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Large Deviations for a Test of Symmetry based on Kernel Density Estimator in $\mathbb{R}^d$ *

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Abstract

Let f_n be a non-parametric kernel density estimator based on a kernel function K and a sequence of independent and identically distributed random variables taking values in $\mathbb{R}^d$. The goal of this article is to extend the large deviations results in He and Gao (2008), i.e., to prove large deviations for the statistic $\sup\limits_{x\in\mathbb{R}^d}|f_n(x)-f_n(-x)|$.

Keywords: Symmetry test, kernel density estimator, large deviations.

AMS Subject Classification: 60F10, 62G07.

§1. Introduction and Main Results

Let $\{X_i, i\geq 1\}$ be a sequence of independent and identically distributed (i.i.d.) random variables taking values in $\mathbb{R}^d$ on probability space $(\Omega,\mathscr{F},\mathsf{P})$ with density function f. Let K be a measurable function. The kernel density estimator of f is defined by

$$f_n(x)=\frac{1}{na_n^d}\sum_{i=1}^n K\Big(\frac{x-X_i}{a_n}\Big),\qquad x\in\mathbb{R}^d, \tag{1.1}$$

where $\{a_n, n\geq 1\}$ is a bandsequence, that is, a sequence of positive numbers sastifying

$$a_n\to 0,\ \ na_n^d\to\infty,\ \ \frac{na_n^d}{\log a_n^{-1}}\to+\infty\qquad \text{as}\ \ n\to\infty. \tag{1.2}$$

The limit properties for the kernel density estimator were studied widely, for recent references on this we refer to He and Gao (2008), Gao (2003), Giné and Guillou (2001), Diallo and Louani (2013), Louani (1998) and references therein. The statistic $\sup\limits_{x\in\mathbb{R}^d}|f_n(x)-f_n(-x)|$ was used to test the hypothesis that the density function $f(x)$ is symmetric about 0. He and Gao (2008) studied moderate deviations and large deviations

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(cf. Dembo and Zeitouni, 1998) for $\sup_{x\in\mathbb{R}^d}|f_n(x)-f_n(-x)|$ of the density f in case $d=1$ by the empirical approach. One could ask whether or not the large deviations results hold for $\sup_{x\in\mathbb{R}^d}|f_n(x)-f_n(-x)|$ in general. In this article, we give an affirmative answer to these, i.e., we establish large deviations for $\sup_{x\in\mathbb{R}^d}|f_n(x)-f_n(-x)|$ under certain conditions by the empirical approach (cf. Giné and Guillou, 2001; Talagrand, 1996; Gao, 2003; He and Gao, 2008).

As usual, we denote by $\|g\|_\infty=\sup_{x\in\mathbb{R}^d}|g(x)|$ and $\|g\|_p=(\int_{x\in\mathbb{R}^d}|g(x)|^p\mathrm{d}x)^{1/p}$ the supremum norm and the L_p-norm of g respectively. The following assumptions will be used in this article.

(A1) f is continuous and symmetric and

$$\lim_{x\to\infty} f(x)=0. \tag{1.3}$$

(A2) $K(x)=P(|ax+b|)$, where $P(\cdot)$ is a bounded real function of bounded variation, a is an $m\times d$ matrix, and $K(x)$ is integrable:

$$\int_{\mathbb{R}^d}|K(x)|\mathrm{d}x<+\infty.$$

(A3) f is differentiable and

$$\sup_x|f'(x)|<\infty. \tag{1.4}$$

By Nolan and Pollard (1987), the class of functions

$$\mathscr{F}=\Big\{K\Big(\frac{x-\cdot}{a_n}\Big);x\in\mathbb{R}^d,a_n\in\mathbb{R}_n^d\setminus\{0\}\Big\}$$

is a bounded measurable VC class of functions(cf. Gao, 2003). It is clear that if (A2) holds, then for any $p\geq1$, $\|K\|_p<\infty$ and

$$\varphi(t)\equiv\int_{\mathbb{R}^d}(\exp\{tK(z)\}-1)\mathrm{d}z<\infty \qquad \text{for any } t\geq0. \tag{1.5}$$

