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The Strong Law of Large Numbers of Random Walk in Random Environments on Half-Line *

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Abstract

The random walk in random environments (RWIRE) on the right half line with reflected barrier on the origin is discussed. The recurrence of RWIRE on the right half line is studied. In the non-recurrent case, we obtain an estimation for the second moment of τ_n, the frist time of RWIRE $\{X_n\}$ from $n-1$ to n, which in turn yields a strong law of large numbers of RWIRE on the right half line.

Keywords: Strong law of large numbers, random walk, random environment, recurrence, moment.

AMS Subject Classification: 60J15, 60F05.

§1. Introduction

Let $\alpha_0 = 1$,$\{\alpha_n\}_{n=1}^{\infty}$ be a fixed sequence of numbers between 0 and 1, we can define a random walk on the nonnegative integers $\mathbb{Z}_+$ with a transition matrix

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & \cdots \\ 1-\alpha_1 & 0 & \alpha_1 & 0 & \cdots & 0 & \cdots \\ 0 & 1-\alpha_1 & 0 & \alpha_2 & \cdots & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 1-\alpha_n & 0 & \alpha_n & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}.$$

By the transition matrix M, we know the random walk has reflected barrier on the origin. Now, suppose that $\{\alpha_n\}_{\alpha=1}^{\infty}$ is a sequence of random variables with $0 \leqslant \alpha_n \leqslant 1$. We can

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still define a stochastic process $\{X_n\}$ on the nonnegative integers $\mathbb{Z}_+$ which satisfies

$$\mathsf{P}(X_0=0)=1, \tag{1.1}$$

$$\mathsf{P}(X_{n+1}=j|X_0=0,X_1=i_1,\ldots,X_{n-1}=i_{n-1},X_n=i,\alpha_n,n\in\mathbb{Z}_+)$$
$$=\begin{cases}\alpha_i, & \text{if } \ j=i+1\\ 1-\alpha_i, & \text{if } \ j=i-1 \qquad \text{a.s..}\\ 0, & \text{otherwise}\end{cases} \tag{1.2}$$

We call the stochastic process $\{X_n\}$ is random walk in random environments (RWIRE) on the right half line with reflected barrier on the origin. $\{X_n\}$ is not, in general, a Markov chain. In fact, the future, given the present, is not independent of the past. Using the same method as [8], we can construct the process on the Cartesian product of the set of environments and the set of paths. Setting $N=\{0,1,2,\ldots\}$, we will define a probability measure on $([0,1]^N\times\mathbb{Z}_+^N,\mathcal{F})$, where $\mathcal{F}$ is the σ-field generated by the cylinder sets. For a fixed environment $e=\{\alpha_n\}_{n=0}^{\infty}$, let P_e be the Markov chain measure on $\mathbb{Z}_+^N$ and $\{X_n\}$ be coordinate process so that

$$\mathsf{P}_e(X_0=0)=1,$$
$$\mathsf{P}_e(X_{n+1}=j|X_n=i)=\begin{cases}\alpha_i, & \text{if } \ j=i+1\\ 1-\alpha_i, & \text{if } \ j=i-1 \qquad \text{for each } \ n\geqslant1,\ i\geqslant0.\\ 0, & \text{otherwise}\end{cases}$$

On the environments $[0,1]^N$, let Q be a product measure so that $\alpha_0=1$ and $\{\alpha_n\}_{n=0}^{\infty}$ is i.i.d.. Now for $A\subset[0,1]^N$ and $F\subset\mathbb{Z}_+^N$ measureable with respect to the σ-fields generated by the cylinder sets, let

$$\mathsf{P}(A\times F)=\int_A\mathsf{P}_e(F)\mathsf{Q}(\mathrm{d}e),$$

The Caratheodory extension theorem shows that P extends to a probability measure on $([0,1]^N\times\mathbb{Z}_+^N,\mathcal{F})$.

