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On the Stochastic Restricted Liu Estimator under Misspecification due to Inclusion of Some Superfluous Variables *

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Abstract

In this paper, we firstly derived the expressions of the well-known ordinary least square estimator (OLSE), the ordinary mixed estimator (OME) introduced by Theil and Golberger (1961) and the stochastic restricted Liu estimator (SRLE) proposed by Yang and Xu (2007) under misspecification due to inclusion of some superfluous variables. Then, performances of these estimators under misspecification are examined. In particular, necessary and sufficient conditions for the superiority of the SRLE over the OLSE and OME with respect to the mean squared error matrix (MSEM) criterion are derived. Furthermore, superiority of the corresponding predictors of these estimators are also investigated.

Keywords: Misspecified linear model, stochastic restrictions, stochastic restricted Liu estimator, mean squared error matrix.

AMS Subject Classification: 62J05.

§1. Introduction

In regression analysis, prior information about the unknown parameters of interest is often available, such as the form of linear stochastic restrictions. In this case, the mixed estimation procedure proposed by Durbin (1953), Theil and Golberger (1961) and Theil (1963, 1971) has received a considerable attention in literature for its simplicity and applications.

However, as pointed out by Madhulike (1999), the regression model may be misspecified for one reason or another in practice and the misspecification of the regression model is a very serious problem in econometric theory. In general, researchers are often concerned with two types of misspecification: excluding some relevant variables and including some superfluous variables, where these two problems are treated separately. For

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the misspecification related to exclusion of some relevant variables, Kadiyala (1986) studied the performance of OME under misspecification and demonstrated that it performs better than the OLSE with respect to the MSEM criterion. Trenkler and Wijekoon (1989) and Wijekoon and Trenkler (1989) derived the conditions under which OME outperforms OLSE in the MSEM sense. Gross et al. (1998) investigated the superiority of the misspecified restricted least squares regression estimator. Hubert and Wijekoon (2004) studied the superiority of the stochastic restricted Liu estimator.

As to the misspecification related to inclusion of some superfluous variables, Fomby (1981) analyzed the impact of such misspecification on the efficiency of the OLSE. Dube. et al. (1991) investigated the efficiency relation between the OLSE and Steil-rule estimator. Madhulike (1999) obtained the conditions for the superiority of OME over the OLSE. The purpose of this paper is to examine the performance of the SRLE by Yang and Xu (2007) in comparison to the OLSE and OME when the regression model is misspecified due to inclusion of some superfluous variables. Furthermore, superiority of the corresponding predictors are also investigated with respect to the MSEM criterion.

The paper is organized as follows. Some notations and lemmas needed in the following discussions are given in Section 2, and model specification and the estimators are derived in Section 3. Then, performances of the SRLE and its corresponding predictor under misspecification respect to the MSEM criterion are examined in Section 4 and Section 5, respectively.

§2. Some Lemmas

In this section, for the convenience of the following proof, we list a few notations and lemmas which are needed in the following discussions. For a matrix M, $M>0$ denotes M is positive semidefinite, and $M\geq 0$ denotes M is positive definite. $\Re(M)$ denotes the range of the matrix M and I_n denotes the $n\times n$ identity matrix.

Lemma 2.1 (Rao and Toutenburg, 1995) Let matrices $A\geq 0$, $B>0$, $\widetilde{\Lambda}=\operatorname{diag}(\lambda_i^B(A))$ denotes the diagonal matrix of the eigenvalues of A in the metric of B, that is $\lambda_i^B(A)$ is the i-th eigenvalue of the matrix $B^{-1}A$, then exists a nonsingular matrix W, such that $A=W'\widetilde{\Lambda}W$, $B=W'W$.

Lemma 2.2 (Wang, 1987) For a partitioned regular matrix $A=\begin{pmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}$, where A_{11} is an $n_1\times n_1$ matrix, A_{12} is an $n_1\times n_2$ matrix, A_{21} is an $n_2\times n_1$ matrix and A_{22} is an $n_2\times n_2$ matrix. If matrices A_{22} and $A_{11.2}=A_{11}-A_{12}A_{22}^{-1}A_{21}$ are regular, then the partitioned inverse is given by $A^{-1}=\begin{pmatrix} A_{11.2}^{-1} & -A_{11.2}^{-1}A_{12}A_{22}^{-1} \\ -A_{22}^{-1}A_{21}A_{11.2}^{-1} & A_{22}^{-1}+A_{22}^{-1}A_{21}A_{11.2}^{-1}A_{12}A_{22}^{-1} \end{pmatrix}$.

