应用概率统计 第三十卷
第二期 2014年4月

Chinese Journal of Applied Probability
and Statistics Vol.30 No.2 Apr. 2014

The Pricing of Credit Securities with Counterparty Risk using a Contagion Model *

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Abstract

In this paper, we study the pricing of defaultable bonds and credit default swaps with counterparty risk using a contagion model. We present a contagion model of correlated defaults in a reduced model. The model assumes the intensities of default processes depend on the stochastic interest rate process driven by a stochastic differential equation and the default process of a counterparty. These are extensions of the models in Jarrow and Yu (2001) and Hao and Ye (2011). Moreover, we derive the explicit formulae for the pricing of defaultable bonds and credit default swap with counterparty risk using the properties of stochastic exponentials and make some numerical analysis on the explicit formulae.

Keywords: Contagion model, defaultable bond, credit default swap, counterparty risk, stochastic exponential.

AMS Subject Classification: 91B24.

§1. Introduction

Defaultable bonds and credit default swap are important credit securities and have been popular for managing and hedging credit risk in the market. There have been mainly two important models dealing with the pricing of these credit securities: the structural model and the reduced-form model. The structural model initially proposed in Black and Scholes (1973), Merton (1974), Black and Cox (1976), could give an intuitive understanding. Comparing with the structural model, the reduced-form model could give a more flexible and tractable model, which is originally proposed by Lando (1994, 1998), and Duffie and Singleton (1999). The default processes occur unexpectably at exogenous intensity processes in their models.

*The project was supported by the Youth Foundation of Suzhou Vocational University (2011SZDQ26).
Received June 27, 2012. Revised October 9, 2013.
doi: 10.3969/j.issn.1001-4268.2014.02.001

In recent years, the reduced-form model has been studied extensively. Jarrow and Yu (2001) present the primary-secondary framework that avoids looping default and simplifies the payoff structure where the protection seller's payments are made only at the maturity of credit default swap. Leung and Kwork (2005) employ the change of measure introduced by Collins-Dufresne et al. (2004) to derive the joint density function in their valuation procedure of the swap rate. Yu (2007) applies the total hazard construction approach to price credit default swap and basket credit default swaps. Hao and Ye (2011) apply the techniques in Park (2008) to the pricing of bonds and credit default swap, where the intensities of default processes depend on the stochastic interest rate process driven by an extended Vasicek model, extending the models in Jarrow and Yu (2001). Our model is similar to Jarrow and Yu (2001) and Hao and Ye (2011). However, we present a more structural specification of the stochastic interest rate process. The stochastic interest rate process is driven by a stochastic differential equation which is the spirit of the approaches of Heath, Jarrow and Morton (1992), Bjork, Kabanov and Runggaldier (1997). Moreover, we derive the pricing of defaultable bonds and credit default swaps with counterparty risk using the properties of stochastic exponentials in Applebaum (2004).

So the aim of this paper is to give a method to price the defaultable bonds and credit default swap with stochastic interest rate process driven by a stochastic differential equation in the reduced-form model. The remainder of this paper is organized as follows. In Section 2 we present the contagion model of correlated defaults in the primary-secondary framework. The model assumes the intensities of default processes depend on the stochastic interest rate process driven by a stochastic differential equation and the default process of a counterparty. These are extensions of the models in Jarrow and Yu (2001) and Hao and Ye (2011). In Section 3 and Section 4, we derive the explicit formulae for the defaultable bonds and credit default swap with counterparty risk using the properties of stochastic exponentials, respectively. In Section 5, we make some numerical analysis on the explicit formulae for the pricing of defaultable bonds and credit default swap with counterparty risk. We conclude in Section 6.

§2. Model Setup

Let $(\Omega, \mathcal{G}, (\mathcal{G}_t)_{t=0}^{T}, \mathsf{P})$ be a filtered probability space satisfying the usual conditions, where $\mathcal{G} = \mathcal{G}_T$, and P is an equivalent martingale measure under which the prices of all discounted securities are martingales. The random default processes are modeled by Cox processes. In Cox processes, the intensities of default, which measure the likelihood of default per unit time, are stochastic processes that depend on a set of economy-wide state variables. On this probability space there is a process X_t, which represents the economy-wide state variables. There are also l point processes N_t^i, $i=1,\ldots,l$, initialized at 0, which represent the default processes of l companies, respectively.

The filtration $\mathcal{G}_t$ is generated collectively by the information contained in the state variables and the default processes companies and defined as follows:

$$\mathcal{G}_t=\mathcal{F}_t^X\vee\mathcal{H}_t^1\vee\cdots\vee\mathcal{H}_t^l, \tag{2.1}$$

where

$$\mathcal{F}_t^X=\sigma(X_s:0\le s\le t) \tag{2.2}$$

and

$$\mathcal{H}_t^i=\sigma(N_s^i:0\le s\le t)\qquad(i=1,\ldots,l) \tag{2.3}$$

are the natural filtrations generated by X_t and N_t^i $(i=1,\ldots,l)$, respectively. Let

$$\mathcal{K}_t^i=\mathcal{F}_T^X\vee\mathcal{H}_T^1\vee\cdots\vee\mathcal{H}_T^{i-1}\vee\mathcal{H}_t^i\vee\mathcal{H}_T^{i+1}\vee\cdots\vee\mathcal{H}_T^l, \tag{2.4}$$

then $\mathcal{K}_0^i=\mathcal{F}_T^X\vee\mathcal{H}_T^1\vee\cdots\vee\mathcal{H}_T^{i-1}\vee\mathcal{H}_T^{i+1}\vee\cdots\vee\mathcal{H}_T^l$.

Denote $N_t^i=I_{\tau^i\le t}$ and assume that τ^i has a strictly positive $\mathcal{K}_0^i$-adapted intensity process λ_t^i satifying $\displaystyle\int_0^t\lambda_s^i\mathrm{d}s<\infty$, a.s., $0\le t\le T$, then the conditional and unconditional distributions of τ^i are given by

$$\mathsf{P}(\tau^i>t|\mathcal{K}_0^i)=\exp\Big(-\int_0^t\lambda_s^i\mathrm{d}s\Big),\qquad 0\le t\le T, \tag{2.5}$$

$$\mathsf{P}(\tau^i>t)=\mathsf{E}\big[\mathrm{e}^{-\int_0^t\lambda_s^i\mathrm{d}s}\big],\qquad i=1,\ldots,l,\ 0\le t\le T. \tag{2.6}$$

We suppose that the existence of an $\mathcal{F}_t^X$-adapted stochastic interest rate process $r(t)$, which is given by a stochastic differential equation

$$\mathrm{d}r(t)=l(t)\mathrm{d}t+b_1(t)\mathrm{d}W_1(t)+b_2(t)\mathrm{d}W_2(t)+\int_E q(t,x)J(\mathrm{d}t\times\mathrm{d}x), \tag{2.7}$$

where $W_1(t)$ and $W_2(t)$ are Wiener processes, J is a Poisson random measure on a measurable space (E,ε) with intensity ν and jump size distribution F. We assume that $W_1(t)$ and $W_2(t)$ are mutually independent, $\nu([0,t]\times E)<\infty$, P-a.s., and $l(t),b_1(t),q(t,x)$ are chosen to satisfy standard conditions, see Heath et al. (1992), Bjork et al. (1997). The stochastic interest rate process is similar to the forward rate process, except that both the time and maturity arguments vary simultaneously.