Theorem 1.1 Let K be nonnegative, $f(\mathbf{0})=0$ and assumptions (A1), (A2) and (A3) hold. Then, for any $\lambda>0$,

$$\lim_{n\to\infty}\frac{1}{na_n^d}\log\mathsf{P}(\|f_n(\cdot)-f_n(-\cdot)\|_\infty>\lambda)=-J(\lambda), \tag{1.6}$$

where

$$J(\lambda)=\inf_{x\in\mathbb{R}^d}\sup_{t\in\mathbb{R}}\Big\{t\lambda-f(x)\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z\Big\}. \tag{1.7}$$

§2. Large Deviations

Let μ be any probability measure on (S,φ) and let $P=\prod_{i\in\mathbb{N}}\mu_i$ be the product probability measure of $\mu_i=\mu$, $i\in\mathbb{N}$. Let $\xi: S^{\mathbb{N}}\mapsto S$, $i\in\mathbb{N}$, be the coordinate functions. The following lemma is taken from Giné and Guillou (cf. Giné and Guillou, 2001; Gao, 2003).

Lemma 2.1 Let $\mathscr{F}$ be a measurable uniformly bounded VC class of functions and let σ^2 and U be any numbers, such that $\sigma^2\geq\sup_{g\in\mathscr{F}}\mathsf{Var}_P(g)$, $U\geq\sup_{g\in\mathscr{F}}\|g\|_\infty$, and $0<\sigma<U/2$. Then, there exist constants C and L depending only on the characteristic (A,v) of the class $\mathscr{F}$, such that the inequality

$$\begin{aligned}&\mathsf{P}\Big(\sup_{g\in\mathscr{F}}\Big|\sum_{i=1}^n(g(\xi_i)-\mathsf{E}g(\xi_i))\Big|>t\Big)\\&<L\exp\Big\{-\frac{t}{LU}\log\Big(1+\frac{tU}{L[\sqrt{n}\sigma+U\sqrt{\log(U/\sigma)}\,]^2}\Big)\Big\}\end{aligned}$$

is valid for all

$$t\geq C\Big(U\log\frac{U}{\sigma}+\sqrt{n}\sigma\sqrt{\log\frac{U}{\sigma}}\Big).\tag{2.1}$$

The following pointwise principle is an extension of He and Gao (2008) by Gärtner-Ellis theorem (Dembo and Zeitouni, 1998).

Proposition 2.1 Let K be nonnegative, $f(\mathbf{0})=0$ and assumptions (A1), (A2) and (A3) hold. Assume $\varphi(t)<\infty$ for all $t>0$. Then, for any $x\in\mathbb{R}^d$, we have that for any closed set $F\subset\mathbb{R}$,

$$\limsup_{n\to\infty}\frac{1}{na_n^d}\log\mathsf{P}((f_n(x)-f_n(-x))\in F)\leq-\inf_{\lambda\in F}J_x(\lambda),\tag{2.2}$$

and for any open set $G\subset\mathbb{R}$

$$\liminf_{n\to\infty}\frac{1}{na_n^d}\log\mathsf{P}((f_n(x)-f_n(-x))\in G)\geq-\inf_{\lambda\in G}J_x(\lambda),\tag{2.3}$$

where

$$J_x(\lambda)=\sup_{t\in\mathbb{R}}\Big\{t\lambda-f(x)\Big(\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z\Big)\Big\}.\tag{2.4}$$