Lemma 2.3 (Baksalary and Kala, 1983) Let matrix $A\geq 0$ and α be a column vector, then the matrix $A-\alpha\alpha'\geq 0$ if and only if $\alpha\in\Re(A)$, $\alpha' A^{-}\alpha\leq 1$, where A^{-} is any g-inverse of A.

§3. Model Specification and the Estimators

We firstly assume the correctly specified multiple linear regression model is given by

$$y=X\beta+u, \tag{3.1}$$

where y is an $n\times 1$ vector of observations on the response variable, X is an $n\times p$ full column rank matrix of n observations on the p explanatory variables, β is a $p\times 1$ vector of regression coefficients, u is an $n\times 1$ disturbance vector assumed to having mean vector 0 and covariance matrix $\sigma^2 I_n$. In this paper, we mainly focus on the case when σ^2 is known, while for the case when σ^2 is unknown, we may replace it with its appropriate estimator for practical use. In addition, suppose that the misspecification relates to the inclusion of q superfluous variables in (3.1) so that the misspecified model is given by

$$y=X\beta+Z\alpha+\varepsilon=D\delta+\varepsilon, \tag{3.2}$$

where Z is an $n\times q$ matrix of n observations on q wrongly included variables and α is the $q\times 1$ coefficient vector associated with them, $D=\begin{pmatrix} X & Z\end{pmatrix}$, $\delta=\begin{pmatrix}\beta\\ \alpha\end{pmatrix}$. The disturbance vector ε in (3.2) is also assumed to have mean vector 0 and covariance matrix $\sigma^2 I_n$.

Furthermore, suppose we have some prior knowledge on the coefficients β in the following form

$$r=R\beta+\nu=\widetilde{R}\delta+\nu, \tag{3.3}$$

where r is a $j\times 1$ stochastic known vector, R is a $j\times p$ matrix of rank $j(\leq p)$ with known elements and $\widetilde{R}=\begin{pmatrix} R & 0\end{pmatrix}$ is a $j\times(p+q)$ matrix. The disturbance vector ν is independent of the disturbance terms u,ε and is assumed to have mean vector 0 and covariance matrix $\psi>0$. Then, the well-known OLSE, OME by Theil and Golberger (1961) and SRLE by Yang and Xu (2007) of the coefficient vector β in (3.1) are respectively given by

$$\widehat{\beta}_{\rm OLSE}=(X'X)^{-1}X'y, \tag{3.4}$$

$$\widehat{\beta}_{\rm OME}=(X'X+\sigma^2R'\psi^{-1}R)^{-1}(X'y+\sigma^2R'\psi^{-1}r), \tag{3.5}$$

$$\widehat{\beta}_{\rm SRLE}=(X'X+\sigma^2R'\psi^{-1}R)^{-1}(F_dX'y+\sigma^2R'\psi^{-1}r), \tag{3.6}$$

where $F_d = (X'X+I)^{-1}(X'X+dI) = I-(1-d)(X'X+I)^{-1}$, $0<d<1$.

In order to derive the corresponding estimators under misspecification due to inclusion of some superfluous variables, we firstly consider the simultaneous spectral decomposition of the matrices $D'D$ and $\widetilde{R}'\psi^{-1}\widetilde{R}$. Since $D'D>0$ and $\widetilde{R}'\psi^{-1}\widetilde{R}\geq 0$, by Lemma 2.1 we know there exists the non-singular matrix $T=(D'D)^{-1/2}P$, such that $T'D'DT=I$, $T'\widetilde{R}'\psi^{-1}\widetilde{R}T=\Lambda$, where P is an orthogonal matrix such that

$$P'[(D'D)^{-1/2}\widetilde{R}'\psi^{-1}\widetilde{R}(D'D)^{-1/2}]P=\Lambda,$$

and Λ is a $(p+q)\times(p+q)$ diagonal matrix with elements $\lambda_i>0$, $i=1,\ldots,j$; $\lambda_i=0$, $i=j+1,\ldots,p+q$. Let $D_*=DT$, $\widetilde{R}_*=\widetilde{R}T$, $\gamma=T^{-1}\delta$, we can get that $D_*'D_*=I$, $\widetilde{R}_*'\psi^{-1}\widetilde{R}_*=\Lambda$, and the model (3.2) and (3.3) can be rewritten as

$$y=D_*\gamma+\varepsilon, \tag{3.7}$$

$$r=\widetilde{R}_*\gamma+\nu. \tag{3.8}$$

Let $\widetilde{F}_d=(D_*'D_*+I)^{-1}(D_*'D_*+dI)=k_1I$, $k_1=(1+d)/2$, then we can get that the OLSE, OME and SRLE of γ for the transformed model (3.7) are

