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Strong Convergence Laws for $\widetilde{\varphi}$-Mixing Sequences of Random Variables *

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Abstract

In this paper, the almost sure convergence and complete convergence for $\widetilde{\varphi}$-mixing random variables are established. The results obtained not only extend and generalize the classical Khintchine-Kolmogorov Convergence Theorem, the Three Series Theorem for independent random variables to the case of $\widetilde{\varphi}$-mixing random variables, but also improve the relevant results without necessarily adding any extra any conditions.

Keywords: $\widetilde{\varphi}$-mixing random variables, almost sure convergence, complete convergence, weighted sums.

AMS Subject Classification: 60F15.

§1. Introduction

In many stochastic models, the assumption of independence among random variables is not plausible. So it is necessary to extend the concept of independence to dependence cases, one of these dependence structures is $\widetilde{\varphi}$-mixing. So we want to know whether the results obtained for independent and identically distributed (i.i.d.) random variables are still true for $\widetilde{\varphi}$-mixing random variables.

Let (Ω, F, P) be a probability space. Let $\{X_n; n \geq 1\}$ be a sequence of random variables that we deal with defined on (Ω, F, P), and let F_n^m denote the σ-algebra generated by the random variables $X_n, X_{n+1}, \ldots, X_m$. Let $S, T \subset N$ be nonempty sets, and define $F_S = \sigma(X_i; i \in S \subset N)$. Given two σ-algebras ψ, ζ in F, note that

$$\varphi(\psi, \zeta) = \sup\{|\mathsf{P}(B|A) - \mathsf{P}(B)|; A \in \psi, \mathsf{P}(A) > 0, B \in \zeta\}, \tag{1.1}$$

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and define the $\widetilde{\varphi}$-mixing coefficients by

$$\widetilde{\varphi}(n) = \sup\{\varphi(F_S, F_T);\ \text{finite subsets } S, T \subset N \text{ such that } \mathrm{dist}(S, T) \ge n\}, \quad n \ge 0. \quad (1.2)$$

Obviously, $0 \le \widetilde{\varphi}(n+1) \le \widetilde{\varphi}(n) \le 1$, $n \ge 0$ and $\widetilde{\varphi}(0) = 1$.

Definition 1.1 A sequence of random variable $\{X_n; n \ge 1\}$ is said to be a $\widetilde{\varphi}$-mixing random variable sequence if there exists $k \in N$ such that $\widetilde{\varphi}(k) < 1$.

Note that if $\{X_n; n \ge 1\}$ is a sequence of independent random variables, then $\widetilde{\varphi}(n) = 0$ for all $n \ge 1$.

$\widetilde{\varphi}$-mixing is similar to φ-mixing, but they are quite different from each other. A large number of limit results for $\widetilde{\varphi}$-mixing sequences of random variables have been established by many researchers. We refer to Wu and Lin (2004) for the complete convergence theorem and strong law of large numbers, Wang and Hu et al. (2008) for the strong law of large numbers and growth rate, Wang and Hu et al. (2009) for the convergence properties about the partial sum, Jiang and Wu (2010) for weak convergence and complete convergence. When these are compared with the corresponding results of independent random variable sequences, there still remains much to be desired.

Here we give an example of the practical application of $\widetilde{\varphi}$-mixing.

Example 1 According to the proof of Theorem 2 in Bradley (1992) and Remark 3 in Bryc and Smolenski (1993), let $\{X_i; i \ge 1\}$ be a strictly stationary Gaussian sequence which has a bounded positive spectral density $f(t)$, then the sequence $\{f(X_i); i \ge 1\}$ has the property that $\widetilde{\varphi}(1) < 1$. Therefore, such a sequence of instantaneous functions $\{f(X_i); i \ge 1\}$ provides a class of examples for $\widetilde{\varphi}$-mixing sequences.

The main purpose of this paper is to establish the almost sure convergence and complete convergence of partial sums for $\widetilde{\varphi}$-mixing random variables. The main results obtained not only extend and generalize the classical Khintchine-Kolmogorov Convergence Theorem, the Three Series Theorem for independent random variables to the case of $\widetilde{\varphi}$-mixing random variables, but also improve the corresponding results without necessarily adding any extra conditions.

§2. Main Results and Proofs

Throughout this paper, c will represent a generic positive constant whose value may change from one appearance to the next, and $a_n = O(b_n)$ will mean $a_n \le c(b_n)$. And $a_n \ll b_n$ will mean $a_n = O(b_n)$.