To be convenient, we use the time-t forward interest rate process $f(t,u)$ instead of the stochastic interest rate process in (2.7). For all $u \ge t$, let

$$\begin{aligned} f(t,u) &= f(0,u) + \int_0^t \alpha(s,u)\mathrm{d}s + \int_0^t \sigma_1(s,u)\mathrm{d}W_1(s) \\ &\quad + \int_0^t \sigma_2(s,u)\mathrm{d}W_2(s) + \int_0^t\int_E \delta(s,x,u)J(\mathrm{d}s\times\mathrm{d}x), \end{aligned} \tag{2.8}$$

then $r(u)$ can be expressed as

$$\begin{aligned} r(u) &= f(u,u) \\ &= f(0,u) + \int_0^u \alpha(s,u)\mathrm{d}s + \int_0^u \sigma_1(s,u)\mathrm{d}W_1(s) \\ &\quad + \int_0^u \sigma_2(s,u)\mathrm{d}W_2(s) + \int_0^u\int_E \delta(s,x,u)J(\mathrm{d}s\times\mathrm{d}x), \end{aligned}$$

where

$$\begin{aligned} l(t) &= \alpha(t,t) + \int_0^t \frac{\partial\alpha(s,T)}{\partial T}\Big|_{T=t}\mathrm{d}s + \int_0^t \frac{\partial\sigma_1(s,T)}{\partial T}\Big|_{T=t}\mathrm{d}W_1(s) \\ &\quad + \int_0^t \frac{\partial\sigma_2(s,T)}{\partial T}\Big|_{T=t}\mathrm{d}W_2(s) + \int_0^t\int_E \frac{\partial\delta(s,x,T)}{\partial T}\Big|_{T=t}J(\mathrm{d}s\times\mathrm{d}x), \\ b_1(t) &= \sigma_1(t,t), \qquad b_2(t) = \sigma_2(t,t), \qquad q(t,x) = \delta(t,x,t). \end{aligned}$$

So

$$\begin{aligned} r(u) &= f(t,u) + \int_t^u \alpha(s,u)\mathrm{d}s + \int_t^u \sigma_1(s,u)\mathrm{d}W_1(s) \\ &\quad + \int_t^u \sigma_2(s,u)\mathrm{d}W_2(s) + \int_t^u\int_E \delta(s,x,u)J(\mathrm{d}s\times\mathrm{d}x). \end{aligned} \tag{2.9}$$

Remark 1 Assuming that $\sigma_1(s,u) = \sigma_1$ and $\sigma_2(s,u) = \sigma_2\mathrm{e}^{l(T-s)/2}$, where σ_1, σ_2 and l are strictly positive constants, $\{W_1(t) : t \in [0,T]\}$ can be interpreted as a "long-run factor" and $\{W_2(t) : t \in [0,T]\}$ can be interpreted as a spread between a "short" and "long term factor"; $\{W_2(t) : t \in [0,T]\}$ affects the forward rates with short maturity more than it does long maturity (See Brenner and Jarrow, 1993).

Supposing that the face value of bond is 1 dollar, the money market account, default-free and defaultable bond prices are, respectively, given by

$$B(t) = \exp\Big[\int_0^t r(s)\mathrm{d}s\Big], \tag{2.10}$$

$$p(t,T) = \mathsf{E}\Big[\frac{B(t)}{B(T)}\Big|\mathcal{G}_t\Big] = \mathsf{E}\big[\mathrm{e}^{-\int_t^T r(s)\mathrm{d}s}\big|\mathcal{G}_t\big], \tag{2.11}$$

$$V^i(\theta^i,t,T) = \mathsf{E}\Big[\frac{B(t)}{B(T)}(\theta^i I_{\tau^i\le T} + I_{\tau^i>T})\Big|\mathcal{G}_t\Big], \qquad i = 1,\dots,l, \tag{2.12}$$

where θ^i is the constant recovery rate of defaultable bond of firm i. Further, see Jarrow and Yu (2001), the defaultable bond prices can also be expressed as

$$V^i(\theta^i,t,T)=\theta^i p(t,T)+I_{\tau_i>t}(1-\theta^i)\mathsf{E}\big[\mathrm{e}^{-\int_t^T(r(s)+\lambda_s^i)\mathrm{d}s}\big|\mathcal{G}_t\big],\qquad i=1,\ldots,l. \tag{2.13}$$

Now we present the contagion model of correlated defaults in the primary-secondary framework, which was originally proposed in Jarrow and Yu (2001). We divide l firms into two mutually exclusive types: n primary firms and $l-n$ secondary firms. Default processes of primary firms only depend on the economy-wide state variables, while default processes of secondary firms depend on the economy-wide state variables and the default processes of the primary firms.

We define the default times of primary firms j as

$$\tau^j=\inf\Big\{t\geq 0:\int_0^t\lambda_s^j\mathrm{d}s\geq E^j\Big\},\qquad j=1,\ldots,n, \tag{2.14}$$

where the intensity λ_t^j is adapted to $\mathcal{F}_t^X$ and $\{E^j:j=1,\ldots,n\}$ is a set of independent unit exponential random variables, which are also independent of X_t, then the conditional and unconditional distributions of τ^j are given by

$$\mathsf{P}(\tau^j>t|\mathcal{F}_T^X)=\exp\Big(-\int_0^t\lambda_s^j\mathrm{d}s\Big),\qquad 0\leq t\leq T, \tag{2.15}$$

$$\mathsf{P}(\tau^j>t)=\mathsf{E}\big[\mathrm{e}^{-\int_0^t\lambda_s^j\mathrm{d}s}\big],\qquad j=1,\ldots,n,\ 0\leq t\leq T. \tag{2.16}$$

For primary firms, we assume that their intensities have the following expression:

$$\lambda_t^j=b_0^j(t)+b^j(t)r(t),\qquad j=1,\ldots,n,\ 0\leq t\leq T,$$

where $b_0^j(t)$ and $b^j(t)$ $(j=1,\ldots,n)$ are deterministic functions.

