应用概率统计　第二十九卷
第四期　2013年8月

Chinese Journal of Applied Probability
and Statistics Vol.29 No.4 Aug. 2013

Asymptotical Stability in Probability for Stochastic Bilinear Systems with Markovian Switching *

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Abstract

This paper deals with asymptotical stability in probability in the large for stochastic bilinear systems. Some new criteria for asymptotical stability of such systems have been established in the inequality of mathematic expectation. A sufficient condition for bilinear stochastic jump systems to be asymptotically stable in probability in the large in Markovian switching laws is derived in a couple of Riccati-like inequalities by introducing a nonlinear state feedback controller. An illustrative example shows the effectiveness of the method.

Keywords: Stochastic bilinear systems, asymptotical stable in probability, Markovian switching, feedback controller.

AMS Subject Classification: 93E15.

§1. Introduction

The stability-analysis of nonlinear stochastic jump systems has attracted much attention in recent years. [1] investigated non-fragile control for a class of uncertain discrete switched fuzzy systems with input delay in linear matrix inequalities (LMIs). Besides, the system was robustly stable by the design of the average length of waiting time on the basis of both the switching signal and the candidated controller[1]. Under the effect of certain continuous excitation, robust H_∞ stabilization of uncertain impulsive switched systems was researched in LMIs[2]. Moreover, asymptotical stability in probability for stochastic nonlinear Markov jump systems was studied by designing backstepping controller[3]. In addition, the state-feedback controller was constructed by regarding Markovian switching as constant such that the closed-loop system had a unique solution[3]. Meantime, H_∞ functional filtering was constituted by the state-estimator for stochastic bilinear systems with

*The research was supported by National Natural Science Foundation of China (11172086) and Natural Science Foundation of Hefei University (13KY03ZD).

Received October 25, 2012. Revised May 7, 2013.

multiplicative noises[4]. Stabilization was analyzed specifically in a class of networked impulsive control systems[6]. Recently, to ensure observability and controllability, an output-feedback stabilizing controller for stochastic bilinear systems was proposed to make the system globally be stable on conditions of noiseless or noisy respectively, where key techniques were proposed by periodic switching of the controller and the dead-beat observer[7]. Bilinear stochastic jump systems play an important role in many practical systems, including controlling of the aviation and chemical process, and even mapping-analysis of gene.The asymptotical stability in probability in the large for stochastic jump systems have good robustness and universality. However, asymptotical stability in probability in the large for bilinear stochastic jump systems has not been absolutely investigated. In the present paper, we consider asymptotical stability in probability for bilinear stochastic jump systems with irreducible homogeneous Markovian switching. A nonlinear state feedback controller is primarily presented by periodic switching (Markovian switching laws) of the controller and the dead-beat observer. The sufficient condition for bilinear stochastic jump systems to be asymptotically stable in probability in the large via the candidated controller is gained in a couple of Riccati-like inequalities. Finally, An illustrative example displays the effectiveness of the method.

§2. Main Problem and Assumption

This paper is based on the underlying complete probability space, which is taken to be the quartet $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathsf{P})$ with a filtration $\mathcal{F}_t$. And $\mathcal{F}_t$ is increasing monotonously and right continuous, while $\mathcal{F}_0$ contains all P-null sets. Let $r(t)$ be a right continuous homogeneous Markov process on the probability space, taking values in a finite state space $\mathcal{S} = \{0, 1, 2, \ldots, N\}$. And there is generator $\Gamma = (\gamma_{pq})_{N\times N}$ given by the following probability distribution,

$$P_{pq}(t) = \mathsf{P}\{r(s+t) = q | r(s) = p\} = \begin{cases} \gamma_{pq}t + o(t), & \text{if } p \neq q; \\ 1 + \gamma_{pq}t + o(t), & \text{if } p = q, \end{cases} \tag{2.1}$$

for any $s, t \geq 0$, where $\gamma_{pq} > 0$ is the transition rate from p to q. If $p = q$, and then satisfying $\gamma_{pq} = -\sum\limits_{q=1, q\neq p}^{N} \gamma_{pq}$.