$$\widehat{\gamma}_{\rm OLSE} = (D_*'D_*)^{-1}D_*'y=T'D'y, \tag{3.9}$$

$$\begin{aligned}\widehat{\gamma}_{\rm OME} &= (D_*'D_*+\sigma^2\widetilde{R}_*'\psi^{-1}\widetilde{R}_*)^{-1}(D_*'y+\sigma^2\widetilde{R}_*'\psi^{-1}r)\\ &= (I+\sigma^2\Lambda)^{-1}T'(D'y+\sigma^2\widetilde{R}'\psi^{-1}r),\end{aligned} \tag{3.10}$$

$$\begin{aligned}\widehat{\gamma}_{\rm SRLE} &= (D_*'D_*+\sigma^2\widetilde{R}_*'\psi^{-1}\widetilde{R}_*)^{-1}(D_*'y+\sigma^2\widetilde{R}_*'\psi^{-1}r)\\ &= (I+\sigma^2\Lambda)^{-1}T'(k_1D'y+\sigma^2\widetilde{R}'\psi^{-1}r).\end{aligned} \tag{3.11}$$

Let $W_1=\begin{pmatrix} I_p & 0\end{pmatrix}_{p\times(p+q)}$, $W_2=\begin{pmatrix} 0 & I_q\end{pmatrix}_{q\times(p+q)}$, we have $\beta=W_1\delta=W_1T\gamma$, $\alpha=W_2\delta=W_2T\gamma$. So the corresponding OLSE, OME and SRLE for the coefficient vector β under misspecification relates to inclusion of some superfluous variable are:

$$\widetilde{\beta}_{\rm OLSE}=W_1T\widehat{\gamma}_{\rm OLSE}=W_1TT'D'y, \tag{3.12}$$

$$\widetilde{\beta}_{\rm OME}=W_1T\widehat{\gamma}_{\rm OME}=W_1T(I+\sigma^2\Lambda)^{-1}T'(D'y+\sigma^2\widetilde{R}'\psi^{-1}r), \tag{3.13}$$

$$\widetilde{\beta}_{\rm SRLE}=W_1T\widehat{\gamma}_{\rm SRLE}=W_1T(I+\sigma^2\Lambda)^{-1}T'(k_1D'y+\sigma^2\widetilde{R}'\psi^{-1}r). \tag{3.14}$$

Since $T=(D'D)^{-1/2}P$, we can get that $TT'=(D'D)^{-1}$, $(T^{-1})'T^{-1}=D'D$ and $P\Lambda P'=(D'D)^{-1/2}\widetilde{R}'\psi^{-1}\widetilde{R}(D'D)^{-1/2}$, $(T^{-1})'\Lambda T^{-1}=(D'D)^{1/2}P\Lambda P'(D'D)^{1/2}=\widetilde{R}'\psi^{-1}\widetilde{R}$. So

$$T(I+\sigma^2\Lambda)^{-1}T'=[(T^{-1})'T^{-1}+\sigma^2(T^{-1})'\Lambda T^{-1}]^{-1}=(D'D+\sigma^2\widetilde{R}'\psi^{-1}\widetilde{R})^{-1}$$

and the estimators (3.12), (3.13) and (3.14) can be rewritten as

$$\begin{aligned}\widetilde{\beta}_{\mathrm{OLSE}} &= W_1TT'D'y = W_1(D'D)^{-1}D'y, && (3.15)\\ \widetilde{\beta}_{\mathrm{OME}} &= W_1T(I+\sigma^2\Lambda)^{-1}T'(D'y+\sigma^2\widetilde{R}'\psi^{-1}r)\\ &= W_1(D'D+\sigma^2\widetilde{R}'\psi^{-1}\widetilde{R})^{-1}(D'y+\sigma^2\widetilde{R}'\psi^{-1}r), && (3.16)\\ \widetilde{\beta}_{\mathrm{SRLE}} &= W_1T(I+\sigma^2\Lambda)^{-1}T'(k_1D'y+\sigma^2\widetilde{R}'\psi^{-1}r)\\ &= W_1(D'D+\sigma^2\widetilde{R}'\psi^{-1}\widetilde{R})^{-1}(k_1D'y+\sigma^2\widetilde{R}'\psi^{-1}r). && (3.17)\end{aligned}$$

