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The Central Limit Theorem for Nonhomogeneous Markov Chains *

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Abstract

In this paper, we prove a new central limit theorem for nonhomogeneous Markov chain by using the martingale central limit theorem under the condition of convergence of transition probability matrices for nonhomogeneous Markov chain in Cesàro sense, which can not be implied by Dobrushin's work.

Keywords: Nonhomogeneous Markov chains, central limit theorem, martingale.

AMS Subject Classification: 60C05, 05C80.

§1. Introduction

Let $\{X_n, n \geq 0\}$ be a nonhomogeneous Markov chain taking values in state space $S = \{1, 2, \ldots, b\}$ with initial distribution

$$\mu = (\mu(1), \mu(2), \ldots, \mu(b)) \tag{1.1}$$

and transition matrices

$$P_n = (p_n(i,j)), \qquad i, j \in S,\ n \geq 1, \tag{1.2}$$

where $p_n(i,j) = \mathsf{P}(X_n = j | X_{n-1} = i)$. Then

$$p(x_0, x_1, \ldots, x_n) = \mu(x_0) \prod_{k=1}^{n} p_k(x_{k-1}, x_k). \tag{1.3}$$

For an arbitrary stochastic square matrix Q whose elements are $q_{i,j}$ we shall set the ergodic δ-coefficient equal to

$$\delta(Q) = \sup_{i,j \in S} \sum_{k \in S} [q_{i,k} - q_{j,k}]^+,$$

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where $[a]^+ = \max\{0, a\}$. Also, define the related α-coefficient $\alpha(Q) = 1 - \delta(Q)$.

For the transition probability matrices $\{P_k : 1 \le k \le n\}$, let

$$\alpha_n = \min_{1 \le k \le n} \alpha(P_k).$$

In addition, let f be any function defined on S. We shall write

$$S_n = \sum_{k=1}^{n} f(X_k).$$

Denote the expectations and variances respectively as follows

$$\mathsf{E}[S_n] = \sum_{k=1}^{n} \mathsf{E}[f(X_k)]$$

and

$$V(S_n) = \mathsf{E}[S_n]^2 - (\mathsf{E}[S_n])^2.$$

Our goal of this work is to describe conditions on X and f under which the central limit theorem holds for S_n. Our conditions are different from which given by Dobrushin (1956) as the following lemma:

Lemma 1.1 Let $\{X_n, n \ge 0\}$ be a nonhomogeneous Markov chain taking values in state space $S = \{1, 2, \dots, b\}$ with initial distribution of equation (1.1) and transition matrices of (1.2). Let f be any function defined on the state space S. If for all $1 \le i \le n$, the variances are bounded below, i.e.

$$V(f(X_i)) = \mathsf{E}[f(X_i)]^2 - (\mathsf{E}[f(X_i)])^2 > c > 0.$$

Then if

$$\lim_{n\to\infty} n^{1/3}\alpha_n = \infty, \tag{1.4}$$

we have the standard normal convergence

$$\frac{S_n - \mathsf{E}[S_n]}{\sqrt{V(S_n)}} \stackrel{D}{\Rightarrow} N(0,1), \tag{1.5}$$

here and thereafter $\stackrel{D}{\Rightarrow}$ denotes convergence in distribution.

The central limit theorem (CLT) for additive functionals of stationary, ergodic Markov chains has been studied intensively during the last decades. A basic approach for proving the CLT, initiated by Gordin and Lifšic (1978) and afterwards pursued by several authors such as Derriennic and Lin (2001, 2003), Gordin and Holzmann (2004), Kipnis and Varadhan (1986), Maxwell and Woodroofe (2000) and Woodroofe (1992), is to construct a martingale approximation to the partial sums. These are decomposed into a sum of a martingale with stationary increments and a remainder term. After showing that the remainder term is negligible in some suitable sense, asymptotic normality follows from a martingale CLT. Recently, Holzmann (2005) also proved the central limit theorems by this method of martingale approximations for continuous-time and discrete-time stationary Markov processes.

