应用概率统计　第二十九卷
第五期　2013年10月

Chinese Journal of Applied Probability
and Statistics Vol.29 No.5 Oct. 2013

Asymptotic Ruin Probability for Cox Risk Model with Variable Premium Rate and Constant Interest Force *

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Abstract

This paper focuses on ruin probability for Cox model with variable premium rate and constant investment return when the claims have heavy tailed distribution. By considering the "skeleton process" of Cox risk model, a recursive equation for finite time ruin probabilities are derived in terms of "renewal techniques" and asymptotic estimation for finite time ruin probabilities and ultimate ruin probability are obtained by inductive method.

Keywords: Cox risk model, heavy tailed distributions, variable premium income, ruin probability.

AMS Subject Classification: 60K05, 62P05, 90A46.

§1. Introduction

Ruin probability is of great importance in risk theory. A good deal of literatures have been presented for solving the ruin probability in classical risk model and some of its generalizations, such as renewal risk model, risk model perturbed diffusions etc., see Asmussen (2000) for comprehensive introduction to this aspect. In the past twenty years, the continuous-time risk processes with interest force or stochastic return on investment received great concern. For example, Cai and Dickson (2002, 2003), Cai (2004), Gerber and Shiu (1997), Paulsen and Gjessing (1997), Grandell (1991), Hipp (2004), Konstantinides et al. (2002), Ng and Yang (2006), Wu and Wei (2004) and references therein. In their risk models, it is often assumed that the premium incomes follow a fixed rate, as an alternative, the risk process with variable premium income rate has recently received an increasing amount of attention in modeling surplus process. For example, Melnikov (2004) studied the ruin probability in a risk model with stochastic premium incomes and all capital of the insurer was invested in stock, like did by Taylor (1980). In this paper, we focus on Cox risk model with variable premium income rate specified by a function of the intensity

*The research was supported by National Natural Science Foundation of China (11126238, 11201006), Humanities and Social Science Projects of Ministry of Education of China (2012YJC2748).

Received October 17, 2012. Revised July 23, 2013.

process of the Cox process, which allows for the dependence of the premium incomes and the claims. This assumption makes our model more reasonable in practice. To our best knowledge, it is the first time to consider the ruin problems under such risk model.

Exponential upper bound estimation for ruin probability is one the main topic in ruin theory, which claims that if the Lundberg coefficient exists, then the ruin probability will decay exponentially w.r.t. the increase of initial surplus. The existence of Lundberg coefficient requires the tail distribution of claims should decay exponentially. Such claims are often named with "small claim" or "light tailed distribution". When the distribution of claims is "heavy tailed", even the mathematical expectation of the claims may not exist, so we can not define the Lundberg coefficient. One common topic on studying the ruin probability for risk model with heavy tailed claims is to find the asymptotic behavior of ruin probability. For example, Embrechts et al. (1997) presented a comprehensive introduction for modeling extremal events in insurance, including asymptotic ruin probabilities when claims are heavy tailed. Cai and Dickson (2004) studied the asymptotic ruin probability for discrete time risk model with dependent investment return when the claims are long-tailed, Wang and Yin (2010) studied the asymptotic ruin probability for risk model with dependent claims, Leipus and Šiaulys (2007) researched the asymptotic ruin probability for Sparre-Anderson risk model with heavy tailed claims, Konstantinides et al. (2002) studied asymptotic ruin probability in the classical risk model with constant interest force in the presence of heavy tails. Relatively, few papers concentrate on the asymptotic ruin probability for Cox risk model with constant interest force and heavy tailed distributed claims and this is the goal of this paper. As results, recursive equations for finite ruin probabilities are derived by "renewal techniques" and asymptotic estimation for finite time ruin probabilities and ultimate ruin probability are obtained by inductive method. This paper is organized as follows. Section 2 presents the introduction to our risk model and the problem to be investigated. Section 3 studies the asymptotic behavior of ruin probability when claims have regularly varying tails with index γ.