Lemma 2.1 Let $\{X_n; n \ge 1\}$ be a sequence of $\widetilde{\varphi}$-mixing random variables with $\mathsf{E}X_n = 0$ and $\mathsf{E}|X_n|^r < \infty$ for some $r \ge 2$ and all $n \ge 1$. Then there exists a constant

$c = c(r, k, \widetilde{\varphi}(k))$ depending only on r, k and $\widetilde{\varphi}(k)$ such that for any $n \geq 1$,

$$\mathsf{E}\Big(\max_{1\leq j\leq n}\Big|\sum_{i=1}^{j} X_i\Big|^r\Big) \leq c\Big[\sum_{i=1}^{n}\mathsf{E}|X_i|^r + \Big(\sum_{i=1}^{n}\mathsf{E}X_i^2\Big)^{r/2}\Big], \tag{2.1}$$

where $\widetilde{\varphi}(k) < 1$.

Proof We can prove the lemma by using the similar method as that for Theorem 2.1 of Utev and Peligrad (2003). The above lemma is a Rosenthal-type inequality for $\widetilde{\varphi}$-mixing random variables. □

Lemma 2.2 (Khintchine-Kolmogorov Convergence Theorem) Let $\{X_n; n \geq 1\}$ be a sequence of $\widetilde{\varphi}$-mixing random variables which satisfies

$$\sum_{n=1}^{\infty}\mathsf{Var}\, X_n < \infty. \tag{2.2}$$

Then, $\sum\limits_{n=1}^{\infty}(X_n - \mathsf{E}X_n)$ converges almost surely and in quadratic mean.

Proof The proof is similar to that of Theorem 3 of Wu and Lin (2004). Without loss of generality, assume that $\mathsf{E}X_n = 0$. For $m \geq n \to \infty$, by the Lemma 2.1 and (2.2), we get that

$$\mathsf{E}(S_m - S_n)^2 \ll \sum_{k=n+1}^{m}\mathsf{E}X_k^2 \to 0. \tag{2.3}$$

Whence, $\{S_n; n \geq 1\}$ is a Cauchy sequence in L_2, according to the Cauchy Convergence Criterion, there exists a random variable S such that $\mathsf{E}(S_n - S)^2 \to 0$, i.e. $S_n \xrightarrow{L_2} S$. A fortiori, $S_n \xrightarrow{\mathsf{P}} S$, and so there exists positive integers $n_k \to \infty$ such that

$$S_{n_k} \to S \quad \text{a.s.} \qquad \text{as} \qquad k \to \infty. \tag{2.4}$$

On the other hand, it follows from Lemma 2.1 and (2.2) that for any given $\varepsilon > 0$ (setting $n_0 = 0$)

$$\begin{aligned}&\sum_{k=1}^{\infty}\mathsf{P}\Big(\max_{n_{k-1}<j\leq n_k}|S_j - S_{n_{k-1}}| > \varepsilon\Big)\\ \ll\ &\sum_{k=1}^{\infty}\mathsf{E}\Big(\max_{n_{k-1}<j\leq n_k}|S_j - S_{n_{k-1}}|\Big)^2 \ll \sum_{k=1}^{\infty}\sum_{j=n_{k-1}+1}^{n_k}\mathsf{E}X_j^2 = \sum_{j=1}^{\infty}\mathsf{E}X_j^2 < \infty.\end{aligned} \tag{2.5}$$

By the Borel-Cantelli Lemma, we obtain that

$$\max_{n_{k-1}<j\leq n_k}|S_j - S_{n_{k-1}}| > \varepsilon \to 0 \quad \text{a.s.} \qquad \text{as} \qquad k \to \infty, \tag{2.6}$$

which together with (2.4), according to the method of subsequence gives

$$S_n \to S \quad \text{a.s.} \qquad \text{as} \qquad k \to \infty. \qquad \square \tag{2.7}$$

Theorem 2.1 Let $\{X_n; n \geq 1\}$ be a sequence of $\widetilde{\varphi}$-mixing random variables for some constants $c > 0$, let $Y_n = X_n I(|X_n| \leq c)$ if the following three series converge, i.e.

$$\sum_{n=1}^{\infty} \mathsf{P}(|X_n| > c) < \infty, \tag{2.8}$$

$$\sum_{n=1}^{\infty} \mathsf{E} Y_n < \infty, \tag{2.9}$$

$$\sum_{n=1}^{\infty} \mathsf{Var}\, Y_n < \infty. \tag{2.10}$$

Then, the series $\sum\limits_{n=1}^{\infty} X_n$ converges almost surely.