Then we add another set of independent unit exponential random variables, which are independent of both X_t and $\{\tau^j:j=1,\ldots,n\}$. If we denote them by $\{E^k:k=n+1,\ldots,l\}$, then the default times of secondary firms can be similarly defined as

$$\tau^k=\inf\Big\{t\geq 0:\int_0^t\lambda_s^k\mathrm{d}s\geq E^k\Big\},\qquad k=n+1,\ldots,l, \tag{2.17}$$

where the intensity λ_t^k is adapted to $\mathcal{F}_t^X\vee\mathcal{H}_t^1\vee\cdots\vee\mathcal{H}_t^n$, then the conditional and unconditional distributions of τ^k are given by

$$\mathsf{P}(\tau^k>t|\mathcal{F}_T^X\vee\mathcal{H}_T^1\vee\cdots\vee\mathcal{H}_T^n)=\exp\Big(-\int_0^t\lambda_s^k\mathrm{d}s\Big),\qquad 0\leq t\leq T, \tag{2.18}$$

$$\mathsf{P}(\tau^k>t)=\mathsf{E}\big[\mathrm{e}^{-\int_0^t\lambda_s^k\mathrm{d}s}\big],\qquad k=n+1,\ldots,l,\ 0\leq t\leq T. \tag{2.19}$$

For secondary firms, we assume that their intensities have the following expression:

$$\lambda_t^k = b_0^k(t) + b^k(t)r(t) + \sum_{j=1}^{n} b_j^k(t) I_{\tau^j \le t}, \qquad k = n+1, \ldots, l, \ 0 \le t \le T,$$

where $b_0^k(t), b^k(t)$ and $b_j^k(t)$ $(j = 1, \ldots, n;\ k = n+1, \ldots, l)$ are deterministic functions.

§3. The Pricing of Defaultable Bonds

In this section, we only consider the case with two firms. Firm A is the primary firm whose default process is independent of the default process of secondary firm B but depends on the stochastic interest rate process $r(t)$, while firm B's default process is correlated with the default process of firm A and the stochastic interest rate process $r(t)$. We assume their default processes, respectively, satisfy the following relations:

$$\lambda_t^A = b_0^A(t) + b_1^A(t)r(t), \qquad 0 \le t \le T, \tag{3.1}$$

$$\lambda_t^B = b_0^B(t) + b_1^B(t)r(t) + b^B(t) I_{\tau^A \le t}, \qquad 0 \le t \le T, \tag{3.2}$$

where $b_0^A(t), b_1^A(t), b_0^B(t), b_1^B(t)$ and $b^B(t)$ are deterministic functions.

Now We provide the following extra assumptions of the model:

(i) $b^B(t)$ is either positive and decreasing or negative and increasing, such that $\lim\limits_{t\to\infty} b^B(t) = 0$;

(ii) Both λ_t^j and λ_t^j are positive processes.

We assume that the bonds issued by firm A and B have the same maturity date T. Denote θ^A and θ^B be the constant recoveries of the bonds issued by firm A and B, respectively, then the prices of defaultable bonds issued by firm A and B can be expressed by

$$V^A(\theta^A, t, T) = \theta^A p(t,T) + I_{\tau_A > t}(1-\theta^A)\mathsf{E}\big[\mathrm{e}^{-\int_t^T (r(s)+\lambda_s^A)\mathrm{d}s}\big|\mathcal{G}_t\big] \tag{3.3}$$

and

$$V^B(\theta^B, t, T) = \theta^B p(t,T) + I_{\tau_B > t}(1-\theta^B)\mathsf{E}\big[\mathrm{e}^{-\int_t^T (r(s)+\lambda_s^B)\mathrm{d}s}\big|\mathcal{G}_t\big]. \tag{3.4}$$

To price the bonds issued by firm A and firm B, we firstly calculate $\mathsf{E}\big[\mathrm{e}^{-\int_t^T a(u)r(u)\mathrm{d}u}\big|\mathcal{G}_t\big]$ using Fubini's theorem and the property of stochastic exponentials, where $a(u)$ $(0 \le u \le T)$ is a deterministic function. About Fubini's theorem and the property of stochastic exponentials, can be found in Applebaum (2004).

Theorem 3.1 Let $h(ar,t,T)=\mathsf{E}\big[\mathrm{e}^{-\int_t^T a(u)r(u)\mathrm{d}u}\big|\mathcal{G}_t\big]$, then $h(ar,t,T)$ has the following expression:

$$\begin{aligned} h(ar,t,T) &= \exp\Big\{-\int_t^T\Big[a(u)f(t,u)+\int_t^u a(u)\alpha(s,u)\mathrm{d}s\Big]\mathrm{d}u+\frac{1}{2}K(a,t,T) \\ &\quad +\int_t^T\int_E\big[\mathrm{e}^{-\int_s^T a(u)\delta(s,x,u)\mathrm{d}u}-1\big]\nu F(\mathrm{d}s\times\mathrm{d}x)\Big\}, \end{aligned}\tag{3.5}$$

where

$$K(a,t,T)=\int_t^T\mathrm{d}s\Big[\Big(\int_s^T a(u)\sigma_1(s,u)\mathrm{d}u\Big)^2+\Big(\int_s^T a(u)\sigma_2(s,u)\mathrm{d}u\Big)^2\Big],$$

for all $0\le t\le T$.

Proof Since $r(t)$ has an explicit expression as (2.9), we get

$$\begin{aligned} & -\int_t^T a(u)r(u)\mathrm{d}u \\ =& -\int_t^T a(u)f(t,u)\mathrm{d}u-\int_t^T\mathrm{d}u\int_t^u a(u)\alpha(s,u)\mathrm{d}s-\int_t^T\mathrm{d}u\int_t^u a(u)\sigma_1(s,u)\mathrm{d}W_1(s) \\ & -\int_t^T\mathrm{d}u\int_t^u a(u)\sigma_2(s,u)\mathrm{d}W_2(s)-\int_t^T\mathrm{d}u\int_t^u\int_E a(u)\delta(s,x,u)J(\mathrm{d}s\times\mathrm{d}x) \\ =& \ g_1(t,T)+g_2(t,T)+g_3(t,T), \end{aligned}\tag{3.6}$$

where

$$\begin{aligned} g_1(t,T) &= -\int_t^T a(u)f(t,u)\mathrm{d}u-\int_t^T\mathrm{d}u\int_t^u a(u)\alpha(s,u)\mathrm{d}s, \\ g_2(t,T) &= -\int_t^T\mathrm{d}u\int_t^u a(u)\sigma_1(s,u)\mathrm{d}W_1(s)-\int_t^T\mathrm{d}u\int_t^u a(u)\sigma_2(s,u)\mathrm{d}W_2(s), \\ g_3(t,T) &= -\int_t^T\mathrm{d}u\int_t^u\int_E a(u)\delta(s,x,u)J(\mathrm{d}s\times\mathrm{d}x). \end{aligned}$$