§2. Model and Problem

Let $(\Omega,\mathcal{F},\mathsf{P})$ be a complete probability space containing all the variables defined in this paper and the Cox risk model with variable premium income is specified by

$$X_t = u + \int_0^t c(\lambda_s)\mathrm{d}s - \sum_{i=1}^{N_t} Y_i, \tag{2.1}$$

where $u > 0$ is the initial value of the surplus process, $\boldsymbol{N} = \{N_t, t \geq 0\}$ is a Cox process with intensity process $\boldsymbol{\lambda} = \{\lambda_t, t \geq 0\}$. N_t denote the number of claims arrived up to time t. $\boldsymbol{Y} = \{Y_i, i \geq 1\}$ are i.i.d. random variables with $F(x)$ the common cumulative distribution function. $\boldsymbol{\lambda}$ is assumed to be a positive-valued, continuous time Markov chain with phase space $E=\{\alpha_i, i=1,2,\ldots,n\}$ and generator $\boldsymbol{Q}=(q_{ij})_{n\times n}$. Define τ_1 the first time that the process λ leave the initial state, i.e. $\tau_1=\inf\{t: t>0, \lambda_t\neq\lambda_0\}$. By the classical results on continuous time Markov chain, if $q_i:=-q_{ii}=\sum\limits_{j\neq i}q_{ij}<\infty$, then we have the following results:

Lemma 2.1 Suppose that $\lambda_0=\alpha_i$, then for any $\alpha_i\in E$, the following properties holds:

$$\mathsf{P}(\tau_1>t)=\mathrm{e}^{-q_it}; \tag{2.2}$$

$$\mathsf{P}(\tau_1\leq t, \lambda_{\tau_1}=\alpha_j)=(1-\mathrm{e}^{-q_it})\frac{q_{ij}}{q_i}; \tag{2.3}$$

$$\mathsf{P}(\lambda_{\tau_1}=\alpha_j)=\frac{q_{ij}}{q_i}. \tag{2.4}$$

The proof for the Lemma 2.1 can be found in Grandell (1991). Denote by $\mathcal{F}_t^\lambda=\sigma\{\lambda_s, 0\leq s\leq t\}$, $\mathcal{F}_t^X=\sigma\{X_t, 0\leq s\leq t\}$ and $\mathcal{F}_t=\sigma\{(\lambda_s, X_s), 0\leq s\leq t\}$. We shall make strong use of Lemma 2.19 in Grandell (1991), which says that

Lemma 2.2 (i) N_t has independent increments relative to $\mathcal{F}_\infty^\lambda$;

(ii) N_t-N_s is Poisson distribution with mean $\displaystyle\int_s^t\lambda_r\mathrm{d}r$ relative to $\mathcal{F}_\infty^\lambda$.

In classical risk model, one basic requirement is "safety loading", which ensures that the expected net income of the insurer is positive in per unit time interval. In our risk model, since the premium income rate is a random variable $c(\lambda_t)$ and the distribution of λ_t is highly dependent on its initial distribution and the generator $\boldsymbol{Q}$. For exposition ease, it is assumed that Process $\boldsymbol{\lambda}$ is stationary with initial distribution $\pi=(\pi_1,\pi_2,\ldots,\pi_n)$, i.e. $\pi_i=\mathsf{P}(\lambda_0=\alpha_i)$, $i=1,2,\ldots,n$. To make "safety loading" hold, the following property is sufficient and natural for our model

$$\mathsf{E}c(\lambda_t)=\mathsf{E}c(\lambda_0)>\mathsf{E}Y\mathsf{E}\lambda_0:=p\mathsf{E}Y. \tag{2.5}$$

Equation (2.5) ensures that for any $t\geq 0$, the expected total premium income is greater than the expected aggregate claims, because

$$\begin{aligned}&\mathsf{E}\int_0^t c(\lambda_s)\mathrm{d}s=\int_0^t\mathsf{E}c(\lambda_s)ds=t\mathsf{E}c(\lambda_0)>\mathsf{E}\Big[\sum_{i=1}^{N(t)}Y_i\Big]=\mathsf{E}Y\mathsf{E}\Big[\int_0^t\lambda_s\mathrm{d}s\Big]=\mathsf{E}Y\mathsf{E}\lambda_0t\\ \Longleftrightarrow\ &\mathsf{E}c(\lambda_t)=\mathsf{E}c(\lambda_0)>\mathsf{E}Y\mathsf{E}\lambda_0:=p\mathsf{E}Y.\end{aligned}$$