Nearly fifty years ago, Dobrushin (1956) proved in his thesis an important central limit theorem for Markov chains in discrete time that are not necessarily homogeneous in time. After Dobrushin's work, some refinements and extensions of his central limit theorem, some of which under more stringent assumptions, were proved by Statulyavichus (1969) and Sarymsakov (1961). Based on Dobrushin's work, Sethuraman and Varadhan (2005) gave a shorter and different proof elucidating more assumptions by using martingale approximation.

In this note, we will consider another central limit theorem for nonhomogeneous Markov chains with finite state space in discrete time which can not be implied by Dobrushin's results. Our main result is as follows:

Theorem 1.1 Let $\{X_n, n \geq 0\}$ be a nonhomogeneous Markov chain taking values in state space $S = \{1, 2, \ldots, b\}$ with initial distribution of (1.1) and transition matrices of (1.2). Let f be any function defined on the state space S. Let $P = (p(i,j))_{b\times b}$ be another transition matrix and P irreducible, $\pi = (\pi_1, \pi_2, \ldots, \pi_b)$ is the unique stationary distribution determined by the transition matrix P. Suppose that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} |p_k(i,j) - p(i,j)| = 0, \qquad \forall\, i, j \in S, \tag{1.6}$$

and

$$\theta = \sum_{i\in S} \pi(i) \Big[f^2(i) - \Big(\sum_{j\in S} f(j) p(i,j) \Big)^2 \Big] > 0. \tag{1.7}$$

Furthermore, if the sequence of δ-coefficients satisfies

$$\lim_{n\to\infty} \frac{\sum_{k=1}^{n} \delta(P_k)}{\sqrt{n}} = 0, \tag{1.8}$$

then we have

$$\frac{S_n - \mathsf{E}[S_n]}{\sqrt{n\theta}} \overset{D}{\Rightarrow} N(0,1), \tag{1.9}$$

where $\overset{D}{\Rightarrow}$ denotes the convergence in distribution.

We will prove Theorem 1.1 in Section 3.

Remark 1 If the transition probability matrix P is strictly positive and there exist two different states $s, t \in S$ such that $f(s) \neq f(t)$, thus it is easy to see that the inequality (1.7) is assured by Jensen's inequality of conditional expectation.

§2. Example

Here, we give an example in which the Dobrushin's condition (1.4) is not satisfied, but our conditions (1.6) and (1.8) are satisfied.

Example 1 Let $S = \{1, 2\}$, we consider the 2×2 transition matrices on S, for $k = 1, 2, \ldots$, let

$$P_n = \begin{cases} \begin{pmatrix} 1 - 1/2^{k+1} & 1/2^{k+1} \\ 1/2^{k+1} & 1 - 1/2^{k+1} \end{pmatrix} & \text{if } \ n = 2^k; \\ \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} & \text{if } \ n \neq 2^k, \end{cases}$$

the δ-coefficients

$$\delta(P_n) = \begin{cases} 1 - 1/2^k & \text{if } \ n = 2^k; \\ 0 & \text{if } \ n \neq 2^k, \end{cases}$$

so that the α-coefficients

$$\alpha(P_n) = \begin{cases} 1/2^k & \text{if } \ n = 2^k; \\ 1 & \text{if } \ n \neq 2^k. \end{cases}$$

On the one hand, let

$$P = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

Obviously, P is ergodic, and its unique stationary distribution is $\pi = (\pi(1), \pi(2)) = (1/2, 1/2)$. Of course, for $\forall\, n = 2, 3, \ldots$, there exists a positive integer number k such that

$$2^k \leq n < 2^{k+1}.$$

For such number n, there is no difficulty to derive that

$$\frac{1}{n}\sum_{t=1}^{n} |p_t(i,j) - p(i,j)| = \frac{1}{n}\sum_{t=1}^{k} \left| \frac{1}{2} - \frac{1}{2^{t+1}} \right| \leq \frac{1}{2^k}\frac{k}{2} \to 0,$$

as n tends to infinity.