Using Fubini's theorem, we have

$$g_2(t,T)=\int_t^T\mathrm{d}W_1(s)\int_s^T[-a(u)\sigma_1(s,u)]\mathrm{d}u+\int_t^T\mathrm{d}W_2(s)\int_s^T[-a(u)\sigma_2(s,u)]\mathrm{d}u$$

and

$$g_3(t,T)=\int_t^T\int_E J(\mathrm{d}s\times\mathrm{d}x)\int_s^T[-a(u)\delta(s,x,u)]\mathrm{d}u.$$

Further by the properties of stochastic exponentials, we know

$$\begin{aligned} &\exp\Big\{\int_0^t\mathrm{d}W_1(s)\int_s^T[-a(u)\sigma_1(s,u)]\mathrm{d}u-\frac{1}{2}\int_0^t\Big[\int_s^T a(u)\sigma_1(s,u)\mathrm{d}u\Big]^2\mathrm{d}s\Big\}, \\ &\exp\Big\{\int_0^t\mathrm{d}W_2(s)\int_s^T[-a(u)\sigma_2(s,u)]\mathrm{d}u-\frac{1}{2}\int_0^t\Big[\int_s^T a(u)\sigma_2(s,u)\mathrm{d}u\Big]^2\mathrm{d}s\Big\} \end{aligned}$$

and

$$\exp\Big\{\int_0^t\int_E J(\mathrm{d}s\times\mathrm{d}x)\int_s^T[-a(u)\delta(s,x,u)]\mathrm{d}u-\int_0^t\int_E\big[\mathrm{e}^{-\int_s^T a(u)\delta(s,x,u)\mathrm{d}u}-1\big]\nu F(\mathrm{d}s\times\mathrm{d}x)\Big\}$$

are $\mathcal{G}_t$-martingales. So

$$\begin{aligned}\mathsf{E}\big[\mathrm{e}^{g_2(t,T)}\big|\mathcal{G}_t\big] &= \exp\Big\{\frac{1}{2}\int_t^T\mathrm{d}s\Big[\Big(\int_s^T a(u)\sigma_1(s,u)\mathrm{d}u\Big)^2+\Big(\int_s^T a(u)\sigma_2(s,u)\mathrm{d}u\Big)^2\Big]\Big\}\\ &\equiv \exp\Big\{\frac{1}{2}K(a,t,T)\Big\}, \qquad (3.7)\\ \mathsf{E}\big[\mathrm{e}^{g_3(t,T)}\big|\mathcal{G}_t\big] &= \exp\Big\{\int_t^T\int_E\big[\mathrm{e}^{-\int_s^T a(u)\delta(s,x,u)\mathrm{d}u}-1\big]\nu F(\mathrm{d}s\times\mathrm{d}x)\Big\}, \qquad (3.8)\end{aligned}$$

where

$$K(a,t,T)=\int_t^T\mathrm{d}s\Big[\Big(\int_s^T a(u)\sigma_1(s,u)\mathrm{d}u\Big)^2+\Big(\int_s^T a(u)\sigma_2(s,u)\mathrm{d}u\Big)^2\Big].$$

From (3.6), (3.7) and (3.8), we complete the proof. □

Theorem 3.2 Assuming no default before t, the pricing of bond issued by primary firm A is

$$V^A(\theta^A,t,T)=\theta^A h(r,t,T)+(1-\theta^A)\mathrm{e}^{-\int_t^T b_0^A(s)\mathrm{d}s}h((1+b_1^A)r,t,T) \qquad (3.9)$$

and the pricing of bond issued by secondary firm B is

$$\begin{aligned}V^B(\theta^B,t,T) &= \theta^B h(r,t,T)+(1-\theta^B)\Big[\mathrm{e}^{-\int_t^T[b_0^B(s)+b^B(s)]\mathrm{d}s}h((1+b_1^B)r,t,T)\\ &\quad+\mathrm{e}^{-\int_t^T b_0^B(s)\mathrm{d}s}\int_t^T b^B(v)\mathrm{e}^{-\int_v^T b^B(s)\mathrm{d}s-\int_t^v b_0^A(s)\mathrm{d}s}L_1(t,v,T)L_2(v,T)\mathrm{d}v\Big], \qquad (3.10)\end{aligned}$$

where

$$\begin{aligned}L_1(t,v,T) &= \exp\Big\{m(t,v,T)+\frac{1}{2}\int_t^v\Big[\int_w^v k_2(s)\sigma_1(w,s)\mathrm{d}s+\int_v^T k_1(s)\sigma_1(w,s)\mathrm{d}s\Big]^2\mathrm{d}w\\ &\quad+\frac{1}{2}\int_t^v\Big[\int_w^v k_2(s)\sigma_2(w,s)\mathrm{d}s+\int_v^T k_1(s)\sigma_2(w,s)\mathrm{d}s\Big]^2\mathrm{d}w\\ &\quad+\int_t^v\int_E\big[\mathrm{e}^{-\int_w^v k_2(s)\delta(w,x,s)\mathrm{d}s-\int_v^T k_1(s)\delta(w,x,s)\mathrm{d}s}-1\big]\nu F(\mathrm{d}w\times\mathrm{d}x)\Big\},\\ L_2(v,T) &= \exp\Big\{-\int_v^T\int_v^s k_1(s)\alpha(w,s)\mathrm{d}w\mathrm{d}s+\frac{1}{2}K(k_1,v,T)\\ &\quad+\int_v^T\int_E\big[\mathrm{e}^{-\int_w^T k_1(s)\delta(w,x,s)\mathrm{d}s}-1\big]\nu F(\mathrm{d}w\times\mathrm{d}x)\Big\},\\ m(t,v,T) &= -\int_t^v k_2(s)f(t,s)\mathrm{d}s-\int_v^T k_1(s)f(t,s)\mathrm{d}s\\ &\quad-\int_t^v\mathrm{d}s\int_t^s k_2(s)\alpha(w,s)\mathrm{d}w-\int_v^T\mathrm{d}s\int_t^v k_1(s)\alpha(w,s)\mathrm{d}w,\end{aligned}$$

$$k_1(s)=1+b_1^B(s),\qquad k_2(s)=1+b_1^A(s)+b_1^B(s),$$

for all $0\le t\le v\le T$ and $0\le s\le T$.