Proof We show Proposition 2.1 by using the Gärtner-Ellis Theorem. By the definition of φ, we have that

$$|\varphi(t)|=\varphi(|t|)<\infty,\qquad t\in\mathbb{R}.$$

Since X_i, $i \geq 1$ are independent and identically distributed, it is easy to get

$$\begin{aligned}
\Psi_x^{(n)}(t) &\equiv \mathsf{E}(\exp\{tna_n^d(f_n(x)-f_n(-x))\}) \\
&= \Big(\mathsf{E}\Big(\exp\Big\{t\Big(K\Big(\frac{x-X_1}{a_n}\Big)-K\Big(\frac{-x-X_1}{a_n}\Big)\Big)\Big\}\Big)\Big)^n \\
&= \Big\{a_n^d\int_{\mathbb{R}^d}\exp\Big\{t\Big(K(z)-K\Big(z-\frac{2x}{a_n}\Big)\Big)\Big\}f(x-a_nz)\mathrm{d}z\Big\}^n.
\end{aligned}$$

Without loss of generality, we always assume that $x \neq \mathbf{0}$, since when $x = \mathbf{0}$, the proposition is obvious.

First, we assume that K has a bounded support. Since $a_n \to 0$ as $n \to \infty$, the supports of $K(z-2x/a_n)$ and $K(z)$ have an empty intersection for n large enough, so

$$\begin{aligned}
\Psi_x^{(n)}(t) = \Big[&a_n^d\int_{\mathbb{R}^d}f(x-a_nz)\mathrm{d}z + a_n^d\Big(\int_{\mathbb{R}^d}(\exp\{tK(z)\}-1)f(x-a_nz)\mathrm{d}z\Big) \\
&+ a_n^d\int_{\mathbb{R}^d}\Big(\exp\Big\{-tK\Big(z-\frac{2x}{a_n}\Big)\Big\}-1\Big)f(x-a_nz)\mathrm{d}z\Big]^n.
\end{aligned}$$

For (A3), $f(x-a_nz) = f(x) + O(a_n^d|z|)$ (cf. Rudin, 2004: page 113, Theorem 5.19) as $n \to \infty$ uniformly with respect to x, therefore,

$$\begin{aligned}
\Psi_x^{(n)}(t) &= \Big[1 + a_n^d\Big(\int_{\mathbb{R}^d}(\exp\{tK(z)\}-1)(f(x)+O(a_n^d|z|))\mathrm{d}z\Big) \\
&\qquad + a_n^d\int_{\mathbb{R}^d}\Big(\exp\Big\{-tK\Big(z-\frac{2x}{a_n}\Big)\Big\}-1\Big)(f(x)+O(a_n^d|z|))\mathrm{d}z\Big]^n \\
&= \Big[1 + a_n^d f(x)\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z + O(a_n^{2d})\Big]^n.
\end{aligned}$$

So

$$\Psi_x(t) \equiv \lim_{n\to\infty}\frac{1}{na_n^d}\log\Psi_x^{(n)}(t) = f(x)\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z,$$

also the limit is uniform with respect to x and t.

Now, we drop the condition of bounded support,

$$\begin{aligned}
\Psi_x^{(n)}(t) &= \Big[a_n^d\int_{\mathbb{R}^d}f(x-a_nz)\mathrm{d}z + a_n^d\Big(\int_{\mathbb{R}^d}(\exp\{tK(z)\}-1)f(x-a_nz)\mathrm{d}z\Big) \\
&\qquad + a_n^d\int_{\mathbb{R}^d}\Big(\exp\Big\{-tK\Big(z-\frac{2x}{a_n}\Big)\Big\}-1\Big)f(x-a_nz)\mathrm{d}z + a_n^d\alpha\Big]^n \\
&= \Big[1 + a_n^d f(x)\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z + O(a_n^{2d}) + a_n^d\alpha\Big]^n,
\end{aligned}$$

where

$$\alpha = \int_{\mathbb{R}^d} \exp\Big\{t\Big(K(z) - K\Big(z - \frac{2x}{a_n}\Big)\Big)\Big\} f(x - a_n z)\mathrm{d}z \\ - \int_{\mathbb{R}^d} f(x - a_n z)\mathrm{d}z - \int_{\mathbb{R}^d} (\exp\{tK(z)\} - 1) f(x - a_n z)\mathrm{d}z \\ - \int_{\mathbb{R}^d} \Big(\exp\Big\{-tK\Big(z - \frac{2x}{a_n}\Big)\Big\} - 1\Big) f(x - a_n z)\mathrm{d}z.$$