Putting $c(\lambda_t)=(1+\rho)p\mathsf{E}Y$ with $\rho>0$, i.e. $c(\lambda_t)$ is linear function of t, then, our model is the one considered in Grandell (1991). Let L_i denote the arrival time of ith claim, then $\{L_i-L_{i-1}, i\geq 1, L_0=0\}$ are i.i.d. random variables relative to $\mathcal{F}_\infty^\lambda$.

In this paper, we assume that the insurer would like to invest all its surplus to a bond market with investment return rate δ, which can also be regard as the interest force. Then, the dynamic of the surplus process is

$$\mathrm{d}X_t = X_t\delta\mathrm{d}t + c(\lambda_t)\mathrm{d}t - \mathrm{d}Z_t, \tag{2.6}$$

where $Z_t = \sum_{i=1}^{N_t} Y_i$ denote aggregate claims up to time t. By Equation (2.6), it is easy to find that

$$X_t = \mathrm{e}^{\delta t}\Big(u + \int_0^t \mathrm{e}^{-\delta r}c(\lambda_r)\mathrm{d}r - \int_0^t \mathrm{e}^{-\delta r}\mathrm{d}Z_r\Big). \tag{2.7}$$

In the rest of this paper, the risk process to be discussed is the one specified by (2.6). Define by $T_i(u) = \inf\{t : X_t < 0|X_0 = u, \lambda_0 = \alpha_i\}$ the ruin time of X_t with $\lambda_0 = \alpha_i$, $X_0 = u$ and $T(u) = \inf\{t : X_t < 0|X_0 = u\}$ the ruin time of process (2.1), with the convention that $\inf\emptyset = \infty$. Denote the ultimate ruin probability with initial surplus u and initial intensity state α_i by $\psi_i(u)$, i.e.

$$\psi_i(u) = \mathsf{P}\{T_i(u) < \infty\} = \mathsf{P}\Big\{\inf_t X_t < 0|X_0 = u, \lambda_0 = \alpha_i\Big\}, \tag{2.8}$$

the ruin probability with initial surplus u by $\psi(u)$, i.e.

$$\psi(u) = \mathsf{P}\{T(u) < \infty\} = \mathsf{P}\Big\{\inf_t X_t < 0|X_0 = u\Big\} = \sum_i^n \psi_i(u)\pi_i, \tag{2.9}$$

the probability that ruin occurs before or on the i_{th} claim with initial tensity α_i by

$$\psi_{i,n}(u) = \mathsf{P}\{T_i(u) \le L_n|X_0 = u, \lambda_0 = \alpha_i\}, \tag{2.10}$$

the probability that ruin occurs before or on the i_{th} claim by

$$\psi_n(u) = \mathsf{P}\{T \le L_n|X_0 = u\}. \tag{2.11}$$

By "differential arguments", a coupled integral equations for ruin probabilities satisfied by vector $(\psi_1(u), \psi_2(u), \ldots, \psi_d(u))$ is obtained in Xu et al. (2014) and the initial value of coupled equations can be determined explicitly when the claims are "light-tailed". This result principally enables us to computing the ruin probability numerically. However, when the claims are heavy tailed, it is difficult to determine the initial value of the coupled integral equations. Alternatively, we try to determine the asymptotic behavior of ruin probability when initial surplus tends to infinity.