We consider the following stochastic bilinear system with the Markovian switching law,

$$\mathrm{d}x(t) = [A(r(t)) + u(t)N(r(t))]x(t)\mathrm{d}t + J(r(t))x(t)\mathrm{d}w(t), \quad x(t) = \varphi(t),\ t \in [-h, 0], \tag{2.2}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input control vector, $w(t)$ is the general Brownian motion. $A(r(t)), N(r(t)), J(r(t))$ are matrices of the appropriate dimension. $\varphi(t)$ is a continuous differentiable vector-valued initial function on R.

Assumption 2.1 Markov process $r(t)$ is independent of the Brownian motion $w(t)$.

Assumption 2.2 There is a symmetric positive definite matrix $Q(r(t))$ in the bilinear stochastic jump system (2.2). If $x^T(t)[Q(r(t))N(r(t))+N^T(r(t))Q(r(t))]x(t)=0$, and then satisfying $x^T(t)[Q(r(t))A(r(t))+A^T(r(t))Q(r(t))]x(t)<0$, for any $x(t)\neq 0$. Therefore, the system is observable from the literature survey.

In this paper, one designs a nonlinear state feedback control law as follows

$$\begin{aligned} u(t) &= -\lambda(r(t))\xi(t)\,\mathrm{sgn}\{\xi^T(t)[Q(r(t))A(r(t))+A^T(r(t))Q(r(t))]\xi(t)\} \\ &= -\lambda(r(t))\xi(t)\,\mathrm{sgn}(\Phi(r(t))), \end{aligned} \tag{2.3}$$

where, $\xi(t)=x(t)/\|x(t)\|$, $\Phi(r(t))=\xi^T(t)[Q(r(t))A(r(t))+A^T(r(t))Q(r(t))]\xi(t)$, $\lambda(r(t))$ is the constant related to $r(t)$, and $Q(r(t))\in\overline{Q}$,

$$\begin{aligned} \overline{Q} &= \{Q(r(t)):[Q(r(t))A(r(t))+A^T(r(t))Q(r(t))]<0, \\ &\quad [Q(r(t))N(r(t))+N^T(r(t))Q(r(t))]=0\}, \end{aligned} \tag{2.4}$$

$$\mathrm{sgn}(x)=\begin{cases} 1 & \text{if } x>0; \\ 0 & \text{if } x=0; \\ -1 & \text{if } x<0. \end{cases} \tag{2.5}$$

Assumption 2.2 illustrates non-empty sets of $\overline{Q}$. The design of nonlinear state feedback controller (2.3) makes unstable part (bilinear section) of the system separated from the original system.

Assumption 2.3 Non-singular matrix $A(r(t))$ is Schur stable, meaning all the eigenvalues of $A(r(t))$ lies in the unit disk.

General stochastic differential equation:

$$\mathrm{d}x(t)=f(x(t),t,r(t))\mathrm{d}t+g(x(t),t,r(t))\mathrm{d}w(t), \tag{2.6}$$

where, the following analysis is imposed on the Borel measurable functions $f: R^n\times R_+\times S\to R^n$ and $g: R^n\times R_+\times S\to R^{n\times p}$. Both f and g are locally Lipschitz in $x\in R^n$ for all $\tau\geq 0$, namely, for any $R>0$, there is a constant $C_R\geq 0$ such that

$|f(x_1,\tau,p)-f(x_2,\tau,p)|+|g(x_1,\tau,p)-g(x_2,\tau,p)|\le C_R|x_1-x_2|$ for any $(\tau,p)\in R_+\times S$ and $(x_1,x_2)\in U_R=\{\xi:|\xi|\le R\}$. Moreover, $f(0,\tau,p)=g(0,\tau,p)=0$.

For $V(x,t,r(t))\in C^{2,1}(R^n\times R_+\times S;R_+)$, $C^{2,1}(R^n\times R_+\times S;R_+)$ expresses all non-negative continuous function on $R^n\times R_+\times S$. $V(x,t,r(t))$ is continuously twice differentiable in $x(t)$ and once differentiable in t.