Proof From (3.1), (3.3) and Theorem 3.1, it is easy to get

$$\begin{aligned}V^A(\theta^A,t,T) &= \theta^A p(t,T)+(1-\theta^A)\mathsf{E}\big[\mathrm{e}^{-\int_t^T(r(s)+\lambda_s^A)\mathrm{d}s}\big|\mathcal{G}_t\big]\\ &= \theta^A h(r,t,T)+(1-\theta^A)\mathrm{e}^{-\int_t^T b_0^A(s)\mathrm{d}s}h((1+b_1^A)r,t,T).\end{aligned}$$

Now we prove Equation (3.10). From (3.2) and (3.4), it is easy to know

$$\begin{aligned}V^B(\theta^B,t,T) &= \theta^B h(r,t,T)+(1-\theta^B)\mathsf{E}\big[\mathrm{e}^{-\int_t^T(r(s)+\lambda_s^B)\mathrm{d}s}\big|\mathcal{G}_t\big]\\ &= \theta^B h(r,t,T)+(1-\theta^B)\mathsf{E}\big[\mathrm{e}^{-\int_t^T b_0^B(s)\mathrm{d}s-\int_t^T(1+b_1^B(s))r(s)\mathrm{d}s-\int_t^T b^B(s)I_{\tau^A\le s}\mathrm{d}s}\big|\mathcal{G}_t\big]\\ &= \theta^B h(r,t,T)+(1-\theta^B)\mathsf{E}\big[\mathrm{e}^{-\int_t^T b_0^B(s)\mathrm{d}s-\int_t^T(1+b_1^B(s))r(s)\mathrm{d}s}\\ &\quad\cdot\mathsf{E}\big[\mathrm{e}^{-I_{\tau^A\le T}\int_{\tau^A}^T b^B(s)\mathrm{d}s}\big|\mathcal{F}_T^X\vee\mathcal{G}_t\big]\big|\mathcal{G}_t\big].\end{aligned}\tag{3.11}$$

Using the property of conditional expectation, we show that

$$\begin{aligned}\mathsf{E}\big[\tau^A\le v\big|\mathcal{F}_T^X\vee\mathcal{G}_t\big] &= 1-\mathsf{E}\big[\tau^A>v\big|\mathcal{F}_T^X\vee\mathcal{G}_t\big]=1-\mathsf{E}\big[\tau^A>v\big|\mathcal{F}_T^X\vee\mathcal{H}_t^A\vee\mathcal{H}_t^B\big]\\ &= 1-I_{\tau^A>t}\frac{\mathsf{E}\big[\tau^A>v\big|\mathcal{F}_T^X\big]}{\mathsf{E}\big[\tau^A>t\big|\mathcal{F}_T^X\big]}=1-I_{\tau^A>t}\exp\Big(-\int_t^v\lambda_s^A\mathrm{d}s\Big)\\ &= 1-\exp\Big(-\int_t^v\lambda_s^A\mathrm{d}s\Big),\qquad t\le v\le T.\end{aligned}$$

So

$$\begin{aligned}&\mathsf{E}\big[\mathrm{e}^{-I_{\tau^A\le T}\int_{\tau^A}^T b^B(s)\mathrm{d}s}\big|\mathcal{F}_T^X\vee\mathcal{G}_t\big]\\ =&\Big(\int_t^T+\int_T^\infty\Big)\mathrm{e}^{-I_{v\le T}\int_v^T b^B(s)\mathrm{d}s}\mathrm{d}\big[1-\mathrm{e}^{-\int_t^v b_0^A(s)\mathrm{d}s-\int_t^v b_1^A(s)r(s)\mathrm{d}s}\big]\\ =&\int_t^T\mathrm{e}^{-\int_v^T b^B(s)\mathrm{d}s}\mathrm{d}\big[1-\mathrm{e}^{-\int_t^v b_0^A(s)\mathrm{d}s-\int_t^v b_1^A(s)r(s)\mathrm{d}s}\big]+\int_T^\infty\mathrm{d}\big[1-\mathrm{e}^{-\int_t^v b_0^A(s)\mathrm{d}s-\int_t^v b_1^A(s)r(s)\mathrm{d}s}\big]\\ =&\mathrm{e}^{-\int_t^T b^B(s)\mathrm{d}s}+\int_t^T b^B(v)\mathrm{e}^{-\int_v^T b^B(s)\mathrm{d}s-\int_t^v b_0^A(s)\mathrm{d}s-\int_t^v b_1^A(s)r(s)\mathrm{d}s}\mathrm{d}v.\end{aligned}\tag{3.12}$$

Substituting (3.12) into (3.11), we easily obtain

$$\begin{aligned}V^B(\theta^B,t,T) &= \theta^B h(r,t,T)+(1-\theta^B)\Big[\mathrm{e}^{-\int_t^T[b_0^B(s)+b^B(s)]\mathrm{d}s}h((1+b_1^B)r,t,T)\\ &\quad+\mathrm{e}^{-\int_t^T b_0^B(s)\mathrm{d}s}\int_t^T b^B(v)\mathrm{e}^{-\int_v^T b^B(s)\mathrm{d}s-\int_t^v b_0^A(s)\mathrm{d}s}\\ &\quad\cdot\mathsf{E}\big[\mathrm{e}^{-\int_v^T(1+b_1^B(s))r(s)\mathrm{d}s-\int_t^v(1+b_1^A(s)+b_1^B(s))r(s)\mathrm{d}s}\big|\mathcal{G}_t\big]dv\Big].\end{aligned}\tag{3.13}$$

Denote $k_1(s) = 1 + b_1^B(s)$ and $k_2(s) = 1 + b_1^A(s) + b_1^B(s)$, we get

$$\begin{aligned}
& \mathsf{E}\big[\mathrm{e}^{-\int_v^T (1+b_1^B(s))r(s)\mathrm{d}s - \int_t^v (1+b_1^A(s)+b_1^B(s))r(s)\mathrm{d}s}\big|\mathcal{G}_t\big] \\
= & \mathsf{E}\big[\mathrm{e}^{-\int_t^v k_2(s)r(s)\mathrm{d}s}\mathsf{E}\big[\mathrm{e}^{-\int_v^T k_1(s)r(s)\mathrm{d}s}\big|\mathcal{G}_v\big]\big|\mathcal{G}_t\big] \\
= & \mathsf{E}[\mathrm{e}^{-\int_t^v k_2(s)r(s)\mathrm{d}s}h(k_1 r, v, T)\big|\mathcal{G}_t\big] \\
= & L_1(t,v,T)L_2(v,T),
\end{aligned} \tag{3.14}$$

where

$$\begin{aligned}
L_1(t,v,T) &= \mathsf{E}[\mathrm{e}^{-\int_t^v k_2(s)r(s)\mathrm{d}s - \int_v^T k_1(s)f(v,s)\mathrm{d}s}\big|\mathcal{G}_t\big], \\
L_2(v,T) &= \exp\Big\{-\int_v^T\int_v^s k_1(s)\alpha(w,s)\mathrm{d}w\mathrm{d}s + \frac{1}{2}K(k_1,v,T) \\
&\quad + \int_v^T\int_E \big[\mathrm{e}^{-\int_w^T k_1(s)\delta(w,x,s)\mathrm{d}s} - 1\big]\nu F(\mathrm{d}w\times\mathrm{d}x)\Big\}.
\end{aligned} \tag{3.15}$$