The random walk in a random environment is formally defined the process $\{X_n\}$ defined on $([0,1]^N\times\mathbb{Z}_+^N,\mathcal{F},\mathsf{P})$, where $X_n(e,\omega)=\omega_n$ and $\overline{\alpha}_n(e,\omega)=\alpha_n$, it is obvious that the distribution of $\{\overline{\alpha}_n\}$ is the same as the distribution of $\{\alpha_n\}$.

When the environment $\{\alpha_n\}_{n=1}^{\infty}$ is fixed, [3], uses systems of difference equations to derive results which we summarize in Lemma 1.1.

Lemma 1.1 Fix $\{\alpha_n\}_{n=1}^{\infty}$ with $0<\alpha_n<1$ for all n; set $\rho_n=(\beta_1\cdots\beta_n)/(\alpha_1\cdots\alpha_n)$, where $\beta_i=1-\alpha_i$, $n\geqslant1$. Then

(i) $\{X_n\}$ is a recurrent chain if and only if

$$\sum_{n=1}^{\infty}\rho_n=\infty. \tag{1.3}$$

(ii) $\{X_n\}$ is a positive recurrent chain if and only if

$$\sum_{n=1}^{\infty}\rho_n=\infty \qquad \text{and} \qquad \sum_{n=1}^{\infty}\frac{1}{\alpha_n\rho_n}<\infty. \tag{1.4}$$

Let

$$\begin{aligned}
&T_0=0,\\
&T_n=\begin{cases}\min\{k>0: X_k=n\}, & \text{if such a } k \text{ exists;}\\ \infty, & \text{if no such } k \text{ exists,}\end{cases}\\
&\tau_n=T_n-T_{n-1}, \qquad n\geqslant 1.
\end{aligned}$$

It is obvious that τ_n is the first time of $\{X_n\}$ from $n-1$ to n. The sequence of ladder variables τ_n is not strictly stationary since RWIRE $\{X_n\}$ on the right half line with reflected barrier on the origin, and the law of large numbers or Birkhoff's ergodic theorem cannot be used for the sequence $\{\tau_n\}$. It is different from the case of [8] (we know that, $\{\tau_n\}$ is strictly stationary in [8]). The weak law of large numbers of random walk in random environments on the right half line is obtained by [2], but the strong law of large numbers is not discussed. We have gotten the strong law of large numbers of RWIRE on the nonnegative integers $\mathbb{Z}_+$ in this paper.

§2. The Second Moments of τ_n

Lemma 2.1 [8] Let $\{Y_n\}$ be a sequence of independent, identically distributed, nondegenerate, finite valued random variables; let $S_n=Y_1+\cdots+Y_n$.

(i) $\sum\limits_{n=1}^{\infty}n^{-1}\mathsf{P}(S_n>0)<\infty$ if and only if $\lim\limits_{n\to\infty}S_n=-\infty$ a.e. in which case $\sum\limits_{n=1}^{\infty}e^{S_n}<\infty$ a.e..

(ii) $\sum\limits_{n=1}^{\infty}n^{-1}\mathsf{P}(S_n>0)=\infty=\sum\limits_{n=1}^{\infty}n^{-1}\mathsf{P}(S_n<0)$ if and only if $-\infty=\liminf\limits_{n\to\infty}S_n<\limsup\limits_{n\to\infty}S_n=\infty$ a.e. in which case $\sum\limits_{n=1}^{\infty}e^{-S_n}=\infty=\sum\limits_{n=1}^{\infty}e^{S_n}$ a.e..

Lemma 2.2 Let $\{X_n\}$ be a RWIRE on $\mathbb{Z}_+$ with random environment $e=\{\alpha_n\}_{n=0}^{\infty}$, where $\alpha_0=1$, $\{\alpha_n\}_{n=1}^{\infty}$ i.i.d. and $0<\alpha_n<1$, $n\geqslant 1$, if $\mathsf{E}(\ln\beta_1/\alpha_1)$ exists, then

(i) $\{X_n\}$ is transient if and only if $\mathsf{E}(\ln\beta_1/\alpha_1)<0$.

(ii) $\{X_n\}$ is positive recurrence if and only if $\mathsf{E}(\ln\beta_1/\alpha_1)>0$.