$L^2_{\mathcal{F}_0}([-h,0];R^n)$ denotes all $\mathcal{F}_0$ measurable function set of stochastic variables $\xi=\{\xi(\theta):-h\le\theta\le 0\}$ on $C([-h,0];R^n)$ and satisfying $\mathsf{E}\{|\xi(\theta)|^p:-h\le\theta\le 0\}<\infty$. τ_1,τ_2 be bounded stopping times, for $0\le\tau_1\le\tau_2$. If both $V(x,t,r(t))$ and $\Im V(x,t,r(t))$ are bounded on $t\in[\tau_1,\tau_2]$ a.s. where, $\Im V(x,t,r(t))$ satisfies Itô's formula conditions, there exists

$$\mathsf{E}\{V(x,\tau_2,r(\tau_2))-V(x,\tau_1,r(\tau_1))\}=\mathsf{E}\int_{\tau_1}^{\tau_2}\Im V(x,t,r(t))\mathrm{d}t. \tag{2.7}$$

Definition 2.1 [5] The equilibrium of $x(t)=0$ is said to be 1) (weakly) stable in probability, for each $\varepsilon>0$, $\delta>0$, there is an r such that if $t>t_0$, $|x_0|<r$ and $i_0\in S$, and then $\mathsf{P}\{|x(t,\xi)|>\varepsilon\}<\delta$.

2) asymptotically stable in probability in the large if it is stable in probability and, for every $\varepsilon>0$, $x_0\in R^n$ and $i_0\in S$, there exists $\lim\limits_{t\to\infty}\mathsf{P}\{|x(t,\xi)|>\varepsilon\}=0$.

§3. The Main Results and Proofs

Lemma 3.1 [2] A given symmetric positive definite matrix P and arbitrary symmetric matrix Q, there is $\lambda_{\min}(P^{-1}Q)x^TPx\le x^TQx\le\lambda_{\max}(P^{-1}Q)x^TPx$ for any vector $x\in L^2_{\mathcal{F}_0}([-h,0];R^n)$.

Lemma 3.2 [3] Assume that for bilinear stochastic jump system (2.2) has a unique equilibrium solution almost surely sense in $t\in[t_0,\infty)$, and then under the control law (2.3), the existence of positive definite function $V\in C^{2,1}(R^n\times R_+\times S;R_+)$, where $c>0$ and the matrix of $D>0$, such that the following formula is hold for the system (2.2)

$$\mathsf{E}\{V(x,t,r(t))\}\le De^{-c(t-t_0)}, \tag{3.1}$$

$$\overline{V}_R=\sup_{t\ge t_0,|x|<R}V(x,t,r(t))\to 0\Leftrightarrow R\to 0, \tag{3.2}$$

for each $x_0\in R^n$ and $i_0\in S$, the equilibrium $x(t)=0$ of bilinear stochastic jump system (2.2) is asymptotically stable in probability in the large.

Lemma 3.3 [3] The character of f and g is hold for system (2.1)-(2.3). For any $l>0$, define the first exit time η_l as $\eta_l=\inf\{t:t\ge t_0,|x(t)|\ge l\}$. Assume that there exists a positive function $V(x,t,r(t)) \in C^{2,1}(R^n \times R_+ \times S; R_+)$ and parameters d and $D \geq 0$, such that

$$\mathsf{E}\{V(x,\eta_1 \wedge t, r(\eta_1 \wedge t))\} \leq De^{-d(\eta_1 \wedge t - t_0)},$$

$$R \to \infty \Rightarrow V_R = \inf_{t \geq t_0, |x| > R} V(x,t,r(t)) \to \infty.$$

Then, for every $x(t_0) = x_0 \in R^n$ and $r(t_0) = i_0 \in S$, there exists a solution $x(t) = x(x_0, i_0; t, r(t))$, unique up to equivalence of system (2.2).

Lemma 3.4[8] Given M, N and $p > 0$, there exists $\mu > 0$, such that

$$(MyN)^T x^T P + Px(MyN) \leq \mu x^T P^2 x + \mu^{-1} M^T M y^T N^T N y$$

for any vector $x, y \in L^2_{\mathcal{F}_0}([-h, 0]; R^n)$.

