_{n+m}(X_{n+m})|X_n, \xi_n] - \mathsf{E}[f_{n+m}(X_{n+m})]| \leq \psi(m)\mathsf{E}|f_n(X_{n+m})| \qquad \text{a.s..} \tag{2.37}$$

定理 2.4 设$\overrightarrow{X}$为随机环境$\overrightarrow{\xi}$中的马氏链, 序列$\{(X_n, \xi_n), n \geq 0\}$为$*$-mixing序列, $f_n(x)$为S上的实可测函数列, $\varphi_n(x)$与a_n如定理2.1中定义. 若

$$\sum_{n=1}^{\infty} \mathsf{E}[\varphi_n(f_n(X_n))]/\varphi_n(a_n) < \infty, \tag{2.38}$$

$$\sup_n \frac{1}{a_n} \sum_{k=1}^{n} \mathsf{E}|f_k(X_k)| < \infty, \tag{2.39}$$

则

$$\lim_{n\to\infty} \frac{1}{a_n} \sum_{k=1}^{n} [f_k(X_k) - \mathsf{E}f_k(X_k)] = 0 \qquad \text{a.s..} \tag{2.40}$$

证明: 注意到

$$\begin{aligned} \Big|\frac{1}{a_n} \sum_{k=1}^{n} \{f_k(X_k) - \mathsf{E}f_k(X_k)\}\Big| \leq & \Big|\frac{1}{a_n} \sum_{k=1}^{n} \{f_k(X_k) - \mathsf{E}[f_k(X_k)|\mathcal{F}_{k-m}]\}\Big| \\ & + \Big|\frac{1}{a_n} \sum_{k=1}^{n} \{\mathsf{E}[f_k(X_k)|\mathcal{F}_{k-m}] - \mathsf{E}f_k(X_k)\}\Big|. \end{aligned} \tag{2.41}$$

由(2.38)与推论2.1, 对$\forall m \geq 1$有

$$\lim_{n\to\infty} \frac{1}{a_n} \sum_{k=1}^{n} \{f_k(X_k) - \mathsf{E}[f_k(X_k)|\mathcal{F}_{k-m}]\} = 0 \qquad \text{a.s..} \tag{2.42}$$

由(2.37)有

$$\Big|\frac{1}{a_n} \sum_{k=1}^{n} \{\mathsf{E}[f_k(X_k)|\mathcal{F}_{k-m}] - \mathsf{E}f_k(X_k)\}\Big| \leq \frac{1}{a_n} \sum_{k=1}^{n} \varphi(m)\mathsf{E}|f_k(X_k)|. \tag{2.43}$$

由(2.39)与$\lim\limits_{m\to\infty} \psi(m) = 0$, 有

$$\lim_{m\to\infty} \Big|\frac{1}{a_n} \sum_{k=1}^{n} \{\mathsf{E}[f_k(X_k)|\mathcal{F}_{k-m}] - \mathsf{E}f_k(X_k)\}\Big| = 0 \qquad \text{a.s..} \tag{2.44}$$

由(2.41), (2.42)与(2.44), 即得(2.40). □

参 考 文 献

[1] Cogburn, R., The ergodic theory of Markov chains in random environments, *Z. Wahrsch. Verw. Gebiete*, **66(2)**(1984), 109–128.

[2] Cogburn, R., Markov chains in random environments: the case of Markovian environment, *Ann. Prob.*, **8(3)**(1980), 908–916.

[3] Cogburn, R., On the central limit theorem for Markov chains in random environments, *Ann. Prob.*, **19(2)**(1991), 587–604.

[4] 王汉兴, 戴永隆, 马氏环境中马氏链的Poisson极限律, 数学学报, **40(2)**(1997), 265–270.

[5] 李应求, 关于马氏环境中马氏链的几个注记, 数学进展, **28(4)**(1999), 358–360.

[6] 李应求, 王苏明, 胡杨利, 随机环境中马氏链与马氏双链间的相互关系, 数学学报, **49(6)**(2006), 1373–1380.

[7] 方大凡, 马氏环境中马氏链的Shannon-McMillan-Breiman定理, 应用概率统计, **16(3)**(2000), 295–298.

[8] 郭明乐, 随机环境中马氏链的强大数定律, 应用概率统计, **20(2)**(2004), 154–160.

[9] 万成高, 马氏环境中马氏链的强大数定律, 应用概率统计, **19(2)**(2003), 155–160.

[10] 郭明乐, 马氏环境中马氏链的强大定律, 应用数学, **16(4)**(2003), 143–148.

[11] 李应求, 状态可数的马氏环境中马氏链的强大数定律, 数学杂志, **23(4)**(2003), 484–490.

[12] Liu, W. and Yang, W.G., A class of strong limit theorems for the sequences of arbitrary random variables, *Stat. Probab. Letts.*, **64**(2003), 121–131.

[13] 王梓坤, 杨向群, 生灭过程与马尔可夫链, 北京: 科学出版社, 2005.

The Strong Law of Large Numbers for Markov Chains in Random Environments

Wu Yanlei Wu Xiaotai

(*Department of Mathematics, Anhui Polytechnic University, Wuhu, 241000*)

In this paper, strong limit theorem of Markov chains is discussed by the martingale difference convergence theorem in the random environments, by which the strong laws of large number of Markov chains are got. Some laws we got broaden the using area of some results already have.

Keywords: Random environments, Markov chains, strong law of large numbers.

AMS Subject Classification: 60F15.