(iii) $\{X_n\}$ is null recurrence if and only if $\mathsf{E}(\ln\beta_1/\alpha_1)=0$.

Theorem 2.1 Suppose $\mu = \mathsf{E}(\beta_1/\alpha_1) < 1$, then each τ_n is finite a.e. and

$$\lim_{n\to\infty} \mathsf{E}\tau_n = \frac{1+\mu}{1-\mu}. \tag{2.1}$$

Furthermore, if $\nu = \mathsf{E}(\beta_1/\alpha_1)^2 < 1$, then

$$\lim_{n\to\infty} \mathsf{E}\tau_n^2 = \frac{1+6\mu+7\nu+2\mu\nu+\mu^2-\mu^2\nu}{(1-\mu)^2(1-\nu)}. \tag{2.2}$$

Proof Note that $\mathsf{P}(\rho_n > 1) \leqslant \mathsf{E}\rho_n = \mu^n$. Thus $\mu < 1$ implies

$$\sum_{n=1}^{\infty} n^{-1}\mathsf{P}(\rho_n > 1) < \infty,$$

equivalently,

$$\sum_{n=1}^{\infty} n^{-1}\mathsf{P}\Big(\ln\Big(\frac{\beta_1}{\alpha_1}\Big) + \cdots + \ln\Big(\frac{\beta_n}{\alpha_n}\Big) > 0\Big) = \sum_{n=1}^{\infty} n^{-1}\mathsf{P}(\ln\rho_n > 0) < \infty.$$

Then by Lemma 2.1 we have

$$\sum_{n=1}^{\infty}\rho_n = \sum_{n=1}^{\infty} e^{\ln\rho_n} < \infty \qquad \text{a.e..}$$

Hence Lemma 1.1 implies $\{X_n\}$ is not recurrent, such that $\lim\limits_{n\to\infty} X_n = \infty$ a.e., so trivially τ_n is finite a.e.. For $e = \{\alpha_n\}$ fixed, noting that $\alpha_0 = 1$, by Markov property we obtain

$$\mathsf{E}^e\tau_1 = 1, \tag{2.3}$$

$$\mathsf{E}^e\tau_n = \alpha_{n-1} + \beta_{n-1}\mathsf{E}^e(1+\tau_{n-1}+\tau_n), \qquad n \geqslant 2. \tag{2.4}$$

Thus, by (2.4) we obtain

$$\mathsf{E}^e\tau_n = \frac{1}{\alpha_{n-1}} + \frac{\beta_{n-1}}{\alpha_{n-1}}\mathsf{E}^e\tau_{n-1} = 1 + \frac{\beta_{n-1}}{\alpha_{n-1}} + \frac{\beta_{n-1}}{\alpha_{n-1}}\mathsf{E}^e\tau_{n-1}, \qquad n \geqslant 2. \tag{2.5}$$

Taking the expectation with respect to Q yields

$$\mathsf{E}\tau_1 = 1, \qquad \mathsf{E}\tau_n = (1+\mu)\mathsf{E}\tau_{n-1}, \quad n \geqslant 2.$$

By induction, we obtain

$$\mathsf{E}\tau_n = \frac{1+\mu}{1-\mu} - \frac{2\mu^n}{1-\mu}, \qquad n \geqslant 1. \tag{2.6}$$

Since $\mu<1$, (2.6) implies (2.1). Next, we prove (2.2). For $e=\{\alpha_n\}$ fixed, by Markov property