Lemma 3.5[10] Given constant matrices $\Omega_1, \Omega_2, \Omega_3$, If $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$, if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0.$$

Let$A(i) = A_i$, $N(i) = N_i$, $J(i) = J_i$, $Q(i) = Q_i$, $i \in S$.

Theorem 3.1 Suppose that all Assumptions 2.1-2.3 hold for the system (2.1)-(2.3). If there are scalar $\mu > 0$, $c = \lambda_{\min}(-P_i S_i) > 0$, and given positive definite matrix $P_i > 0$, satisfy that

$$S_i = \begin{bmatrix} \Theta_i & N_i^T & \lambda_i P_i \\ N_i & -\mu I & 0 \\ \lambda_i^T P_i & 0 & -\mu^{-1} I \end{bmatrix} < 0, \tag{3.3}$$

where $\Theta_i = A_i^T P_i + P_i A_i + J_i^T P_i J_i + \sum\limits_{j=1}^{n} \gamma_{ij} P_j$. Then, the equilibrium $x(t) = 0$ of bilinear stochastic jump system (2.2) is asymptotically stable in probability in the large.

Proof Choosing a Lyapunov function candidate

$$V(x,t,i) = x^T(t) P_i x(t). \tag{3.4}$$

The stochastic derivative of V along a given trajectory of (2.2) is obtained from Itô's formula as follows:

$$\begin{aligned} & \frac{\mathrm{d}}{\mathrm{d}t} V(x(t),t,i) = \Im V(x(t),t,i) \\ = \ & ([A_i - \lambda_i \xi(t)\,\mathrm{sgn}(\Phi) N_i] x(t))^T P_i x(t) + x^T(t) P_i [A_i - \lambda_i \xi(t)\,\mathrm{sgn}(\Phi) N_i] x(t) \\ & + x^T(t) J_i^T P_i J_i x(t) + \sum_{j=1}^{N} \gamma_{ij} P_j + 2x^T(t) P_i J_i x(t) \mathrm{d}w(t). \end{aligned} \tag{3.5}$$

It follows from Lemma 3.4, there exists $\mu > 0$, such that

$$\begin{aligned} & N_i^T \xi^T(t)\,\mathrm{sgn}(\Phi_i)\lambda_i^T P_i + P_i\lambda_i\xi^T(t)\,\mathrm{sgn}(\Phi_i)N_i \\ \leq\ & \mu\lambda_i^T P_i^T P_i\lambda_i + \mu^{-1}\,\mathrm{sgn}(\Phi_i)^2 N_i^T\xi^T(t)\xi(t)N_i \leq \mu\lambda_i^T P_i^2\lambda_i + \mu^{-1}N_i^T N_i. \end{aligned} \tag{3.6}$$

Learning from the Lemma 3.5, (3.3) is equivalent to the following inequality

$$S_i = A_i^T P_i + P_i A_i + \mu\lambda_i^T P_i^2\lambda_i + \mu^{-1}N_i^T N_i + J_i^T P_i J_i + \sum_{j=1}^{N}\gamma_{ij}P_j < 0. \tag{3.7}$$

The equality (3.5) is taken integral on interval $[t_0, t]$, $t \in [t_0, \infty)$ taking expectations on both sides of the equality and combining with (2.7), one gets

$$\begin{aligned} \mathsf{E}\int_{t_0}^{t}\Im V(x,t,r(t))\mathrm{d}t &= \mathsf{E}\{V(x,t,r(t)) - V(x,t_0,r(t_0))\} \\ &\leq \mathsf{E}\{x^T(t)S_i x(t)\} = -\mathsf{E}\{x^T(t)(-S_i)x(t)\}. \end{aligned} \tag{3.8}$$

According to the Lemma 3.1, one obtains

$$\begin{aligned} \mathsf{E}\int_{t_0}^{t}\Im V(x,t,r(t))\mathrm{d}t &\leq \mathsf{E}\{x^T(t)(-S_i)x(t)\} \\ &\leq \lambda_{\max}[-(P_i S_i)]\mathsf{E}\{x^T(t)(-P_i)x(t)\} \\ &\leq -\lambda_{\min}[-(P_i S_i)]\mathsf{E}\{x^T(t)P_i x(t)\} \\ &= -c\mathsf{E}\{V(x,t,r(t))\}. \end{aligned} \tag{3.9}$$