On the other hand, for such the same number n, we have

$$\alpha_n = \min_{1\le t\le n} \alpha(P_t) = \frac{1}{2^k}.$$

Then we have

$$\lim_{n\to\infty} n^{1/3}\alpha_n = 0.$$

But

$$0 \le \frac{\sum\limits_{t=1}^{n}\delta(P_t)}{\sqrt{n}} \le \frac{\sum\limits_{t=1}^{k}\Big(1-\dfrac{1}{2^t}\Big)}{\sqrt{2^k}} \le \frac{k}{\sqrt{2^k}} \to 0, \qquad \text{as } n\to\infty.$$

That is, our conditions (1.6) and (1.8) hold, but the Dobrushin's condition (1.4) is not satisfied.

§3. Proof of Theorem 1.1

Denote

$$D_n = f(X_n) - \mathsf{E}[f(X_n)|X_{n-1}], \quad n\ge 1, \qquad D_0 = 0. \tag{3.1}$$

$$W_n = \sum_{k=1}^{n} D_k. \tag{3.2}$$

Let $\mathcal{F}_n = \sigma(X_k, 0\le k\le n)$, obviously, $\{W_n, \mathcal{F}_n, n\ge 1\}$ is a martingale, so that $\{D_n, \mathcal{F}_n, n\ge 0\}$ is the related martingale difference. For $n=1,2,\ldots$, denote

$$V(W_n) = \sum_{k=1}^{n} \mathsf{E}[D_k^2|\mathcal{F}_{k-1}],$$

and

$$v(W_n) = \mathsf{E}[V(W_n)].$$

It is easy to see that

$$v(W_n) = \mathsf{E}[W_n^2] = \mathsf{E}[V(W_n)].$$

In order to prove Theorem 1.1, we at first state the central limit theorem associated with the stochastic sequence of $\{W_n\}_{n\ge 1}$, which is a key step to prove our main result Theorem 1.1.

Lemma 3.1 Let $\{X_n, n\ge 0\}$ be a nonhomogeneous Markov chain taking values in state space $S=\{1,2,\ldots,b\}$ with initial distribution of equation (1.1) and transition matrices of (1.2). Let f be any function defined on the state space S. Let $P=(p(i,j))_{b\times b}$

be another transition matrix and P irreducible, $\pi = (\pi_1, \pi_2, \ldots, \pi_b)$ is the unique stationary distribution determined by the transition matrix P. Suppose that the conditions (1.6) and (1.7) are satisfied and $\{W_n, n \geq 0\}$ is defined as equation (3.2), then we have

$$\frac{W_n}{\sqrt{n\theta}} \stackrel{D}{\Rightarrow} N(0,1), \tag{3.3}$$

where $\stackrel{D}{\Rightarrow}$ denotes the convergence in distribution.

Our job to prove Lemma 3.1 is based on the following two important statements such as Lemma 3.2 (see Brown (1971) or Hall and Heyde (1980)) and Lemma 3.3 (see Yang (1998)).

Lemma 3.2 Let $(\Omega, \mathcal{F}, \mathsf{P})$ be a probability space and $\{\mathcal{F}_n, n = 1, 2, \ldots\}$ an increasing sequence of σ-algebras. Suppose that $\{M_n, \mathcal{F}_n, n = 1, 2, \ldots\}$ is a martingale, denote its related martingale difference by $\xi_0 = 0$, $\xi_n = M_n - M_{n-1}$ $(n = 1, 2, \ldots)$. For $n = 1, 2, \ldots$, we denote

$$V(M_n) := \sum_{j=1}^{n} \mathsf{E}[\xi_j^2 | \mathcal{F}_{j-1}],$$
$$v(M_n) := \mathsf{E}[V(M_n)],$$

where $\mathcal{F}_0$ is the trivial σ-algebra. Suppose the following conditions are satisfied:

(i)

$$\frac{V(M_n)}{v(M_n)} \stackrel{\mathsf{P}}{\Rightarrow} 1, \tag{3.4}$$

(ii) the Lindeberg condition holds, i.e. for any $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{\sum_{j=1}^{n} \mathsf{E}[\xi_j^2 I(|\xi_j| \geq \epsilon\sqrt{v(M_n)})]}{v(M_n)} = 0,$$

where $I(\cdot)$ denotes the indicator function.