§3. Asymptotic Estimation for Ruin Probability

Note that ruin only takes place when a claim arrived, thus ensuring us to consider the so-called "skeleton-process" of process (2.7) for studying ruin probability. Denote the "discounted skeleton risk process" of process (2.7) by

$$
\begin{aligned}
M_n &:= \mathrm{e}^{-\delta L_n} X_{L_n} = \mathrm{e}^{-\delta L_n}\Big[X_{L_{n-1}}\mathrm{e}^{\delta(L_n-L_{n-1})} + \int_{L_{n-1}}^{L_n} \mathrm{e}^{\delta r} c(\lambda_r)\mathrm{d}r - Y_n\Big] \\
&= u + \int_0^{L_n} \mathrm{e}^{-\delta r} c(\lambda_r)\mathrm{d}r - \sum_{i=1}^{n} Y_i \mathrm{e}^{-\delta L_i} \\
&= u + \sum_{i=1}^{n} \Big[\int_{L_{i-1}}^{L_i} \mathrm{e}^{-\delta r} c(\lambda_r)\mathrm{d}r - Y_i \mathrm{e}^{-\delta L_i}\Big] \qquad (3.1) \\
&= M_{n-1} + \mathrm{e}^{-\delta L_{n-1}}\Big[\int_{L_{n-1}}^{L_n} \mathrm{e}^{-\delta(r-L_{n-1})} c(\lambda_r)\mathrm{d}r - Y_n \mathrm{e}^{-\delta(L_n-L_{n-1})}\Big] \qquad (3.2)
\end{aligned}
$$

with the convention that $L_0 = 0$. The following theorem is basic for this paper.

Theorem 3.1 Finite time ruin probability $\psi_n(u)$ and $\psi_{i,n}(u)$ satisfy the following recursive equation respectively

$$
\begin{aligned}
\psi_n(u) &= \mathsf{E}\Big[\overline{F}(G(L_1)) + \int_0^{G(L_1)} \psi_{n-1}(G(L_1)-y)\mathrm{d}F(y)\Big], \\
\psi_{i,n}(u) &= \mathsf{E}\Big[\overline{F}(G(L_1)) + \sum_{j=1}^{d} H_{ij} \int_0^{G(L_1)} \psi_{j,n-1}(G(L_1)-y)\mathrm{d}F(y)|\lambda_0 = \alpha_i\Big], \\
&\qquad i = 1, 2, \ldots, d, \qquad (3.3)
\end{aligned}
$$

where

$$
\begin{aligned}
G(L_1) &\triangleq u\mathrm{e}^{\delta L_1} + \int_0^{L_1} \mathrm{e}^{\delta(L_1-r)} c(\lambda_r)\mathrm{d}r, \qquad (3.4) \\
H_{ij} &= \mathsf{P}(\lambda_{L_1} = \alpha_j|\lambda_0 = \alpha_i). \qquad (3.5)
\end{aligned}
$$

Proof By calculating whether the first claim cause ruin or not, it follows that

$$
\begin{aligned}
&\psi_n(u) = \mathsf{P}(T \le L_n) = \mathsf{P}\Big(\bigcup_{k=1}^{n}\{M_k < 0\}|M_0 = u\Big) \\
=\;& \mathsf{P}\Big(\bigcup_{k=1}^{n}\{M_k < 0\}|M_1 < 0\Big)\mathsf{P}(M_1 < 0) + \mathsf{P}\Big(\bigcup_{k=1}^{n}\{M_k < 0\}|M_1 > 0\Big)\mathsf{P}(M_1 > 0) \\
=\;& \mathsf{E}\Big[\mathsf{P}(Y_1 > G(L_1)) + \mathsf{P}\Big(\bigcup_{k=2}^{n}\{M_k < 0\}|Y_1 < G(L_1)\Big)\mathsf{P}(Y_1 < G(L_1))\Big] \\
=\;& \mathsf{E}\Big[\overline{F}(G(L_1)) + \int_0^{G(L_1)} \psi_{n-1}(G(L_1)-y)\mathrm{d}F(y)\Big]. \qquad (3.6)
\end{aligned}
$$

With similar discussion to Equation (3.6), we have Equation (3.3) immediately. □

Note that for any $n \ge 1$, $\psi_n(u)$ is decreasing w.r.t. u when u approaches to infinity. Then we have the following lower bound for ruin probability. Letting $n \to \infty$ in Equation