Using Fubini's theorem, we have

$$\begin{aligned}
& -\int_t^v k_2(s)r(s)\mathrm{d}s - \int_v^T k_1(s)f(v,s)\mathrm{d}s \\
= & m(t,v,T) - \int_t^v \mathrm{d}W_1(w)\Big[\int_w^v k_2(s)\sigma_1(w,s)\mathrm{d}s + \int_v^T k_1(s)\sigma_1(w,s)\mathrm{d}s\Big] \\
& -\int_t^v \mathrm{d}W_2(w)\Big[\int_w^v k_2(s)\sigma_2(w,s)\mathrm{d}s + \int_v^T k_1(s)\sigma_2(w,s)\mathrm{d}s\Big] \\
& -\int_t^v\int_E J(\mathrm{d}w\times\mathrm{d}x)\Big[\int_w^v k_2(s)\delta(w,x,s)\mathrm{d}s + \int_v^T k_1(s)\delta(w,x,s)\mathrm{d}s\Big],
\end{aligned}$$

where

$$\begin{aligned}
m(t,v,T) &= -\int_t^v k_2(s)f(t,s)\mathrm{d}s - \int_v^T k_1(s)f(t,s)\mathrm{d}s \\
&\quad -\int_t^v \mathrm{d}s\int_t^s k_2(s)\alpha(w,s)\mathrm{d}w - \int_v^T \mathrm{d}s\int_t^v k_1(s)\alpha(w,s)\mathrm{d}w.
\end{aligned} \tag{3.16}$$

Further by the properties of stochastic exponential, we find that

$$\begin{aligned}
L_1(t,v,T) &= \exp\Big\{m(t,v,T) + \frac{1}{2}\int_t^v\Big[\int_w^v k_2(s)\sigma_1(w,s)\mathrm{d}s + \int_v^T k_1(s)\sigma_1(w,s)\mathrm{d}s\Big]^2\mathrm{d}w \\
&\quad + \frac{1}{2}\int_t^v\Big[\int_w^v k_2(s)\sigma_2(w,s)\mathrm{d}s + \int_v^T k_1(s)\sigma_2(w,s)\mathrm{d}s\Big]^2\mathrm{d}w \\
&\quad + \int_t^v\int_E\big[\mathrm{e}^{-\int_w^v k_2(s)\delta(w,x,s)\mathrm{d}s - \int_v^T k_1(s)\delta(w,x,s)\mathrm{d}s} - 1\big]\nu F(\mathrm{d}w\times\mathrm{d}x)\Big\}.
\end{aligned} \tag{3.17}$$

From (3.13)-(3.17), we complete the proof. $\square$

§4. The Pricing of Credit Default Swap

In a credit default swap, we assume firm C holds bonds issued by the reference firm A with the constant recovery rate θ^A and the maturity date T. To hedge this credit risk, firm C buys a protection from firm B with the maturity date T. To make the calculation convenient, we provide the following assumption:

(i) Firm A is the primary party and firm B is the secondary party.

(ii) The swap premium payments are made continuously at a constant swap rate S.

(iii) The protection seller's payments are made only at the maturity of the credit default swap.

Making use of the results of previous sections, we give the pricing of credit default swap with counterparty risk.

Theorem 4.1 Assuming that the stochastic interest rate process $r(t)$ satisfies (2.9) and the intensities λ^A and λ^B satisfy (3.1) and (3.2), respectively, the swap rate S has the following expression:

$$S=(1-\theta^A)\frac{V^B(0,0,T)-\exp\Big\{-\displaystyle\int_0^T[b_0^A(u)+b_0^B(u)]\mathrm{d}u\Big\}h(k_2r,0,T)}{\displaystyle\int_0^T h(r,0,s)\mathrm{d}s}. \tag{4.1}$$

Proof It is easy to get that the time-0 market value of buyer C's payments to seller B is

$$\mathsf{E}\Big[\int_0^T S\mathrm{e}^{-\int_0^s r(u)\mathrm{d}u}\mathrm{d}s\Big]=S\int_0^T\mathsf{E}\big[\mathrm{e}^{-\int_0^s r(u)\mathrm{d}u}\big]\mathrm{d}s=S\int_0^T h(r,0,s)\mathrm{d}s$$

and the time-0 market value of seller B's payments to buyer C at the time of default of reference firm A is $(1-\theta^A)\mathsf{E}\big[I_{\tau^A\le T}\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}I_{\tau^B>T}\big]$. By the arbitrage-free principle, we find that

$$\begin{aligned}
S &= \frac{(1-\theta^A)\mathsf{E}\big[I_{\tau^A\le T}\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}I_{\tau^B>T}\big]}{\displaystyle\int_0^T h(r,0,s)\mathrm{d}s}\\
&= (1-\theta^A)\frac{\mathsf{E}\big[\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}I_{\tau^B>T}\big]-\mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}I_{\tau^B>T}\big]}{\displaystyle\int_0^T h(r,0,s)\mathrm{d}s}\\
&= (1-\theta^A)\frac{\mathsf{E}\big[\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}I_{\tau^B>T}\big]-\mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}\mathsf{E}\big(I_{\tau^B>T}\big|\mathcal{F}_T^X\vee\mathcal{H}_T^A\big)\big]}{\displaystyle\int_0^T h(r,0,s)\mathrm{d}s}\\
&= (1-\theta^A)\frac{V^B(0,0,T)-\mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T(r(u)+\lambda_u^B)\mathrm{d}u}\big]}{\displaystyle\int_0^T h(r,0,s)\mathrm{d}s}.
\end{aligned} \tag{4.2}$$