Then we have

$$\frac{M_n}{\sqrt{v(M_n)}} \stackrel{D}{\Rightarrow} N(0,1), \tag{3.5}$$

where $\stackrel{\mathsf{P}}{\Rightarrow}$ and $\stackrel{D}{\Rightarrow}$ denote convergence in probability and in distribution respectively.

Let $\delta_i(\cdot)$ be the Kronecker delta function, that is, $\delta_i(j) = \delta_{ij}$, $(i, j \in S)$. Denote

$$L_n(i) = \sum_{k=0}^{n-1} \delta_i(X_k).$$

Lemma 3.3 (The strong law of large numbers of nonhomogeneous Markov chain) Let $\{X_n, n \geq 0\}$ be a nonhomogeneous Markov chain taking values in state space $S = \{1, 2, \ldots, b\}$ with initial distribution of equation (1.1) and transition matrices of (1.2). Let $P = (p(i,j))_{b\times b}$ be a transition matrix and P irreducible. If (1.6) holds, then

$$\lim_{n\to\infty} \frac{1}{n} L_n(i) = \pi_i \qquad \text{a.e.}, \tag{3.6}$$

where $\pi = (\pi_1, \pi_2, \ldots, \pi_b)$ is the unique stationary distribution determined by the transition matrix P.

Now let's come to prove Lemma 3.1.

Proof of Lemma 3.1 Noting that by using Markov property and the property of the conditional expectation again, we have

$$\begin{aligned}\frac{V(W_n)}{n} &= \frac{1}{n}\sum_{k=1}^{n} \mathsf{E}[D_k^2|\mathcal{F}_{k-1}] \\ &= \frac{1}{n}\sum_{k=1}^{n} \{\mathsf{E}[f^2(X_k)|X_{k-1}] - (\mathsf{E}[f(X_k)|X_{k-1}])^2\} \\ &:= I_1(n) - I_2(n),\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}I_1(n) &= \frac{1}{n}\sum_{k=1}^{n} \mathsf{E}[f^2(X_k)|X_{k-1}] \\ &= \sum_{j\in S}\sum_{i\in S} f^2(j)\frac{1}{n}\sum_{k=1}^{n} p_k(i,j)\delta_i(X_{k-1})\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}I_2(n) &= \frac{1}{n}\sum_{k=1}^{n} (\mathsf{E}[f(X_k)|X_{k-1}])^2 \\ &= \sum_{i\in S}\sum_{j,l\in S} f(j)f(l)\frac{1}{n}\sum_{k=1}^{n} p_k(i,j)p_k(i,l)\delta_i(X_{k-1}).\end{aligned} \tag{3.9}$$

Noting that, on the one hand, by using equation (1.6) we can easily get

$$\lim_{n\to\infty}\left|\frac{1}{n}\sum_{k=1}^{n}\delta_i(X_{k-1})(p_k(i,j)-p(i,j))\right| \leq \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}|p_k(i,j)-p(i,j)| = 0,$$

thus we have

$$\begin{aligned}\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\delta_i(X_{k-1})p_k(i,j) &= \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\delta_i(X_{k-1})p(i,j) \\ &= \lim_{n\to\infty}\frac{1}{n}L_n(i)p(i,j) \\ &= \pi_i p(i,j) \qquad \text{a.e.},\end{aligned} \tag{3.10}$$

where the third equation holds because of (3.6). Combining equations (3.8) and (3.10), we get