(3.6) yields

$$\begin{aligned}\psi(u) &= \mathsf{E}\Big[\overline{F}(G(L_1)) + \int_0^{G(L_1)} \psi(G(L_1)-y)\mathrm{d}F(y)\Big] \\ &\geq \mathsf{E}\Big[\overline{F}(G(L_1)) + \int_{G(L_1)-u}^{G(L_1)} \psi(u)\mathrm{d}F(y)\Big], \end{aligned} \tag{3.7}$$

which implies that

$$\psi(u) \geq \frac{\mathsf{E}[\overline{F}(G(L_1))]}{1-\mathsf{E}[F(G(L_1))-F(G(L_1)-u)]} = \frac{\mathsf{E}[\overline{F}(G(L_1))]}{\mathsf{E}[\overline{F}(G(L_1))+F(G(L_1-u))]}. \tag{3.8}$$

Definition 3.1 A distribution B defined on $(-\infty,\infty)$ is said to have a regularly varying tail with index γ, or $B\in\mathfrak{R}_\gamma$, if there exists some constant $\gamma>0$ such that for any $y>0$,

$$\lim_{x\to\infty}\frac{\overline{B}(xy)}{\overline{B}(x)} = y^{-\gamma}. \tag{3.9}$$

Definition 3.2 A distribution B is said to have Dominant-tail, or $B\in\mathfrak{D}$, if for any $0<y<1$,

$$\limsup\frac{\overline{B}(xy)}{\overline{B}(x)} < \infty. \tag{3.10}$$

Definition 3.3 A distribution B is said to have long-tailed distribution, or $B\in\mathfrak{L}$, if for any $y>0$

$$\lim\frac{\overline{B}(x+y)}{\overline{B}(x)} = 1. \tag{3.11}$$

It is well known that

$$\mathfrak{R}_{-\gamma}\subset\mathfrak{D}\cap\mathfrak{L} \tag{3.12}$$

and distributions in all the three classes of distributions are heavy tailed. For a review of heavy tailed distributions and their applications, see Embrechts et al. (1997). It is well known that if $F_1\in\mathfrak{R}_{-\gamma}$ and $F_2\in\mathfrak{R}_{-\gamma}$ then $F_1*F_2\in\mathfrak{R}_{-\gamma}$ and

$$\overline{F_1*F_2}(x)\sim\overline{F_1}(x)+\overline{F_2}(x). \tag{3.13}$$

Further, we know that for a distribution F_1 supported on $[0,\infty)$ and a distribution F_2 supported on $(-\infty,\infty)$, the tail of convolution F_1*F_2 satisfies

$$\overline{F_1*F_2} = \overline{F}_1(x) + \int_{-\infty}^{x} F_1(x-y)\mathrm{d}F_2(y). \tag{3.14}$$

We also have, the class $\mathfrak{R}_{-\gamma}$ is closed under tail-equivalences, namely, for two distributions B_1 and B_2, if $B_1(x)\sim CB_2(x)$ for some $c>0$ and $B_1(x)\in\mathfrak{R}_{-\gamma}$, then $B_2(x)\in\mathfrak{R}_{-\gamma}$. Further, it is easy to see that if $B\in\mathfrak{D}$, then for any constant $c>0$, the function $B(cu)/B(u)$ is uniformly bounded in $u\in(-\infty,\infty)$. Moreover, it is obvious that for two positive functions f and g, if $f(x)\sim g(x)$ then there exist a constant $C>0$ such that $f(x)\leq Cg(x)$, $x\geq 0$.

Theorem 3.2 Let $F \in \mathfrak{R}_{-\gamma}$ for some $\gamma > 0$, then for $n = 1, 2, 3, \ldots$,

$$\psi_n(u) \sim \mathsf{E}\Big[\sum_{k=1}^{n} \mathrm{e}^{-k\delta\gamma L_1}\Big]\overline{F}(u), \qquad u \to \infty. \tag{3.15}$$

$$\psi(u) \sim \mathsf{E}\Big[\sum_{k=1}^{\infty} \mathrm{e}^{-k\delta\gamma L_1}\Big]\overline{F}(u), \qquad u \to \infty. \tag{3.16}$$