$$\begin{aligned}
\mathsf{E}^e\tau_n^2 &= \sum_{k=1}^{\infty}k^2\mathsf{P}_e(\tau_n=k)=\mathsf{P}_e(\tau_n=1)+\sum_{k=2}^{\infty}k^2\mathsf{P}_e(\tau_n=k)\\
&= \alpha_{n-1}+\beta_{n-1}\sum_{k=2}^{\infty}k^2\sum_{i=2}^{k-1}\mathsf{P}_e(\tau_{n-1}=i-1)\mathsf{P}_e(\tau_n=k-i)\\
&= \alpha_{n-1}+\beta_{n-1}\sum_{i=2}^{\infty}\Big(\sum_{k=i+1}^{\infty}k^2\mathsf{P}_e(\tau_n=k-i)\Big)\mathsf{P}_e(\tau_{n-1}=i-1)\\
&= \alpha_{n-1}+\beta_{n-1}(\mathsf{E}^e\tau_n^2+2i\mathsf{E}^e\tau_n+i^2)\mathsf{P}_e(\tau_{n-1}=i-1)\\
&= \alpha_{n-1}+\beta_{n-1}(\mathsf{E}^e\tau_n^2+2\mathsf{E}^e\tau_n\cdot\mathsf{E}^e\tau_{n-1}+2\mathsf{E}^e\tau_n+\mathsf{E}^e\tau_{n-1}^2+2\mathsf{E}^e\tau_{n-1}+1)\\
&= 1+\beta_{n-1}\mathsf{E}^e\tau_n^2+\beta_{n-1}\mathsf{E}^e\tau_{n-1}^2+2\beta_{n-1}(1+\mathsf{E}^e\tau_{n-1})\mathsf{E}^e\tau_n+2\beta_{n-1}\mathsf{E}^e\tau_{n-1}\\
&= 1+\beta_{n-1}\mathsf{E}^e\tau_n^2+\beta_{n-1}\mathsf{E}^e\tau_{n-1}^2\\
&\quad+2\beta_{n-1}(1+\mathsf{E}^e\tau_{n-1})\Big(1+\frac{\beta_{n-1}}{\alpha_{n-1}}+\frac{\beta_{n-1}}{\alpha_{n-1}}\mathsf{E}^e\tau_{n-1}\Big)+2\beta_{n-1}\mathsf{E}^e\tau_{n-1}\\
&= 1+\beta_{n-1}\mathsf{E}^e\tau_n^2+\beta_{n-1}\mathsf{E}^e\tau_{n-1}^2+2\beta_{n-1}+\frac{2\beta_{n-1}^2}{\alpha_{n-1}}+4\Big(\beta_{n-1}+\frac{\beta_{n-1}^2}{\alpha_{n-1}}\Big)\mathsf{E}^e\tau_{n-1}\\
&\quad+\frac{2\beta_{n-1}^2}{\alpha_{n-1}}(\mathsf{E}^e\tau_{n-1})^2.
\end{aligned}$$

So

$$\begin{aligned}
\mathsf{E}^e\tau_n^2 &= \frac{\beta_{n-1}}{\alpha_{n-1}}\mathsf{E}^e\tau_{n-1}^2+1+3\frac{\beta_{n-1}}{\alpha_{n-1}}+\frac{2\beta_{n-1}^2}{\alpha_{n-1}^2}+4\Big(\frac{\beta_{n-1}}{\alpha_{n-1}}+\frac{\beta_{n-1}^2}{\alpha_{n-1}^2}\Big)\mathsf{E}^e\tau_{n-1}\\
&\quad+\frac{2\beta_{n-1}^2}{\alpha_{n-1}^2}(\mathsf{E}^e\tau_{n-1})^2. \qquad (2.7)
\end{aligned}$$

Taking the expectation with respect to Q yields

$$\mathsf{E}\tau_n^2=\mu\mathsf{E}\tau_{n-1}^2+1+3\mu+2\nu+4(\mu+\nu)\Big(\frac{1+\mu}{1-\mu}-\frac{2\mu^{n-1}}{1-\mu}\Big)+2\nu\mathsf{E}(\mathsf{E}^e\tau_{n-1})^2. \qquad (2.8)$$

According to (2.8), noting that $\mathsf{E}^e\tau_1=1$, we obtain

$$\mathsf{E}^e\tau_n=1+2\Big(\frac{\beta_{n-1}}{\alpha_{n-1}}+\frac{\beta_{n-1}\beta_{n-2}}{\alpha_{n-1}\alpha_{n-2}}+\cdots+\frac{\beta_{n-1}\beta_{n-2}\cdots\beta_1}{\alpha_{n-1}\alpha_{n-2}\cdots\alpha_1}\Big).$$