$$\begin{aligned}\lim_{n\to\infty} I_1(n) &= \sum_{j\in S}\sum_{i\in S}\pi_i p(i,j) f^2(j) \\ &= \sum_{i\in S}\pi_i f^2(i) \qquad \text{a.e.,}\end{aligned} \tag{3.11}$$

where the second equation holds since $\pi P=\pi$. On the other hand we claim that

$$\lim_{n\to\infty} I_2(n) = \sum_{i\in S}\pi_i\Big[\sum_{j\in S} f(j)p(i,j)\Big]^2 \qquad \text{a.e..} \tag{3.12}$$

In fact, by using equation (1.6) again, denote

$$M=\sup_{j\in S} f(j). \tag{3.13}$$

We can approximate $I_2(n)$ according to the following procedure

$$\begin{aligned}&\Big|I_2(n)-\sum_{i\in S}\sum_{j,l\in S} f(j)f(l)\frac{1}{n}\sum_{k=1}^{n} p(i,j)p(i,l)\delta_i(X_{k-1})\Big| \\ \le\ & M^2\sum_{i\in S}\sum_{j,l\in S}\frac{1}{n}\sum_{k=1}^{n}|p_k(i,j)p_k(i,l)-p(i,j)p(i,l)| \\ \le\ & M^2\sum_{i\in S}\sum_{j,l\in S}\frac{\sum_{k=1}^{n}|p_k(i,j)-p(i,j)|p_k(i,l)+\sum_{k=1}^{n}|p_k(i,l)-p(i,l)|p(i,j)}{n} \\ \le\ & M^2\sum_{i\in S}\sum_{j,l\in S}\frac{\sum_{k=1}^{n}|p_k(i,j)-p(i,j)|+\sum_{k=1}^{n}|p_k(i,l)-p(i,l)|}{n}\to 0, \qquad \text{as}\ \ n\to\infty.\end{aligned}$$

Thus by using Lemma 3.3 again, we obtain

$$\begin{aligned}\lim_{n\to\infty} I_2(n) &= \sum_{i\in S}\sum_{j,l\in S} f(j)f(l)p(i,j)p(i,l)\frac{1}{n}\sum_{k=1}^{n}\delta_i(X_{k-1}) \\ &= \sum_{i\in S}\sum_{j,l\in S}\pi_i f(j)f(l)p(i,j)p(i,l) \qquad \text{a.e.} \\ &= \sum_{i\in S}\pi_i\Big[\sum_{j\in S} f(j)p(i,j)\Big]^2 \qquad \text{a.e..}\end{aligned}$$

Therefore equation (3.12) is true. Combining (3.7), (3.11) and (3.12), we arrive at

$$\lim_{n\to\infty}\frac{V(W_n)}{n} = \sum_{i\in S}\pi_i\Big[f^2(i)-\Big[\sum_{j\in S} f(j)p(i,j)\Big]^2\Big] \qquad \text{a.e.,} \tag{3.14}$$

which implies that

$$\lim_{n\to\infty}\frac{V(W_n)}{n} = \sum_{i\in S}\pi_i\Big[f^2(i)-\Big[\sum_{j\in S} f(j)p(i,j)\Big]^2\Big] \qquad \text{in probability.} \tag{3.15}$$

Note that

$$\begin{aligned}\frac{V(W_n)}{n} &\le \max_{1\le k\le n} \mathsf{E}[D_k^2|X_{k-1}] \\ &= \max_{1\le k\le n} \{\mathsf{E}[f^2(X_k)|X_{k-1}] - (\mathsf{E}[f(X_k)|X_{k-1}])^2\} \\ &\le \max_{i\in S} f^2(i), \end{aligned} \tag{3.16}$$

and S is a finite set, then the random sequence $\{V(W_n)/n, n \ge 1\}$ is uniformly integrable. Combining above two facts, we arrive at by (1.7)

$$\lim_{n\to\infty} \frac{\mathsf{E}[V(W_n)]}{n} = \sum_{i\in S} \pi_i \Big[f^2(i) - \Big[\sum_{j\in S} f(j)p(i,j)\Big]^2\Big] > 0.$$