Let $(D'D)^{-1}=\begin{pmatrix} D^{11} & D^{12}\\ D^{21} & D^{22}\end{pmatrix}$, $(D'D+\sigma^2\widetilde{R}'\psi^{-1}\widetilde{R})^{-1}=\begin{pmatrix} \widetilde{D}^{11} & \widetilde{D}^{12}\\ \widetilde{D}^{21} & \widetilde{D}^{22}\end{pmatrix}$, then by Lemma 2.2 we can compute that

$$\begin{aligned}&D^{11}=(X'\overline{P}_ZX)^{-1},\\ &D^{12}=-(X'\overline{P}_ZX)^{-1}X'Z(Z'Z)^{-1},\\ &D^{21}=-(Z'Z)^{-1}Z'X(X'\overline{P}_ZX)^{-1},\\ &D^{22}=(Z'Z)^{-1}+(Z'Z)^{-1}Z'X(X'\overline{P}_ZX)^{-1}X'Z(Z'Z)^{-1},\\ &\widetilde{D}^{11}=(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1},\\ &\widetilde{D}^{12}=-(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}X'Z(Z'Z)^{-1},\\ &\widetilde{D}^{21}=-(Z'Z)^{-1}Z'X(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1},\\ &\widetilde{D}^{22}=(Z'Z)^{-1}+(Z'Z)^{-1}Z'X(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}X'Z(Z'Z)^{-1},\end{aligned}$$

where $\overline{P}_Z=I-P_Z$, $P_Z=Z(Z'Z)^{-1}Z'$. Therefore, from (3.15), (3.16) and (3.17) we can get that

$$\begin{aligned}\widetilde{\beta}_{\mathrm{OLSE}} &= W_1\begin{pmatrix} D^{11} & D^{12}\\ D^{21} & D^{22}\end{pmatrix}\begin{pmatrix} X'y\\ Z'y\end{pmatrix}\\ &= W_1\begin{pmatrix} (X'\overline{P}_ZX)^{-1}X'P_Zy\\ (Z'Z)^{-1}Z'[y-X(X'\overline{P}_ZX)^{-1}X'\overline{P}_Zy]\end{pmatrix}\\ &= (X'\overline{P}_ZX)^{-1}X'\overline{P}_Zy, && (3.18)\\ \widetilde{\beta}_{\mathrm{OME}} &= W_1\begin{pmatrix} \widetilde{D}^{11} & \widetilde{D}^{12}\\ \widetilde{D}^{21} & \widetilde{D}^{22}\end{pmatrix}\begin{pmatrix} X'y+\sigma^2R'\psi^{-1}r\\ Z'y\end{pmatrix}\\ &= W_1\begin{pmatrix} (X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}(X'\overline{P}_Zy+\sigma^2R'\psi^{-1}r)\\ (Z'Z)^{-1}Z'[y-X(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}(X'\overline{P}_Zy+\sigma^2R'\psi^{-1}r)]\end{pmatrix}\\ &= (X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}(X'\overline{P}_Zy+\sigma^2R'\psi^{-1}r), && (3.19)\end{aligned}$$

$$\begin{aligned}
\widetilde{\beta}_{\mathrm{SRLE}} &= W_1 \begin{pmatrix} \widetilde{D}^{11} & \widetilde{D}^{12} \\ \widetilde{D}^{21} & \widetilde{D}^{22} \end{pmatrix} \begin{pmatrix} k_1 X'y + \sigma^2 R'\psi^{-1} r \\ k_1 Z'y \end{pmatrix} \\
&= W_1 \begin{pmatrix} (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}(k_1 X'\overline{P}_Z y + \sigma^2 R'\psi^{-1} r) \\ (Z'Z)^{-1}Z'[y - X(X'\overline{P}_Z X + \sigma^2 \widetilde{R}'\psi^{-1}\widetilde{R})^{-1}(k_1 X'\overline{P}_Z y + \sigma^2 R'\psi^{-1} r)] \end{pmatrix} \\
&= (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}(k_1 X'\overline{P}_Z y + \sigma^2 R'\psi^{-1} r). \qquad (3.20)
\end{aligned}$$