According to the Lemma 3.2, (3.1) is set up by the differential knowledge, in which $D = \varphi^T P_i\varphi$. Obviously, if

$$R \to 0,$$

one gets

$$\sup_{t\geq t_0,|x|<R} V(x,t,r(t)) = \sup_{t\geq t_0,|x|<R} x^T(t)P_i x(t) \to 0. \tag{3.10}$$

Similarly, if

$$\overline{V}_R = \sup_{t\geq t_0,|x|<R} V(x,t,r(t)) \to 0,$$

that is $x^T(t)P_i x(t) \to 0$. Since $P_i > 0$, then $x(t) \to 0$.

Lemma 3.3 state clearly that

$$R \to \infty \Rightarrow V_R = \inf_{t\geq t_0,|x|>R} V(x,r(t)) \to \infty. \tag{3.11}$$

The expression of (3.11) is apparently implied that

$$R \to 0.$$

Moreover, one obtains

$$\overline{V}_R = \sup_{t \geq t_0, |x| < R} V(x, t, r(t)) \to 0 \Leftrightarrow R \to 0.$$

Therefore, the equilibrium state of bilinear stochastic jump system (2.2) is asymptotically stable in probability in the large. □

§4. Simulation

We consider stochastic bilinear system (2.2) as follows:

$$\mathrm{d}x(t) = [A(r(t)) + u(t)N(r(t))]x(t)\mathrm{d}t + J(r(t))x(t)\mathrm{d}w(t),$$

system parameters are as follows:

$$A_i = \begin{bmatrix} 1.5 & 1 \\ -0.5 & 2.5 \end{bmatrix}, \qquad N_i = \begin{bmatrix} -0.01 & 0.1 \\ 0 & -0.06 \end{bmatrix}, \qquad J_i = \begin{bmatrix} -1 & 0 \\ 0.5 & -0.4 \end{bmatrix}.$$

Choose $x_1(0) = [0.3 \quad -0.2]$, $\lambda_i = 0.815$, $\mu = 0.095$, $\varphi = I$(unit matrix), $t_0 = 0.1$, $u_{1\,\min} = -4 \leq u(t) \leq u_{1\,\max} = 5$, $c = [0.8 \ \ 1.6]$, one obtains from (3.3)

$$P_i = \begin{bmatrix} 8.2636 & 0.2826 \\ 0.2826 & 5.3355 \end{bmatrix}.$$

Numerical simulation shows that the system state eventually is driven to be the stable equilibrium state (Figure 1).

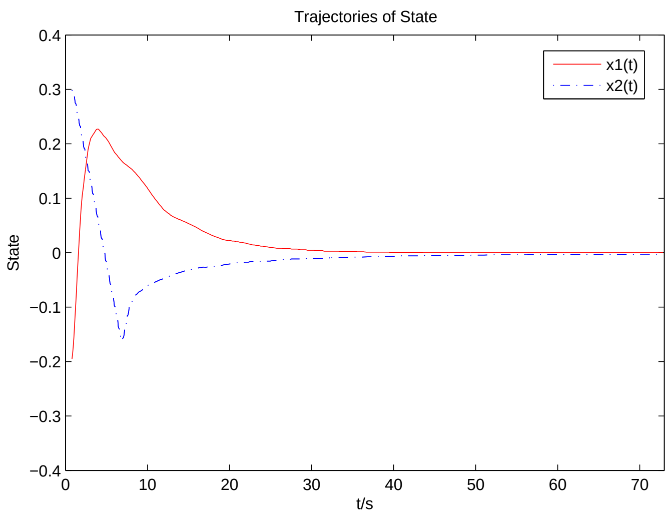

Figure 1 Trajectories of the state

Numerical example displays the evolvement in the expectation of V function to be rapidly converged to the definite zero-value (Figure 2).