Thus it follows that

$$\frac{V(W_n)}{v(W_n)} \overset{\mathsf{P}}{\Rightarrow} 1.$$

Similarly to the analysis of inequality (3.16), we also obtain that

$$\{D_n^2 = [f(X_n) - \mathsf{E}[f(X_n)|X_{n-1}]]^2\}$$

is uniformly integrable, so that

$$\lim_{n\to\infty} \frac{\sum\limits_{j=1}^{n} \mathsf{E}D_j^2 I(|D_j| \ge \epsilon\sqrt{n})}{n} = 0,$$

which implies that the Lindeberg condition holds, then we can easily get our conclusion (3.3) by using Lemma 3.2. Thus we complete the proof of Lemma 3.1. □

Now let's come to prove our main result Theorem 1.1 based on Lemma 3.1.

Proof of Theorem 1.1 Noting that

$$S_n - \mathsf{E}[S_n] = W_n + \sum_{k=1}^{n} [\mathsf{E}[f(X_k)|X_{k-1}] - \mathsf{E}[f(X_k)]]. \tag{3.17}$$

Denote

$$\mathsf{P}(X_k = j) = P_k(j), \qquad j \in S,$$

and $M = \sup\limits_{j\in S} f(j)$. Let's come to evaluate the upper bound of $|\mathsf{E}[f(X_k)|X_{k-1}] - \mathsf{E}[f(X_k)]|$.

In fact, by using the C-K formula of Markov chain we can get

$$
\begin{aligned}
|\mathsf{E}[f(X_k)|X_{k-1}] - \mathsf{E}[f(X_k)]| &= \Big|\sum_{j\in S} f(j)P_k(j|X_{k-1}) - \sum_{j\in S} f(j)P_k(j)\Big| \\
&\le \sup_i \Big|\sum_{j\in S} f(j)\Big[P_k(j|i) - \sum_s P_{k-1}(s)P_k(j|s)\Big]\Big| \\
&\le M\sup_i \sum_j \Big|P_k(j|i) - \sum_s P_{k-1}(s)P_k(j|s)\Big| \\
&= M\sup_i \sum_j \Big|\sum_{s\in S} P_{k-1}(s)P_k(j|i) - \sum_{s\in S} P_{k-1}(s)P_k(j|s)\Big| \\
&\le M\sup_i \sum_s P_{k-1}(s) \sup_s \sum_{j\in S} |P_k(j|i) - P_k(j|s)| \\
&= M\sup_{i,s} \sum_{j\in S} |P_k(j|i) - P_k(j|s)| \\
&= 2M\delta(P_k),
\end{aligned}
\tag{3.18}
$$

here

$$\delta(P_k) = \sup_{i,s}\sum_{j\in S}[P_k(j|i) - P_k(j|s)]^+ = \frac{1}{2}\sup_{i,s}\sum_{j\in S}|P_k(j|i) - P_k(j|s)|.$$

Applying condition (1.8), we get

$$\lim_{n\to\infty} \frac{\sum\limits_{k=1}^{n} [\mathsf{E}[f(X_k)|X_{k-1}] - \mathsf{E}[f(X_k)]]}{\sqrt{n}} = 0. \tag{3.19}$$

Then, by using (1.7), (3.3), (3.17) and (3.19), we can arrive at our conclusion (1.9). Thus the proof of Theorem 1.1 is completed. $\square$

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非齐次马氏链的中心极限定理

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本文将针对非齐次马氏链的转移矩阵列在Cesàro收敛意义下, 利用鞅的中心极限定理证明一个不同于Dobrushin结果的非齐次马氏链的中心极限定理.

关键词: 非齐次马氏链, 中心极限定理, 鞅.

学科分类号: O211.6, O211.4.