For $i = 1, 2, \ldots, d$ and $n = 1, 2, 3, \ldots$ we have

$$\psi_{i,n}(u) \sim \mathsf{E}\Big[\sum_{k=1}^{n} \mathrm{e}^{-k\delta\gamma L_1}\big|\lambda_0 = \alpha_i\Big]\overline{F}(u), \qquad u \to \infty. \tag{3.17}$$

$$\psi_i(u) \sim \mathsf{E}\Big[\sum_{k=1}^{\infty} \mathrm{e}^{-k\delta\gamma L_1}\big|\lambda_0 = \alpha_i\Big]\overline{F}(u), \qquad u \to \infty. \tag{3.18}$$

Proof For understanding ease, we prove Equation (3.15) and Equation (3.16) firstly. By Equation (3.6), it follows that

$$\begin{aligned}\lim_{u\to\infty} \frac{\psi_1(u)}{\overline{F}(u)} &= \lim_{u\to\infty} \frac{\mathsf{P}(T(u) \le 1)}{\overline{F}(u)} = \lim_{u\to\infty} \mathsf{E}\Big[\frac{\overline{F}(G(L_1))}{\overline{F}(u)}\Big] \\ &= \lim_{u\to\infty} \mathsf{E}\Big[\frac{\overline{F}(G(L_1))}{\overline{F}(u\mathrm{e}^{\delta L_1})}\frac{\overline{F}(u\mathrm{e}^{\delta L_1})}{\overline{F}(u)}\Big]. \end{aligned} \tag{3.19}$$

By dominated convergence theorem, Equation (3.11) and Equation (3.9), it follows that the limit of Equation (3.19) is

$$\mathsf{E}\Big[\lim_{u\to\infty} \frac{\overline{F}(G(L_1))}{\overline{F}(u\mathrm{e}^{\delta L_1})}\frac{\overline{F}(u\mathrm{e}^{\delta L_1})}{\overline{F}(u)}\Big] = \mathsf{E}\Big[\mathrm{e}^{-\delta\gamma L_1}\Big] < 1. \tag{3.20}$$

Note that

$$\psi_{n+1}(u) = \mathsf{E}\Big[\overline{F}(G(L_1)) + \int_0^{G(L_1)} \psi_n(G(L_1) - y)\mathrm{d}F(y)\Big].$$

Let $\Upsilon_n = 1 - \psi_n(u)$, by property (3.14), one can easily find that

$$\overline{\Upsilon}_n(u) = \mathsf{E}[\overline{\Upsilon_{n-1} * F}(G(L_1))] \sim \mathsf{E}[\overline{\Upsilon}_{n-1}(G(L_1)) + \overline{F}(G(L_1))], \qquad u \to \infty.$$

Then, obviously,

$$\begin{aligned}\lim_{u\to\infty} \frac{\psi_2(u)}{\overline{F}(u)} &= \lim_{u\to\infty} \frac{\overline{\Upsilon}_2(u)}{\overline{F}(u)} = \lim_{u\to\infty} \frac{\mathsf{E}[\overline{\Upsilon}_1(G(L_1)) + \overline{F}(G(L_1))]}{\overline{F}(u)} \\ &= \mathsf{E}\big[\mathrm{e}^{-\delta\gamma L_1}(1 + \mathrm{e}^{-\delta\gamma L_1})\big]. \end{aligned} \tag{3.21}$$

$$\begin{aligned}\lim_{u\to\infty} \frac{\psi_3(u)}{\overline{F}(u)} &= \lim_{u\to\infty} \frac{\overline{\Upsilon}_3(u)}{\overline{F}(u)} = \lim_{u\to\infty} \frac{\mathsf{E}[\overline{\Upsilon}_2(G(L_1)) + \overline{F}(G(L_1))]}{\overline{F}(u)} \\ &= \mathsf{E}\big[\mathrm{e}^{-\delta\gamma L_1}(1 + \mathrm{e}^{-\delta\gamma L_1} + 2\mathrm{e}^{-\delta\gamma L_1})\big]. \end{aligned} \tag{3.22}$$