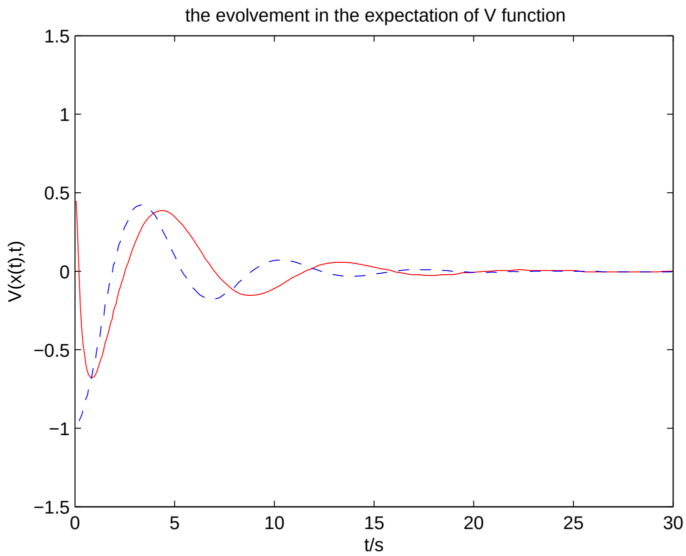

Figure 2 The evolvement in the expectation of V function

§5. Conclusion

In this paper, some new criteria for the asymptotical stability of the jump system have been established in the inequality of mathematic expectation of a Lyapunov function. Sufficient conditions are derived in a couple of Riccati-like inequalities, such that bilinear stochastic jump system is asymptotically stable in probability in the above-mentioned controller. Finally, an example validates effectiveness of the proposed method. In the future, an output feedback controller will be introduced to study asymptotically stable in probability in the large for stochastic bilinear system.

References

[1] Du, H.B., Lin, X.Z. and Li, S.H., Robust exponential stabilization for a class of uncertain switched systems with input delay, *Control and Decision*, **24(9)**(2009), 1316–1320.

[2] Xu, H.L., Liu, X.Z. and Teo, K.L., Robust H_∞ stabilization with definite attenuance of an uncertain impulsive switched system, *The ANZIAM Journal*, **46(4)**(2005), 471–484.

[3] Wu, Z.J., Xie, X.J., Shi, P. and Xia, Y.Q., Backstepping controller design for a class of stochastic nonlinear systems with Markovian switching, *Automatica*, **45(4)**(2009), 997–1004.

[4] Halabi, S., Souley Ali, H., Rafaralahy, H. and Zasadzinski, M., H_∞ functional filtering for stochastic bilinear systems with multiplicative noises, *Automatica*, **45(4)**(2009), 1038–1045.

[5] Liu, S.J., Zhang, J.F. and Jiang, Z.P., Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems, *Automatica*, **43(2)**(2007), 238–251.

[6] Guan, Z.H., Huang, J. and Chen, G.R., Stability analysis of networked impulsive control systems, *Proceedings of the 25th Chinese Control Conference*, 7–11, 2006.

[7] Hanba, S. and Miyasato, Y., Output feedback stabilization of bilinear systems using dead-beat observers, *Automatica*, **37(6)**(2001), 915–920.

[8] Wang, Y., Xie, L. and de Souza, C.E., Robust control of a class of uncertain nonlinear systems, *Systems & Control Letters*, **19(2)**(1992), 139–149.

[9] Khas'minskii, R.Z., *Stochastic Stability of Nonlinear Uncertain Systems*, New York, Springer, 1980.

[10] Li, H. and Fu, M., A linear matrix inequality approach to robust H_∞ filtering, *IEEE Transactions on Signal Processing*, **45(9)**(1997), 2338–2350.

Markov切换随机双线性系统依概率渐进稳定性

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本文研究随机双线性系统大范围渐进稳定性. 利用数学期望不等式给出了随机双线性系统渐进稳定的新标准. 设计了一种非线性状态反馈控制器, 利用类Riccati不等式推出了Markov切换随机双线性系统大范围依概率渐进稳定的充分条件. 数值算例表明本文提出的方法是可行的.

关 键 词: 随机双线性系统, 依概率渐进稳定, Markov切换, 反馈控制器.

学 科 分 类 号: O211.6.