It implies

$$\begin{aligned}
(\mathsf{E}^e\tau_n)^2 &= 1+4\Big(\frac{\beta_{n-1}}{\alpha_{n-1}}+\frac{\beta_{n-1}\beta_{n-2}}{\alpha_{n-1}\alpha_{n-2}}+\cdots+\frac{\beta_{n-1}\beta_{n-2}\cdots\beta_1}{\alpha_{n-1}\alpha_{n-2}\cdots\alpha_1}\Big)\\
&\quad+4\Big(\frac{\beta_{n-1}}{\alpha_{n-1}}+\frac{\beta_{n-1}\beta_{n-2}}{\alpha_{n-1}\alpha_{n-2}}+\cdots+\frac{\beta_{n-1}\beta_{n-2}\cdots\beta_1}{\alpha_{n-1}\alpha_{n-2}\cdots\alpha_1}\Big)^2.
\end{aligned}$$

Taking the expectation with respect to Q yields

$$\begin{aligned}\mathsf{E}(\mathsf{E}^e\tau_n)^2 &= 1+4(\mu+\mu^2+\cdots+\mu^{n-1})+4(\nu+\nu^2+\cdots+\nu^{n-1})+8\sum_{k=2}^{n-2}\sum_{i=1}^{k-1}\mu^i\nu^{k-i}\\ &= 8\sum_{k=0}^{n-1}\sum_{i=0}^{k}\mu^i\nu^{k-i}-4(\mu+\mu^2+\cdots+\mu^{n-1})-4(\nu+\nu^2+\cdots+\nu^{n-1})-7.\end{aligned}$$

So we see

$$\begin{aligned}\lim_{n\to\infty}\mathsf{E}(\mathsf{E}^e\tau_n)^2 &= \frac{8}{(1-\mu)(1-\nu)}-\frac{4\mu}{1-\mu}-\frac{4\nu}{1-\nu}-7\\ &= \frac{1+3\mu+3\nu+\mu\nu}{(1-\mu)(1-\nu)}.\end{aligned}\tag{2.9}$$

(2.8) implies

$$\begin{aligned}\lim_{n\to\infty}\mathsf{E}\tau_n^2 &= \mu\lim_{n\to\infty}\mathsf{E}\tau_n^2+1+3\mu+2\nu+\frac{4(\mu+\nu)(1+\mu)}{1-\mu}+\frac{2\nu(1+3\mu+3\nu+\mu\nu)}{(1-\mu)(1-\nu)}\\ &= \mu\lim_{n\to\infty}\mathsf{E}\tau_n^2+\frac{1+6\mu+7\nu+2\mu\nu+\mu^2-\mu^2\nu}{(1-\mu)(1-\nu)}.\end{aligned}$$

It is obvious that

$$\lim_{n\to\infty}\mathsf{E}\tau_n^2=\frac{1+6\mu+7\nu+2\mu\nu+\mu^2-\mu^2\nu}{(1-\mu)^2(1-\nu)}. \qquad \square$$

§3. The Strong Law of Large Numbers

In this section we give the almost sure limits of the averages X_n/n in terms of the environments.

Lemma 3.1 [1] When an environment $e=\{\alpha_n\}$ is fixed, under probability P_e, the random variables τ_n, $n\geqslant 1$, are independent. When $\alpha_0=1$, $\{\alpha_n\}_{n=1}^{\infty}$ be a sequence of independent, identically distributed, random variables with $0<\alpha_n<1$ for all n, if MCIRE $\{X_n\}$ satisfies $\limsup X_n=+\infty$ a.e., then $\{\tau_n\}$ is strong mixing under probability P.

Theorem 3.1 Suppose $\mu=\mathsf{E}(\beta_1/\alpha_1)<1$ and $\nu=\mathsf{E}(\beta_1/\alpha_1)^2<1$, let $r(n)=\sup\limits_k|\mathsf{E}(\tau_k\tau_{k+n})-\mathsf{E}\tau_k\mathsf{E}\tau_{k+n}|$, $n\geqslant 1$, then $r(n)\leqslant C\mu^n$, $n\geqslant 1$.