By an inductive method, we assume that for $n = k \ge 1$,

$$\psi_k(u) \sim \mathsf{E}\Big[\sum_{i=1}^{k} \mathrm{e}^{-i\delta\gamma L_1}\Big]\overline{F}(u), \qquad u \to \infty. \tag{3.23}$$

Then, for $n = k+1$, we have

$$\begin{aligned}
\lim_{u\to\infty}\frac{\psi_{k+1}(u)}{\overline{F}(u)} &= \lim_{u\to\infty}\frac{\overline{\Upsilon}_{k+1}(u)}{\overline{F}(u)} \\
&= \lim_{u\to\infty}\frac{\mathsf{E}[\overline{\Upsilon}_k(G(L_1))+\overline{F}(G(L_1))]}{\overline{F}(u)} = \lim_{u\to\infty}\frac{\mathsf{E}[\psi_k(G(L_1))+\overline{F}(G(L_1))]}{\overline{F}(u)} \\
&= \mathsf{E}\Big[\mathrm{e}^{-\delta\gamma L_1}\Big(1+\sum_{i=1}^{k}\mathrm{e}^{-i\delta\gamma L_1}\Big)\Big] = \mathsf{E}\Big[\sum_{i=1}^{k+1}\mathrm{e}^{-i\delta\gamma L_1}\Big]. \qquad (3.24)
\end{aligned}$$

Letting $k\to\infty$ in Equation (3.24) yields

$$\psi(u) \sim \mathsf{E}\Big[\frac{1}{\mathrm{e}^{\delta\gamma L_1}-1}\Big]\overline{F}(u). \qquad (3.25)$$

Denote $1-\psi_{i,n}(u)$ by $\Upsilon_{i,n}(u)$, similar to Equation (3.21), we have

$$\overline{\Upsilon}_{i,n} = \mathsf{E}\Big[\sum_{j=1}^{d}H_{ij}(\overline{\Upsilon}_{j,n-1}(G(\widetilde{L}_1))+\overline{F}(G(\widetilde{L}_1)))|\lambda_0=\alpha_i\Big], \qquad (3.26)$$

where $\widetilde{L}_1$ is a copy of L_1 (thus resembling the same distribution with L_1) and independent of λ_t. With a similar discussion to Equation (3.19) and (3.23), we have

$$\lim_{u\to\infty}\frac{\psi_{i,1}(u)}{\overline{F}(u)} = \mathsf{E}\big[\mathrm{e}^{-\delta\gamma L_1}|\lambda_0=\alpha_i\big] \qquad (3.27)$$

and assume that

$$\psi_{i,k}(u) \sim \mathsf{E}\Big[\sum_{i=1}^{k}\mathrm{e}^{-i\delta\gamma L_1}|\lambda_0=\alpha_i\Big]\overline{F}(u), \qquad u\to\infty. \qquad (3.28)$$

Then

$$\begin{aligned}
\lim_{u\to\infty}\frac{\psi_{i,k+1}(u)}{\overline{F}(u)} &= \mathsf{E}\Big[\Big(\mathrm{e}^{-\delta\gamma L_1}+\sum_{j=1}^{d}H_{ij}\mathsf{E}\Big[\sum_{l=1}^{k}\mathrm{e}^{-l\delta\gamma L_1}|\lambda_{L_1}=\alpha_j\Big]\Big)\Big|\lambda_0=\alpha_i\Big] \\
&= \mathsf{E}\Big[\sum_{l=1}^{k+1}\mathrm{e}^{-l\delta\gamma L_1}|\lambda_0=\alpha_i\Big]. \qquad (3.29)
\end{aligned}$$

This completes the proof. □

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带有常值利息力的可变保费Cox风险模型下破产概率的渐近估计

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本文研究了一类Cox模型下的理赔为重尾分布时破产概率的渐近估计. 假设保费收取费率是Cox计数过程的强度过程的函数, 通过更新技巧得到了有限时间破产概率的递推方程和终极破产概率的积分方程, 利用归纳递推的方法, 得到了终极破产概率的渐近估计.

关键词: Cox风险模型, 重尾分布, 可变保费, 破产概率.
学科分类号: O212.3.