Proof If (2.10) holds true, then $\sum\limits_{n=1}^{\infty} (Y_n - \mathsf{E} Y_n)$ converges almost surely by Lemma 2.2, it follows from (2.9) that $\sum\limits_{n=1}^{\infty} Y_n$ converges almost surely. According to (2.8),

$$\sum_{n=1}^{\infty} \mathsf{P}(|X_n| > c) = \sum_{n=1}^{\infty} \mathsf{P}(X_n \neq Y_n) < \infty. \tag{2.11}$$

So, the sequence of $\{X_n; n \geq 1\}$ random variables and the sequence of $\{Y_n; n \geq 1\}$ random variables are equivalent. It follows from the Borel-Cantelli Lemma that

$$\mathsf{P}(X_n \neq Y_n;\ \text{i.o.}) = 0, \tag{2.12}$$

which together with $\sum\limits_{n=1}^{\infty} Y_n$ converges almost surely, we can obtain that $\sum\limits_{n=1}^{\infty} X_n$ converges almost surely. $\square$

Corollary 2.1 Let $\{X_n; n \geq 1\}$ be a sequence of $\widetilde{\varphi}$-mixing random variables with $\mathsf{E} X_n = 0$, $n \geq 1$, and for some constants $c > 0$ such that

$$\sum_{n=1}^{\infty} \mathsf{E}[X_n^2 I(|X_n| \leq c) + |X_n| I(|X_n| > c)] < \infty. \tag{2.13}$$

Then, the series $\sum\limits_{n=1}^{\infty} X_n$ converges almost surely.

Corollary 2.2 Let $\{X_n; n \geq 1\}$ be a sequence of $\widetilde{\varphi}$-mixing random variables, for $0 < p \leq 2$, $\sum\limits_{n=1}^{\infty} \mathsf{E}|X_n|^p < \infty$. Furthermore, when $1 < p \leq 2$, assume that $\mathsf{E} X_n = 0$. Then, the series $\sum\limits_{n=1}^{\infty} X_n$ converges almost surely.

Theorem 2.2 Let $\{X_n; n \geq 1\}$ be a $\widetilde{\varphi}$-mixing sequence of random variables with $\mathsf{E} X_n = 0$. Let $\{a_n; n \geq 1\}$ be a sequence of positive real numbers such that $0 < a_n \uparrow \infty$. Let $\{g_n(t); n \geq 1\}$ be a sequence of nonnegative and even functions such that for each

$n \geq 1$, $g_n(t) > 0$ as $t > 0$, $g_n(|t|)/|t|$ is an increasing function of $|t|$ and $g_n(|t|)/|t|^2$ is a decreasing function of $|t|$, respectively, that is,

$$\frac{g_n(|t|)}{|t|} \uparrow \quad \text{and} \quad \frac{g_n(|t|)}{|t|^2} \downarrow, \qquad \text{as} \qquad |t| \uparrow . \tag{2.14}$$

If

$$\sum_{n=1}^{\infty} \sum_{i=1}^{n} \mathsf{E} \frac{g_i(|X_i|)}{g_i(a_n)} < \infty \qquad \text{as} \qquad n \to \infty. \tag{2.15}$$

Then,

$$\sum_{n=1}^{\infty} \mathsf{P}\Big(\Big|a_n^{-1} \sum_{i=1}^{n} X_i\Big| > \varepsilon\Big) < \infty \qquad \text{for any } \ \varepsilon > 0. \tag{2.16}$$

Proof For all $n \geq 1$, $1 \leq i \leq n$, define

$$X_n i = X_i I(|X_i| \leq a_n) + a_n I(X_i > a_n) - a_n I(X_i < -a_n);$$
$$X_{ni}^1 = (X_i - a_n) I(X_i > a_n) + (X_i + a_n) I(X_i < -a_n).$$

Clearly, $X_i = X_{ni} + X_{ni}^1$ for all $n \geq 1$, $1 \leq i \leq n$.