From the expression of λ_u^A, λ_u^B and the property of conditional expectation, we have

$$
\begin{aligned}
& \mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T(r(u)+\lambda_u^B)\mathrm{d}u}\big] \\
= \;& \mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T[b_0^B(u)+(1+b_1^B(u))r(u)+b^B(u)I_{\tau^A\le u}]\mathrm{d}u}\big] \\
= \;& \mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T[b_0^B(u)+(1+b_1^B(u))r(u)]\mathrm{d}u-\int_0^T b^B(u)I_{\tau^A\le u}\mathrm{d}u}\big] \\
= \;& \mathsf{E}\big[I_{\tau^A>T}\mathrm{e}^{-\int_0^T b_0^B(u)\mathrm{d}u-\int_0^T(1+b_1^B(u))r(u)\mathrm{d}u}\big] \\
= \;& \mathsf{E}\big[\mathrm{e}^{-\int_0^T b_0^B(u)\mathrm{d}u-\int_0^T(1+b_1^B(u))r(u)\mathrm{d}u}\mathsf{E}\big[I_{\tau^A>T}\big|\mathcal{F}_T^X\big]\big] \\
= \;& \mathsf{E}\big[\mathrm{e}^{-\int_0^T b_0^B(u)\mathrm{d}u-\int_0^T(1+b_1^B(u))r(u)\mathrm{d}u}\mathrm{e}^{-\int_0^T\lambda_u^A\mathrm{d}u}\big] \\
= \;& \mathsf{E}\big[\mathrm{e}^{-\int_0^T[b_0^A(u)+b_0^B(u)]\mathrm{d}u-\int_0^T[1+b_1^A(u)+b_1^B(u)]r(u)\mathrm{d}u}\big] \\
= \;& \mathrm{e}^{-\int_0^T[b_0^A(u)+b_0^B(u)]\mathrm{d}u}h(k_2r,0,T). \qquad (4.3)
\end{aligned}
$$

Substituting (4.3) into (4.2), we show (4.1) holds. □

§5. Numerical Analysis

In this section, using the explicit formulae obtained in the previous sections, we make some numerical analysis on the pricing of defaultable bonds and credit default swap with counterparty risk. For simplicity, we choose the $F(x)=(1-\mathrm{e}^{-x})I_{x>0}$ and the parameters $T=3$, $\sigma_1(s,u)=0.02$, $\sigma_2(s,u)=0.05$, $f(t,u)=0.3$, $\alpha(s,u)=0.1$, $\delta(s,x,u)=0.02$, $\nu=0.3$, $b^B(t)=0.5$, where $0\le t\le s\le u$. Unless otherwise noted, the $F(x)$ and the parameters are the same as the ones given above.

To present the impact of parameters b_0^A and b_1^A on the pricing $V^A(\theta^A,t,T)$ of defaultable bond issued by firm A, we choose $\theta^A=0.5$ and consider the following cases:

$$
\begin{aligned}
&\text{Case 1}: \; b_0^A=0.1,\; b_1^A=0.2; \qquad \text{Case 2}: \; b_0^A=0.1,\; b_1^A=0.6; \\
&\text{Case 3}: \; b_0^A=0.4,\; b_1^A=0.2; \qquad \text{Case 4}: \; b_0^A=0.4,\; b_1^A=0.6.
\end{aligned}
$$

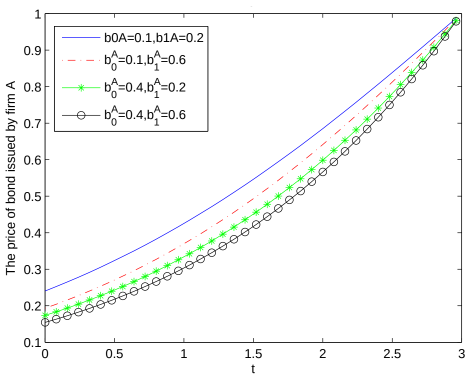

Figure 1

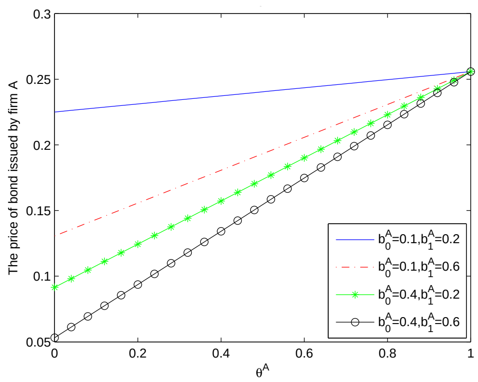

Figure 2

Figure 1 displays $V^A(\theta^A,t,T)$ against the parameters b_0^A and b_1^A in the contagion model. $V^A(\theta^A,t,T)$ will decrease as b_0^A increases, which means the buyer is willing to pay a lower price to the firm A with more risky underlying assets. Be same as b_0^A, $V^A(\theta^A,t,T)$ will also decrease as b_1^A increases. Besides, Figure 1 presents that $V^A(\theta^A,t,T)$ increases with t increasing. Fixed $t=0$, we consider the change of $V^A(\theta^A,t,T)$ with θ^A. Figure 2 displays that $V^A(\theta^A,t,T)$ increases with θ^A increasing in the contagion model. Furthermore, Figure 2 also displays that $V^A(\theta^A,t,T)$ will decrease when b_0^A and b_1^A increase. In addition, it is noted that $V^A(\theta^A,t,T)$ is a fixed value when $\theta^A=1$. That is, $V^A(\theta^A,t,T)$ does not change along with the change of parameters b_0^A and b_1^A when $\theta^A=1$.

To present the impact of parameters b_0^A, b_1^A, b_0^B and b_1^B on the pricing $V^B(\theta^B,t,T)$ of defaultable bond issued by firm B, we choose $t=0$ and consider the following cases:

$$
\begin{aligned}
&\text{Case 5}: \ b_0^A=0.1,\ b_1^A=0.2,\ b_0^B=0.05,\ b_1^B=0.05;\\
&\text{Case 6}: \ b_0^A=0.4,\ b_1^A=0.6,\ b_0^B=0.05,\ b_1^B=0.05;\\
&\text{Case 7}: \ b_0^A=0.1,\ b_1^A=0.2,\ b_0^B=0.1,\ b_1^B=0.2;\\
&\text{Case 8}: \ b_0^A=0.4,\ b_1^A=0.6,\ b_0^B=0.1,\ b_1^B=0.2.
\end{aligned}
$$