Similarly, we can compute the OLSE, OME and SRLE of the coefficient vector α under misspecification as follows:

$$\begin{aligned}
\widetilde{\alpha}_{\mathrm{OLSE}} &= W_2 (D'D)^{-1} D'y \\
&= W_2 \begin{pmatrix} \widetilde{\beta}_{\mathrm{OLSE}} \\ (Z'Z)^{-1}Z'(y - X\widetilde{\beta}_{\mathrm{OLSE}}) \end{pmatrix} = (Z'Z)^{-1}Z'(y - X\widetilde{\beta}_{\mathrm{OLSE}}), \quad (3.21) \\
\widetilde{\alpha}_{\mathrm{OME}} &= W_2 (D'D + \sigma^2 \widetilde{R}'\psi^{-1}\widetilde{R})^{-1}(D'y + \sigma^2 \widetilde{R}'\psi^{-1} r) \\
&= W_2 \begin{pmatrix} \widetilde{\beta}_{\mathrm{OME}} \\ (Z'Z)^{-1}Z'(y - X\widetilde{\beta}_{\mathrm{OME}}) \end{pmatrix} = (Z'Z)^{-1}Z'(y - X\widetilde{\beta}_{\mathrm{OME}}), \quad (3.22) \\
\widetilde{\alpha}_{\mathrm{SRLE}} &= W_2 (D'D + \sigma^2 \widetilde{R}'\psi^{-1}\widetilde{R})^{-1}(k_1 D'y + \sigma^2 \widetilde{R}'\psi^{-1} r) \\
&= W_2 \begin{pmatrix} \widetilde{\beta}_{\mathrm{SRLE}} \\ (Z'Z)^{-1}Z'(y - X\widetilde{\beta}_{\mathrm{SRLE}}) \end{pmatrix} = (Z'Z)^{-1}Z'(y - X\widetilde{\beta}_{\mathrm{SRLE}}). \quad (3.23)
\end{aligned}$$

By some straightforward calculations, we can get the expectation vectors and covariance matrices of the OLSE, OME and SRLE for β under misspecification are

$$\begin{aligned}
\mathsf{E}(\widetilde{\beta}_{\mathrm{OLSE}}) &= (X'\overline{P}_Z X)^{-1} X'\overline{P}_Z \mathsf{E}(y) = \beta, \\
\mathsf{E}(\widetilde{\beta}_{\mathrm{OME}}) &= (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}[X'\overline{P}_Z \mathsf{E}(y) + \sigma^2 R'\psi^{-1}\mathsf{E}(r)] = \beta, \\
\mathsf{E}(\widetilde{\beta}_{\mathrm{SRLE}}) &= (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}[k_1 X'\overline{P}_Z \mathsf{E}(y) + \sigma^2 R'\psi^{-1}\mathsf{E}(r)] \\
&= (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}(k_1 X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)\beta \\
&= \beta - (1 - k_1)(X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1} X'\overline{P}_Z X\beta, \\
\mathsf{D}(\widetilde{\beta}_{\mathrm{OLSE}}) &= (X'\overline{P}_Z X)^{-1} X'\overline{P}_Z \cdot \sigma^2 I_n \cdot \overline{P}_Z X (X'\overline{P}_Z X)^{-1} = \sigma^2 (X'\overline{P}_Z X)^{-1}, \\
\mathsf{D}(\widetilde{\beta}_{\mathrm{OME}}) &= \sigma^2 (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}, \\
\mathsf{D}(\widetilde{\beta}_{\mathrm{SRLE}}) &= \sigma^2 (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}(k_1^2 X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R) \\
&\quad \cdot (X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1}.
\end{aligned}$$

So the bias vectors and the MSEM of the OLSE, OME and SRLE are

$$\mathrm{Bias}(\widetilde{\beta}_{\mathrm{OLSE}}) = \mathrm{Bias}(\widetilde{\beta}_{\mathrm{OME}}) = 0,$$

$$b_1 = \mathrm{Bias}(\widetilde{\beta}_{\mathrm{SRLE}}) = -(1 - k_1)(X'\overline{P}_Z X + \sigma^2 R'\psi^{-1} R)^{-1} X'\overline{P}_Z X\beta,$$

and

$$\begin{aligned}
\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{OLSE}}) &= \sigma^2(X'\overline{P}_ZX)^{-1}, && (3.24)\\
\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{OME}}) &= \sigma^2(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}, && (3.25)\\
\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{SRLE}}) &= \sigma^2(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}(k_1^2X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)\\
&\quad\cdot(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}+b_1b_1'. && (3.26)
\end{aligned}$$