By the conditions of the theorem, for any $\varepsilon > 0$, for n large enough, $|\alpha| \le M\varepsilon(2\exp\{tK_0\} + 4)$, where $M = \|f\|_\infty$, $K_0 = \sup\limits_z K(z)$. Therefore, $\alpha = o(1)$ as $n \to \infty$, it is uniform to x and t. Hence we get

$$\Psi_x(t) \equiv \lim_{n\to\infty} \frac{1}{na_n^d} \log \Psi_x^{(n)}(t) = f(x) \int_{\mathbb{R}^d} (\exp\{tK(z)\} + \exp\{-tK(z)\} - 2)\mathrm{d}z.$$

Since $\Psi_x(t)$ is differentiable with respect to $t \in \mathbb{R}$, Application of Gärtner-Ellis theorem yields (2.3) and (2.4) immediately. □

Lemma 2.2 Let assumptions (A1) and (A2) hold. For any $0 < \delta < 1$, let $B_{n,k}$, $k = 1, 2, \ldots, l_n$, be l_n cubes with side length δa_n, such that $\{B_{n,k}, k = 1, 2, \ldots, l_n\}$ is a covering of $[-a_n^{-1}, a_n^{-1}]^d$ and

$$l_n \ge 2^d(\delta a_n^2)^{-d} + 1.$$

Take $z_{n,k} \in B_{n,k}$, $1 \le k \le l_n$, $n \ge 1$. Then, for any $\varepsilon > 0$,

$$\lim_{\delta\to 0}\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}\Big(\sup_{1\le k\le l_n}\sup_{x\in B_{n,k}} |f_{n,k}(x) - f_{n,k}(-x)| \ge \varepsilon\Big) = -\infty, \tag{2.5}$$

where $f_{n,k}(x) = f_n(x) - f_n(z_{n,k})$, $f_{n,k}(-x) = f_n(-x) - f_n(-z_{n,k})$.

Proof we use Lemma 2.2 in Gao (2003) to see that for any $\eta \in (0, \varepsilon)$, there exists $\delta_0 > 0$, such that for any $\delta < \delta_0$ and for any $x \in B_{n,k}$,

$$\int_{\mathbb{R}^d} \Big[K\Big(\frac{x-y}{a_n}\Big) - K\Big(\frac{z_{n,k}-y}{a_n}\Big) - \Big(K\Big(\frac{-x-y}{a_n}\Big) - K\Big(\frac{-z_{n,k}-y}{a_n}\Big)\Big)\Big]^2 f(y)\mathrm{d}y \\ \le 4a_n^d \|f\|_\infty \eta.$$

Now, take $U = 4\|K\|_\infty$, $\sigma^2 = 4a_n^d\|f\|_\infty\eta$, then by Lemma 2.1, for n large enough,

$$\mathsf{P}\Big(\sup_{1\le k\le l_n}\sup_{x\in B_{n,k}} |f_{n,k}(x) - f_{n,k}(-x) - \mathsf{E}(f_{n,k}(x) - f_{n,k}(-x))| \ge \varepsilon\Big) \\ \le Ll_n \exp\Big\{-\frac{na_n^d\varepsilon}{4\|K\|_\infty L} \log\Big(1 + \frac{\varepsilon\|K\|_\infty}{4L\|f\|_\infty\eta}\Big)\Big\},$$

which implies (2.5) by letting $\eta \to 0$. □

Lemma 2.3 Suppose that (A1) and (A2) hold. Then for any $\varepsilon > 0$,

$$\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}\Big(\sup_{x \notin [-a_n^{-1}, a_n^{-1}]^d} |f_n(x) - f_n(-x)| \geq \varepsilon \Big) = -\infty. \tag{2.6}$$