Proof For a fixed environment $e=\{\alpha_n\}_{n=0}^{\infty}$ with $\alpha_0=1$, by (2.8) we obtain

$$\begin{aligned}\mathsf{E}^e\tau_{k+n} &= 1+2\Big(\frac{\beta_{k+n-1}}{\alpha_{k+n-1}}+\frac{\beta_{k+n-1}\beta_{k+n-2}}{\alpha_{k+n-1}\alpha_{k+n-2}}+\cdots+\frac{\beta_{k+n-1}\beta_{k+n-2}\cdots\beta_{k+1}}{\alpha_{k+n-1}\alpha_{k+n-2}\cdots\alpha_{k+1}}\Big)\\ &\quad+\frac{\beta_{k+n-1}\beta_{k+n-2}\cdots\beta_k}{\alpha_{k+n-1}\alpha_{k+n-2}\cdots\alpha_k}+\frac{\beta_{k+n-1}\beta_{k+n-2}\cdots\beta_k}{\alpha_{k+n-1}\alpha_{k+n-2}\cdots\alpha_k}\mathsf{E}^e\tau_k.\end{aligned}$$

According to Lemma 3.1, noting that $\{\alpha_n\}_{n=1}^{\infty}$ is i.i.d., and τ_k is measurable with respect to the σ-fields $\mathcal{F}_{k-1} = \sigma(\alpha_i, 1 \leqslant i \leqslant k-1)$, one can easily check that

$$\begin{aligned} \mathsf{E}(\tau_k\tau_{k+n}) &= \mathsf{E}(\mathsf{E}^e(\tau_k\tau_{k+n})) = \mathsf{E}(\mathsf{E}^e\tau_k\mathsf{E}^e\tau_{k+n}), \\ &= (1+2(\mu+\mu^2+\cdots+\mu^{n-1})+\mu^n)\mathsf{E}\tau_k + \mu^n\mathsf{E}(\mathsf{E}^e\tau_k)^2, \\ \mathsf{E}\tau_k\mathsf{E}\tau_{k+n} &= \mathsf{E}\tau_k(1+2(\mu+\mu^2+\cdots+\mu^{n-1})+\mu^n+\mu^n\mathsf{E}\tau_k). \end{aligned}$$

Then we have

$$|\mathsf{E}(\tau_k\tau_{k+n}) - \mathsf{E}\tau_k\mathsf{E}\tau_{k+n}| = |\mu^n\mathsf{E}(\mathsf{E}^e\tau_k)^2 - \mu^n(\mathsf{E}\tau_k)^2| = \mu^n|\mathsf{E}(\mathsf{E}^e\tau_k)^2 - (\mathsf{E}\tau_k)^2|.$$

If $\nu = \mathsf{E}(\beta_1/\alpha_1)^2 < 1$, (2.5) and (2.6) imply

$$r(n) = \sup_k |\mathsf{E}(\tau_k\tau_{k+n}) - \mathsf{E}\tau_k\mathsf{E}\tau_{k+n}| \leqslant C\mu^n, \qquad n \geqslant 1. \qquad \square$$

Theorem 3.2 Suppose $\mu = \mathsf{E}(\beta_1/\alpha_1) < 1$ and $\nu = \mathsf{E}(\beta_1/\alpha_1)^2 < 1$, then

(i) $\lim\limits_{n\to\infty} T_n/n = (1+\mu)/(1-\mu)$ a.e.,

(ii) $\lim\limits_{n\to\infty} X_n/n = (1-\mu)/(1+\mu)$ a.e..