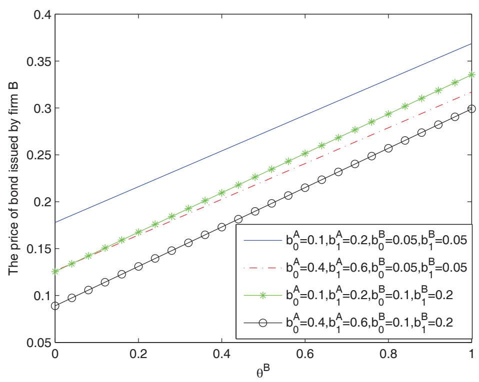

Figure 3

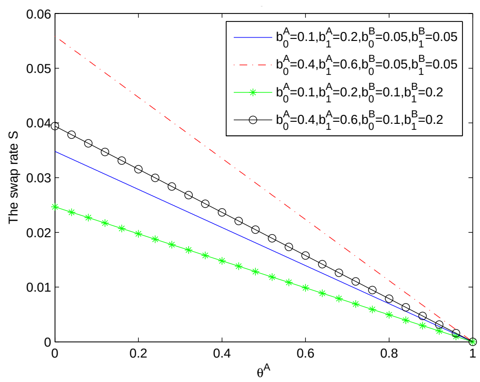

Figure 4

From Figure 3, when b_0^A and b_1^A increase, fixed b_0^B and b_1^B, $V^B(\theta^B,t,T)$ will decrease; when b_0^B and b_1^B increase, fixed b_0^A and b_1^A, $V^B(\theta^B,t,T)$ will decrease. In addition, $V^B(\theta^B,t,T)$ increases with θ^B. Particularly, if $b_0^A=0.1$, $b_1^A=0.2$, $b_0^B=0.05$ and $b_1^B=0.05$, then $V^B(0,0,3)=0.1779$; If $b_0^A=0.4$, $b_1^A=0.6$, $b_0^B=0.05$ and $b_1^B=0.05$, then $V^B(0,0,3)=0.1263$; If $b_0^A=0.1$, $b_1^A=0.2$, $b_0^B=0.1$ and $b_1^B=0.2$, then $V^B(0,0,3)=0.1255$; If $b_0^A=0.4$, $b_1^A=0.6$, $b_0^B=0.1$ and $b_1^B=0.2$, then $V^B(0,0,3)=0.0891$.

From Figure 4, when b_0^A and b_1^A increase, fixed b_0^B and b_1^B, the swap rate S will increase; when b_0^B and b_1^B increase, fixed b_0^A and b_1^A, the swap rate S will decrease. Furthermore, it is consistent with the fact that the protection buyer will pay a higher swap rate when holding a greater risk of the underlying asset; while the protection buyer will pay a lower swap rate when counterparty defaults easier. In addition, it is noted that the swap rate S is equal to zero when $\theta^A = 1$. That is, S does not change along with the change of parameters b_0^A, b_1^A, b_0^B and b_1^B when $\theta^A = 1$.

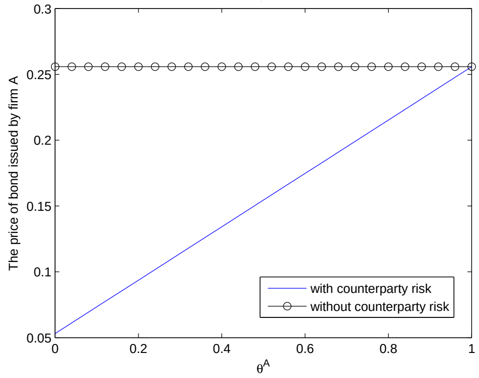

Figure 5

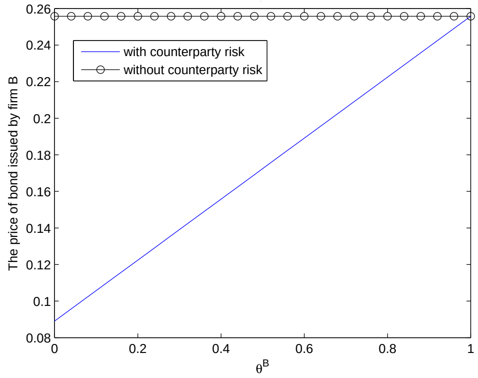

Figure 6

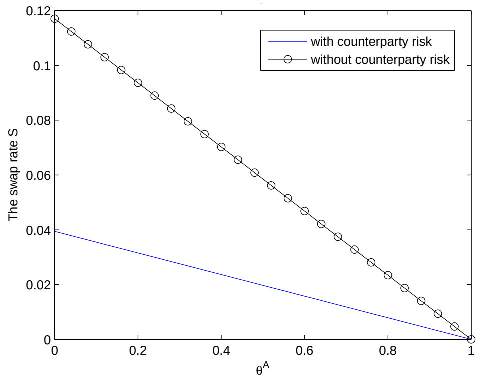

Figure 7

To compare the prices of credit securities with counterparty risk and without counterparty risk, we choose $b_0^A = 0.4$, $b_1^A = 0.6$, $b_0^B = 0.1$ and $b_1^B = 0.2$. Figure 5 present the prices of bond issued by firm A with counterparty risk and without counterparty risk, respectively. Figure 6 present the prices of bond issued by firm B with counterparty risk and without counterparty risk, respectively. Figure 7 present the swap rates S of the credit default swap with counterparty risk and without counterparty risk, respectively. From Figure 5, 6 and 7, we can easily conclude that the prices of credit securities with counterparty risk are less than the prices of credit securities without counterparty risk.

§6. Conclusion

In this paper, we present a contagion model of correlated defaults in a reduced model. The model assumes the intensities of default processes depend on the stochastic interest rate process driven by a stochastic differential equation and the default process of a counterparty. The model assumes the intensities of default processes depend on the stochastic interest rate driven by a stochastic differential equation and the default process of a counterparty, extending the models in Jarrow and Yu (2001) and Hao and Ye (2011). We derive the explicit formulae for the pricing of defaultable bonds and credit default swap with counterparty risk using the properties of stochastic exponentials and make some numerical analysis on the explicit formulae. Specifically, we consider the case that the relevant recovery rates are constants and we can further study more general models. Hence, another important topic for further research is the pricing formulas of defaultable bonds and credit default swap with counterparty risk under the stochastic recovery rates.

Acknowledgement The author is very grateful to the referee for valuable comments and criticisms to improve the earlier version of the paper.

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利用传染模型对含有对手风险的信用证券定价

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本文利用传染模型研究了可违约债券和含有对手风险的信用违约互换的定价. 我们在约化模型中引入具有违约相关性的传染模型, 该模型假设违约过程的强度依赖于由随机微分方程驱动的随机利率过程和交易对手的违约过程. 本文模型可视为Jarrow和Yu (2001)及Hao和Ye (2011)中模型的推广. 进一步地, 我们利用随机指数的性质导出了可违约债券和含有对手风险的信用违约互换的定价公式并进行了数值分析.

关键词: 传染模型, 可违约债券, 信用违约互换, 对手风险, 随机指数.

学科分类号: O211.6.