$$\begin{aligned}
\mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n} X_i\Big| > \varepsilon\Big) &= \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{k}(X_{ni} + X_{ni}^1)\Big| > \varepsilon\Big) \\
&\leq \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n} X_{ni}^1\Big| > \frac{\varepsilon}{2}\Big) + \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n} X_n i\Big| > \frac{\varepsilon}{2}\Big) \\
&\leq \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n} X_{ni}^1\Big| > \frac{\varepsilon}{2}\Big) \\
&\quad + \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n}(X_{ni} - \mathsf{E}X_{ni})\Big| > \frac{\varepsilon}{2} - \Big|\frac{1}{a_n}\sum_{i=1}^{n}\mathsf{E}X_{ni}\Big|\Big).
\end{aligned}$$

It suffices to prove the following inequalities for any $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n} X_{ni}^1\Big| > \varepsilon/2\Big) < \infty; \tag{2.17}$$

$$\sum_{n=1}^{\infty} \mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^{n}(X_{ni} - \mathsf{E}X_{ni})\Big| > \varepsilon/2\Big) < \infty; \tag{2.18}$$

$$\Big|\frac{1}{a_n}\sum_{i=1}^{n}\mathsf{E}X_{ni}\Big| \to 0 \qquad \text{as} \qquad n \to \infty. \tag{2.19}$$

First, we show that

$$\Big|\frac{1}{a_n}\sum_{i=1}^{n}\mathsf{E}X_{ni}\Big| \to 0 \qquad \text{as} \qquad n \to \infty.$$

In fact, by $\mathsf{E}X_n=0$, then $\mathsf{E}X_{ni}=-\mathsf{E}X^1_{ni}$. Note that $|X^1_{ni}|\le|X_i|$, $g_n(|t|)/|t|\uparrow$ as $|t|\uparrow$, then we have

$$\begin{aligned}\Big|\frac{1}{a_n}\sum_{i=1}^n\mathsf{E}X_{ni}\Big| &\le \frac{1}{a_n}\sum_{i=1}^n\mathsf{E}|X^1_{ni}|\le\frac{1}{a_n}\sum_{i=1}^n\mathsf{E}|X_i|I(|X_i|>a_n)\\ &= \sum_{i=1}^n\frac{\mathsf{E}X_i}{a_n}\times\frac{g_i(a_n)}{g_i(a_n)}\times\frac{\mathsf{E}g_i(|X_i|)}{\mathsf{E}g_i(|X_i|)}I(|X_i|>a_n)\\ &\le \sum_{i=1}^n\frac{\mathsf{E}g_i(|X_i|)}{g_i(a_n)}I(|X_i|>a_n)\\ &\le \sum_{i=1}^n\frac{\mathsf{E}g_i(|X_i|)}{g_i(a_n)}\to 0 \qquad \text{as} \qquad n\to\infty. \end{aligned}\tag{2.20}$$

Second, note that for each $n\ge 2$, by Markov inequality and Lemma 2.1, it follows that

$$\begin{aligned}&\sum_{n=1}^\infty\mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^n(X_{ni}-\mathsf{E}X_{ni})\Big|>\varepsilon/2\Big)\\ \le\ & c\sum_{n=1}^\infty\sum_{i=1}^n a_n^{-2}\mathsf{E}X_i^2I(|X_i|\le a_n)+c\sum_{n=1}^\infty a_n^{-2}\sum_{i=1}^n a_n^2\mathsf{P}(|X_i|>a_n).\end{aligned}\tag{2.21}$$

Hence, we need only to prove that

$$\mathrm{I}\triangleq\sum_{n=1}^\infty\sum_{i=1}^n a_n^{-2}\mathsf{E}X_i^2I(|X_i|\le a_n)<\infty;\tag{2.22}$$

$$\mathrm{II}\triangleq\sum_{n=1}^\infty\sum_{i=1}^n\mathsf{P}(|X_i|>a_n)<\infty.\tag{2.23}$$

It follows from (2.14) and (2.15) that

$$\begin{aligned}\mathrm{I}&\triangleq\sum_{n=1}^\infty\sum_{i=1}^n a_n^{-2}\mathsf{E}X_i^2I(|X_i|\le a_n)\\ &= C\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}g_i(|X_i|)}{g_i(a_n)}\times\frac{g_i(a_n)}{a_n^2}\times\frac{\mathsf{E}|X_i|^2}{\mathsf{E}g_i(|X_i|)}I(|X_i|\le a_n)\\ &\le C\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}g_i(|X_i|)}{g_i(a_n)}<\infty.\end{aligned}\tag{2.24}$$