§4. Performances of the Estimators under Misspecification

To examine superiority of the SRLE $\widetilde{\beta}_{\mathrm{SRLE}}$ over the OLSE $\widetilde{\beta}_{\mathrm{OSLE}}$ and the OME $\widetilde{\beta}_{\mathrm{OME}}$ for the misspecified model related to inclusion of some superfluous variables, we investigate the following differences:

$$\Delta_1=\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{OLSE}})-\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{SRLE}})=D_1-b_1b_1', \tag{4.1}$$

$$\Delta_2=\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{OME}})-\mathrm{MSEM}(\widetilde{\beta}_{\mathrm{SRLE}})=D_2-b_1b_1', \tag{4.2}$$

where

$$\begin{aligned}
D_1 &= \sigma^2[(X'\overline{P}_ZX)^{-1}-(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}\\
&\quad\cdot(k_1^2X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}]\\
&= \sigma^2(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}A_1(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1},\\
D_2 &= \sigma^2[(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}-(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}\\
&\quad\cdot(k_1^2X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}]\\
&= \sigma^2(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1}A_2(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)^{-1},
\end{aligned}$$

and

$$\begin{aligned}
A_1 &= (X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)(X'\overline{P}_ZX)^{-1}(X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)\\
&\quad-(k_1^2X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)\\
&= (1-k_1^2)X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R+\sigma^4R'\psi^{-1}R(X'\overline{P}_ZX)^{-1}R'\psi^{-1}R,\\
A_2 &= (X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)-(k_1^2X'\overline{P}_ZX+\sigma^2R'\psi^{-1}R)\\
&= (1-k_1^2)X'\overline{P}_ZX.
\end{aligned}$$

For $0<d<1$ and $k_1=(1+d)/2$, we have that $1/2<k_1<1$, $0<1-k_1^2<3/4$, so it's obvious that $A_1>0$, $A_2>0$, which implies that $D_1>0$, $D_2>0$.

4.1 Comparison between the SRLE and OLSE

Since $b_1 \in \Re(D_1)$, by Lemma 2.3, we know that $\Delta_1 = D_1 - b_1 b_1' \geq 0$ if and only if $b_1' D_1^{-1} b_1 \leq 1$, so the following theorem is obtained:

Theorem 4.1 The SRLE dominates the OLSE under misspecification respect to the MSEM criterion if and only if $b_1' D_1^{-1} b_1 \leq 1$.

4.2 Comparison between the SRLE and OME

Since $b_1 \in \Re(D_2)$, by Lemma 2.3, we know that $\Delta_2 = D_2 - b_1 b_1' \geq 0$ if and only if $b_1' D_2^{-1} b_1 \leq 1$, so the following theorem is obtained:

Theorem 4.2 The SRLE dominates the OME under misspecification respect to the MSEM criterion if and only if $b_1' D_2^{-1} b_1 \leq 1$.

Remark 1 From Theorem 4.1 and Theorem 4.2, we can conclude that there are situations where the SRLE outperforms the OLSE and OME with respect to the MSEM criterion under misspecification due to inclusion of some superfluous variables.

§5. Performances of the Predictors under Misspecification

In order to examine performances of the predictors, we assume that the model (3.1) holds for further realizations of the dependent variable, that is y_0, u_0 are $m \times 1$ vectors and X_0 is an $m \times p$ matrix such that

$$y_0 = X_0 \beta + u_0, \tag{5.1}$$

where X_0 is known and the disturbance vectors u_0 and u are independent. Then, the misspeicified model due to inclusion of q superfluous variables can be written as

$$y_0 = X_0 \beta + Z_0 \alpha + \varepsilon_0, \tag{5.2}$$

where Z_0 is known and both the disturbance vectors ε_0 and u_0 are assumed to have mean vector 0 and variance covariance matrix $\sigma^2 I_m$.

Now from (3.18)-(3.23), we can get that the ordinary least square predictor (OLSP), the ordinary mixed predictor (OMP) and the stochastic restricted Liu predictor (SRLP) under misspecification can be defined as

$$\widetilde{y}_{\mathrm{OLSP}} = X_0 \widetilde{\beta}_{\mathrm{OLSE}} + Z_0 \widetilde{\alpha}_{\mathrm{OLSE}} = W_0 \widetilde{\beta}_{\mathrm{OLSE}} + Z_0 (Z'Z)^{-1} Z' y, \tag{5.3}$$

$$\widetilde{y}_{\mathrm{OMP}} = X_0 \widetilde{\beta}_{\mathrm{OME}} + Z_0 \widetilde{\alpha}_{\mathrm{OME}} = W_0 \widetilde{\beta}_{\mathrm{OME}} + Z_0 (Z'Z)^{-1} Z' y, \tag{5.4}$$

$$\widetilde{y}_{\mathrm{SRLP}} = X_0 \widetilde{\beta}_{\mathrm{SRLE}} + Z_0 \widetilde{\alpha}_{\mathrm{SRLE}} = W_0 \widetilde{\beta}_{\mathrm{SRLE}} + Z_0 (Z'Z)^{-1} Z' y, \tag{5.5}$$