Proof For any $\eta \in (0, \varepsilon)$, there exists $n_0 > 0$, such that, for $n \geq n_0$,

$$\sup_{x \notin [-a_n^{-1}, a_n^{-1}]^d} \int_{\mathbb{R}^d} \Big(K\Big(\frac{x-y}{a_n}\Big) - K\Big(\frac{-x-y}{a_n}\Big)\Big)^2 f(y) \mathrm{d}y \leq \eta a_n^d.$$

Now, let us take $U = 2\|K\|_\infty$, $\sigma^2 = a_n^d \eta$, it follows from Lemma 2.1 that for any n large enough,

$$\begin{aligned}
&\mathsf{P}\Big(\sup_{x \notin [-a_n^{-1}, a_n^{-1}]^d} |f_n(x) - f_n(-x) - \mathsf{E}(f_n(x) - f_n(-x))| \geq \varepsilon \Big) \\
\leq\ & L \exp\Big\{ -\frac{\varepsilon n a_n^d}{2L\|K\|_\infty} \log\Big(1 + \frac{\varepsilon n a_n^d \|K\|_\infty}{2Lna_n^d \eta}\Big)\Big\}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}\Big(\sup_{x \notin [-a_n^{-1}, a_n^{-1}]^d} |f_n(x) - f_n(-x)| \geq \varepsilon \Big) \\
\leq\ & -\frac{\varepsilon}{2L\|K\|_\infty} \log\Big(1 + \frac{\varepsilon \|K\|_\infty}{2L\eta}\Big).
\end{aligned}$$

Letting $\eta \to 0$, we get (2.6). $\Box$

Lemma 2.4 Let K be nonnegative, and let (A1) and (A2) hold. Then for any $\lambda > 0$,

$$\sup_{t\in\mathbb{R}} \inf_{x\in\mathbb{R}^d} \{t\lambda - \Psi_x(t)\} = \inf_{x\in\mathbb{R}^d} \sup_{t\in\mathbb{R}} \{t\lambda - \Psi_x(t)\}. \tag{2.7}$$

Proof Set $M = \|f\|_\infty$ and let

$$G(t, y) = t\lambda - y \int_{\mathbb{R}^d} (\exp\{tK(z)\} + \exp\{-tK(z)\} - 2) \mathrm{d}z.$$

Then $G : \mathbb{R} \times [0, M] \mapsto \mathbb{R}$ satisfies the property that for t fixed, $G(t, y)$ is convex as a function of y, and for y fixed, $G(t, y)$ is concave as a function of t. It follows from the minimax theorem (cf. Sion, 1958) that

$$\sup_{t\in\mathbb{R}} \inf_{y\in[0,M]} G(t, y) = \inf_{y\in[0,M]} \sup_{t\in\mathbb{R}} G(t, y),$$

that is, (2.7) holds. $\Box$

Lemma 2.5 Let K be nonnegative, and let (A1) and (A2) hold. Set

$$h(t)=\int_{\mathbb{R}^d}(\exp\{tK(z)\}-\exp\{-tK(z)\})K(z)\mathrm{d}z.$$

Then, for $\lambda\geq 0$,

$$J(\lambda)=\lambda h^{-1}(\lambda/M)-M\int_{\mathbb{R}^d}(\exp\{h^{-1}(\lambda/M)K(z)\}+\exp\{-h^{-1}(\lambda/M)K(z)\}-2)\mathrm{d}z,$$

where $M=\|f\|_\infty$ and h^{-1} denotes the inverse of h. In particular, J is continuous on $[0,\infty)$.