It follows from (2.14), (2.15) and Markov inequality that

$$\begin{aligned}\mathrm{II}&\triangleq\sum_{n=1}^\infty\sum_{i=1}^n\mathsf{P}(|X_i|>a_n)\le\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}|X_i|}{a_n}\\ &=\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}|X_i|}{a\quad n}\times\frac{\mathsf{E}g_i(|X_i|)}{\mathsf{E}g_i(|X_i|)}\times\frac{g_i(a_n)}{g_i(a_n)}\le\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}g_i(|X_i|)}{g_i(a_n)}<\infty.\end{aligned}\tag{2.25}$$

Finally, since $g_n(|t|)/|t|\uparrow$ as $|t|\uparrow$, then $g_n(|t|)\uparrow$ as $|t|\uparrow$. So,

$$\begin{aligned}&\sum_{n=1}^\infty\mathsf{P}\Big(\Big|\frac{1}{a_n}\sum_{i=1}^nX^1_{ni}\Big|>\varepsilon/2\Big)\\ \le\ &\sum_{n=1}^\infty\sum_{i=1}^n\mathsf{P}(|X_i|>a_n)\le\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}|X_i|}{a_n}\le\sum_{n=1}^\infty\sum_{i=1}^n\frac{\mathsf{E}g_i(|X_i|)}{g_i(a_n)}<\infty.\qquad\square\end{aligned}\tag{2.26}$$

Corollary 2.3 Under the conditions of Theorem 2.2, then

$$\frac{1}{a_n}\sum_{i=1}^{n} X_i \to 0 \qquad \text{a.s.}. \tag{2.27}$$

Proof By Theorem 2.2, it follows from Borel-Cantelli Lemma that

$$\Big|\frac{1}{a_n}\sum_{i=1}^{n} X_i\Big| \to 0 \qquad \text{a.s.}. \tag{2.28}$$

The proof of Corollary 2.3 is obvious. □

By taking $g_n(t)=|t|^p$, $0<p\leq 2$ in Theorem 2.2, we can immediately obtain the following corollary.

Corollary 2.4 Under the conditions of Theorem 2.2, if

$$\sum_{n=1}^{\infty}\sum_{i=1}^{n}\frac{\mathsf{E}(|X_i|^p)}{a_n^p}<\infty \qquad \text{as} \qquad n\to\infty, \tag{2.29}$$

then

$$\frac{1}{a_n}\sum_{i=1}^{n} X_i \to 0 \quad \text{a.s.} \qquad \text{as} \qquad n\to\infty.$$

Corollary 2.5 Under the conditions of Theorem 2.2, If

$$\sum_{i=1}^{n}\mathsf{E}\frac{g_i(|X_i|)}{g_i(a_n)}\to 0 \qquad \text{as} \qquad n\to\infty, \tag{2.30}$$

then

$$a_n^{-1}\sum_{i=1}^{n} X_i \to 0 \quad \text{in probability}. \tag{2.31}$$

Proof The proof can be accomplished in a similar way as Theorem 2.2. Here, we omit the proof of this corollary. □

Remark 1 Corollary 2.3 holds true under the conditions of Theorem 2.2, the fact that

$$\sum_{n=1}^{\infty}\sum_{i=1}^{n}\mathsf{E}\frac{g_i(|X_i|)}{g_i(a_n)}<\infty$$

is stronger than the fact that

$$\sum_{i=1}^{n}\mathsf{E}\frac{g_i(|X_i|)}{g_i(a_n)}\to 0 \qquad \text{as} \qquad n\to\infty.$$

Hence, Corollary 2.3 not only generalizes the result of Corollary 2.5, but also improves it with necessarily adding a stronger condition.

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$\widetilde{\varphi}$混合随机变量序列的强收敛定律

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本文建立了$\widetilde{\varphi}$混合随机变量序列的几乎处处收敛性和完全收敛性的结果. 所获结果不仅把独立随机变量经典的Khintchine-Kolmogorov收敛定理和三级数收敛定理推广至$\widetilde{\varphi}$混合随机变量情形下, 并在没有增加任何附加条件下改进了相关结果.

关键词: $\widetilde{\varphi}$混合随机变量, 几乎处处收敛性, 完全收敛性, 加权和.

学科分类号: O211.4.