Proof Let $\tau_n' = \tau_n - \mathsf{E}\tau_n$, $T_n' = \sum\limits_{k=1}^{n} \tau_k' = T_n - \mathsf{E}T_n$, then $\mathsf{E}\tau_n' = 0$, Theorem 2.1 and Theorem 3.1 imply

$$\begin{aligned} \mathsf{E}(T_n')^2 &= \mathsf{D}T_n = \sum_{k=1}^{n} \mathsf{D}\tau_k + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \mathsf{Cov}\,(\tau_i,\tau_j) \\ &\leqslant \sum_{k=1}^{n} \mathsf{E}\tau_k^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} r(j-i) \\ &\leqslant Cn + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C\mu^{j-i} \\ &\leqslant Cn, \end{aligned}$$

where C is a constant, line and line may be different. Chebyshev's inequality implies that if $\varepsilon > 0$.

$$\sum_{n=1}^{\infty} \mathsf{P}(|T_{n^2} - \mathsf{E}T_{n^2}| > \varepsilon n^2) \leqslant \varepsilon^{-2}\sum_{n=1}^{\infty}\frac{\mathsf{D}T_{n^2}}{n^4} \leqslant \varepsilon^{-2}C\sum_{n=1}^{\infty}\frac{1}{n^2} < \infty.$$

Borel-Cantelli lemma implies

$$\mathsf{P}(|T_{n^2} - \mathsf{E}T_{n^2}| > \varepsilon n^2 \ \text{ i.o.}) = 0.$$

Since ε is arbitrary, it shows

$$\frac{T_{n^2} - \mathsf{E}T_{n^2}}{n^2} \to 0 \qquad \text{a.e..} \tag{3.1}$$

For each n, let $D_n = \max\limits_{n^2 \leqslant k < (n+1)^2} |T'_k - T'_{n^2}|$, note that $D_n^2 \leqslant \sum\limits_{k=n^2+1}^{(n+1)^2} |T'_k - T'_{n^2}|^2$, we obtain

$$\mathsf{E}D_n^2 \leqslant \sum_{k=n^2+1}^{(n+1)^2} \mathsf{E}(T'_k - T'_{n^2})^2 \leqslant C \sum_{k=n^2+1}^{n^2+2n+1} (k - n^2) = Cn(2n+1).$$

Chebsyshev's inequality implies

$$\sum_{n=1}^{\infty} \mathsf{P}(D_n > n^2\varepsilon) \leqslant \sum_{n=1}^{\infty} \frac{\mathsf{E}D_n^2}{n^4\varepsilon^2} \leqslant C\varepsilon^{-2} \sum_{n=1}^{\infty} \frac{2n+1}{n^3} < \infty.$$

It is similar to (3.2),

$$\frac{D_n}{n^2} \to 0 \qquad \text{a.e..} \tag{3.2}$$

We observe that if $n^2 \leqslant k < (n+1)^2$, then

$$\frac{|T'_k|}{k} \leqslant \frac{|T'_{n^2}| + D_n}{n^2}.$$

So (3.1) and (3.2) imply

$$\frac{T'_k}{k} \to 0 \qquad \text{a.e.,}$$

i.e.

$$\lim_{n\to\infty} \frac{T_n - \mathsf{E}T_n}{n} = 0 \qquad \text{a.e..}$$

Since $\lim\limits_{n\to\infty} \mathsf{E}\tau_n = (1+\mu)/(1-\mu)$, it follows that

$$\lim_{n\to\infty} \frac{\mathsf{E}T_n}{n} = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \mathsf{E}\tau_k = \frac{1+\mu}{1-\mu}.$$

The proof of (i) is completed.

The result for X_n is followed by a simple argument used in [8], which shows that

$$\lim_{n\to\infty} \frac{X_n}{n} = \lim_{n\to\infty} \frac{n}{T_n} \qquad \text{a.e..}$$

The proof of (ii) is complete. $\square$

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半直线上随机环境中随机游动的强大数定律

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本文讨论了右半直线上带有原点反射壁的随机环境中随机游动, 并给出了其常返性的充要条件. 在非常返的情况下, 获得了$\{X_n\}$从$n-1$到n首中时τ_n的二阶矩$\mathsf{E}\tau_n^2$的一个估计. 同时, 给出了右半直线上随机环境中随机游动的强大数定律.

关 键 词: 强大数定律, 随机游动, 随机环境, 常返, 矩.

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