where $W_0 = X_0 - Z_0(Z'Z)^{-1}Z'X$. Since $\overline{P}_Z Z = Z - Z(Z'Z)^{-1}Z'Z = 0$, we have

$$\mathsf{Cov}\,(\widetilde{\beta}_{\rm OLSE}, Z_0(Z'Z)^{-1}Z'y) = \sigma^2(X'\overline{P}_Z X)^{-1}X'\overline{P}_Z Z(Z'Z)^{-1}Z_0' = 0,$$
$$\mathsf{Cov}\,(\widetilde{\beta}_{\rm OME}, Z_0(Z'Z)^{-1}Z'y) = \sigma^2(X'\overline{P}_Z X + \sigma^2 R'\psi^{-1}R)^{-1}X'\overline{P}_Z Z(Z'Z)^{-1}Z_0' = 0,$$
$$\mathsf{Cov}\,(\widetilde{\beta}_{\rm SRLE}, Z_0(Z'Z)^{-1}Z'y) = \sigma^2 k_1(X'\overline{P}_Z X + \sigma^2 R'\psi^{-1}R)^{-1}X'\overline{P}_Z Z(Z'Z)^{-1}Z_0' = 0.$$

So from (5.3), (5.4) and (5.5) we can calculate the covariance matrices of the OLSP, OMP and SRLP are

$$\mathsf{D}(\widetilde{y}_{\rm OLSP}) = W_0\mathsf{D}(\widetilde{\beta}_{\rm OLSE})W_0' + \sigma^2 Z_0(Z'Z)^{-1}Z_0', \tag{5.6}$$
$$\mathsf{D}(\widetilde{y}_{\rm OMP}) = W_0\mathsf{D}(\widetilde{\beta}_{\rm OME})W_0' + \sigma^2 Z_0(Z'Z)^{-1}Z_0', \tag{5.7}$$
$$\mathsf{D}(\widetilde{y}_{\rm SRLP}) = W_0\mathsf{D}(\widetilde{\beta}_{\rm SRLE})W_0' + \sigma^2 Z_0(Z'Z)^{-1}Z_0'. \tag{5.8}$$

On the other hand, since

$$\begin{aligned}
\mathsf{E}(\widetilde{y}_{\rm OLSP}) &= X_0\mathsf{E}(\widetilde{\beta}_{\rm OLSE}) + Z_0\mathsf{E}(\widetilde{\alpha}_{\rm OLSE}) \\
&= X_0\mathsf{E}(\widetilde{\beta}_{\rm OLSE}) + Z_0(Z'Z)^{-1}Z'[\mathsf{E}(y) - X\mathsf{E}(\widetilde{\beta}_{\rm OLSE})] = X_0\mathsf{E}(\widetilde{\beta}_{\rm OLSE}), \\
\mathsf{E}(\widetilde{y}_{\rm OMP}) &= X_0\mathsf{E}(\widetilde{\beta}_{\rm OME}) + Z_0\mathsf{E}(\widetilde{\alpha}_{\rm OME}) \\
&= X_0\mathsf{E}(\widetilde{\beta}_{\rm OME}) + Z_0(Z'Z)^{-1}Z'[\mathsf{E}(y) - X\mathsf{E}(\widetilde{\beta}_{\rm OME})] = X_0\mathsf{E}(\widetilde{\beta}_{\rm OME}), \\
\mathsf{E}(\widetilde{y}_{\rm SRLP}) &= X_0\mathsf{E}(\widetilde{\beta}_{\rm SRLE}) + Z_0\mathsf{E}(\widetilde{\alpha}_{\rm SRLE}) \\
&= X_0\mathsf{E}(\widetilde{\beta}_{\rm SRLE}) + (Z'Z)^{-1}Z'[\mathsf{E}(y) - X\mathsf{E}(\widetilde{\beta}_{\rm SRLE})] \\
&= X_0\mathsf{E}(\widetilde{\beta}_{\rm SRLE}) - (Z'Z)^{-1}Z'X\mathrm{Bias}(\widetilde{\beta}_{\rm SRLE}),
\end{aligned}$$

we have

$$\mathrm{Bias}(\widetilde{y}_{\rm OLSP}) = X_0\mathsf{E}(\widetilde{\beta}_{\rm OLSE}) - X_0\beta = X_0\mathrm{Bias}(\widetilde{\beta}_{\rm OLSE}) = W_0\mathrm{Bias}(\widetilde{\beta}_{\rm OLSE}),$$
$$\mathrm{Bias}(\widetilde{y}_{\rm OMP}) = X_0\mathsf{E}(\widetilde{\beta}_{\rm OME}) - X_0\beta = X_0\mathrm{Bias}(\widetilde{\beta}_{\rm OME}) = W_0\mathrm{Bias}(\widetilde{\beta}_{\rm OME}),$$
$$\mathrm{Bias}(\widetilde{y}_{\rm SRLP}) = X_0\mathrm{Bias}(\widetilde{\beta}_{\rm SRLE}) - (Z'Z)^{-1}Z'X\mathrm{Bias}(\widetilde{\beta}_{\rm SRLE}) = W_0\mathrm{Bias}(\widetilde{\beta}_{\rm SRLE}).$$

Therefore, the MSEM of the OLSP, OMP and SRLP are

$$\begin{aligned}
\mathrm{MSEM}(\widetilde{y}_{\rm OLSP}) &= \mathsf{D}(\widetilde{y}_{\rm OLSP}) + \mathrm{Bias}(\widetilde{y}_{\rm OLSP})\mathrm{Bias}(\widetilde{y}_{\rm OLSP})' \\
&= W_0\mathrm{MSEM}(\widetilde{\beta}_{\rm OLSE})W_0' + \sigma^2 Z_0(Z'Z)^{-1}Z_0', \\
\mathrm{MSEM}(\widetilde{y}_{\rm OMP}) &= \mathsf{D}(\widetilde{y}_{\rm OMP}) + \mathrm{Bias}(\widetilde{y}_{\rm OMP})\mathrm{Bias}(\widetilde{y}_{\rm OMP})' \\
&= W_0\mathrm{MSEM}(\widetilde{\beta}_{\rm OME})W_0' + \sigma^2 Z_0(Z'Z)^{-1}Z_0', \\
\mathrm{MSEM}(\widetilde{y}_{\rm SRLP}) &= \mathsf{D}(\widetilde{y}_{\rm SRLP}) + \mathrm{Bias}(\widetilde{y}_{\rm SRLP})\mathrm{Bias}(\widetilde{y}_{\rm SRLP})' \\
&= W_0\mathrm{MSEM}(\widetilde{\beta}_{\rm SRLE})W_0' + \sigma^2 Z_0(Z'Z)^{-1}Z_0'.
\end{aligned}$$

5.1 Comparison between the SRLP and OLSP

In order to compare the SRLP and OLSP with respect to the MSEM criterion, we consider the following difference

$$\Delta_3 = \mathrm{MSEM}(\widetilde{y}_{\mathrm{OLSP}}) - \mathrm{MSEM}(\widetilde{y}_{\mathrm{SRLP}}) = W_0\Delta_1 W_0'.$$

So it's obvious that if $\Delta_1 \geq 0$, then $\Delta_3 \geq 0$ and the following results is obtained:

Theorem 5.1 Under the conditions derived in Theorem 4.1, namely if the SRLE dominates the OLSE with respect to the MSEM criterion, then the SRLP $\widehat{y}_{\mathrm{SRLP}}$ dominates the OLSP $\widehat{y}_{\mathrm{OLSP}}$ with respect to the MSEM criterion.

5.2 Comparison between the SRLP and OMP

Let's similarly investigate the following difference

$$\Delta_4 = \mathrm{MSEM}(\widetilde{y}_{\mathrm{OMP}}) - \mathrm{MSEM}(\widetilde{y}_{\mathrm{SRLP}}) = W_0\Delta_2 W_0'.$$

Obviously if $\Delta_2 \geq 0$, then $\Delta_4 \geq 0$ and we may obtain:

Theorem 5.2 Under the conditions derived in Theorem 4.2, namely if the SRLE dominates the OME with respect to the MSEM criterion, then the SRLP $\widehat{y}_{\mathrm{SRLP}}$ dominates the OMP $\widehat{y}_{\mathrm{OMP}}$ with respect to the MSEM criterion.

Remark 2 The results obtained in Theorem 5.1 and Theorem 5.2 show that when the SRLE is potentially superior the OLSE and OME in the sense of MSEM, then the corresponding SRLP will also dominate the OLSP and OMP under the same conditions.

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包含多余回归变量的错误指定模型的随机约束Liu估计

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对由于包含多余回归自变量而导致的错误指定线性回归模型, 本文导出了回归系数的最小二乘估计, 普通混合估计以及随机约束Liu估计, 并在均方误差矩阵准则下对这三个估计的优良性进行了比较, 给出了随机约束Liu估计优于最小二乘估计和普通混合估计的充要条件. 此外, 对它们所对应的经典预测值的优良性也进行了讨论.

关键词: 错误指定线性模型, 随机约束, 随机约束Liu估计, 均方误差矩阵.

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