Proof It is trivial that h is strictly increasing on $[0,\infty)$ and $h(0)=0$, $\lim\limits_{t\to\infty}h(t)=\infty$, hence, h^{-1} exists, and it is strictly increasing and continuous on $[0,\infty)$. Let

$$G(t,y)=t\lambda-y\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z,\qquad t\in\mathbb{R},\ y\in[0,M].$$

Then, $\partial G(t,y)/\partial t=\lambda-yh(t)$, and so

$$\sup_{t\in\mathbb{R}}G(t,y)=\begin{cases}G(h^{-1}(\lambda/y),y) & \text{if } y\neq 0;\\ +\infty & \text{if } y=0.\end{cases}$$

Since

$$G(h^{-1}(\lambda/y),y)=\sup_{t\geq 0}G(t,y)=\sup_{t\geq 0}\Big\{t\lambda-y\int_{\mathbb{R}^d}(\exp\{tK(z)\}+\exp\{-tK(z)\}-2)\mathrm{d}z\Big\}\quad(2.8)$$

is decreasing with respect to $y\in[0,M]$, we have

$$J(\lambda)=\inf_{y\in[0,M]}\sup_{t\geq 0}G(t,y)=G(h^{-1}(\lambda/M),M).$$

In particular, J is continuous on $[0,\infty)$. □

Proof of Theorem 1.1 Now we prove (1.6). The proof of Theorem 1.1 comes from that of Theorem 1.2 in Gao (2003). For sake of convenience, we give a complete proof here. First, for any $x\in\mathbb{R}^d$, by Proposition 2.1, we have

$$\begin{aligned}&\liminf_{n\to\infty}\frac{1}{na_n^d}\log\mathsf{P}(\|f_n(\cdot)-f_n(-\cdot)\|_\infty>\lambda)\\ \geq&\liminf_{n\to\infty}\frac{1}{na_n^d}\log\mathsf{P}(\|f_n(x)-f_n(-x)\|_\infty>\lambda)\geq -J_x(\lambda).\end{aligned}$$

Hence

$$\liminf_{n\to\infty}\frac{1}{na_n^d}\log\mathsf{P}(\|f_n(\cdot)-f_n(-\cdot)\|_\infty>\lambda)\geq -J(\lambda).$$

To prove the reverse inequality, we note

$$\begin{aligned}
&\|f_n(\cdot) - f_n(-\cdot)\|_\infty \\
&= \max\Big\{ \sup_{x\in[-a_n^{-1},a_n^{-1}]^d} |f_n(x) - f_n(-x)|, \sup_{x\notin[-a_n^{-1},a_n^{-1}]^d} |f_n(x) - f_n(-x)| \Big\}
\end{aligned}$$

and

$$\begin{aligned}
&\sup_{x\in[-a_n^{-1},a_n^{-1}]^d} |f_n(x) - f_n(-x)| \\
&\le \max_{1\le k\le l_n} \Big\{ \sup_{x\in B_{n,k}} |f_{n,k}(x) - f_{n,k}(-x)| + |f_n(z_{n,k}) - f_n(-z_{n,k})| \Big\},
\end{aligned}$$

by Lemmas 2.2 and 2.3, we have that for any $0 < \varepsilon < \lambda/2$,

$$\begin{aligned}
&\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}(\|f_n(\cdot) - f_n(-\cdot)\|_\infty > \lambda) \\
&= \limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}\Big(\sup_{x\in[-a_n^{-1},a_n^{-1}]^d} |f_n(x) - f_n(-x)| > \lambda \Big) \\
&\le \lim_{\delta\to 0}\limsup_{n\to\infty} \frac{1}{na_n^d} \log \Big(\mathsf{P}\Big(\max_{1\le k\le l_n} \sup_{x\in B_{n,k}} |f_{n,k}(x) - f_{n,k}(-x)| \ge \varepsilon \Big) \\
&\quad + \mathsf{P}\Big(\max_{1\le k\le l_n} |f_n(z_{n,k}) - f_n(-z_{n,k})| > \lambda - \varepsilon \Big)\Big) \\
&= \lim_{\delta\to 0}\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}\Big(\max_{1\le k\le l_n} |f_n(z_{n,k}) - f_n(-z_{n,k})| > \lambda - \varepsilon \Big). \qquad (2.9)
\end{aligned}$$

On the other hand, by the Chebyshev's inequality,

$$\begin{aligned}
&\mathsf{P}\Big(\max_{1\le k\le l_n} |f_n(z_{n,k}) - f_n(-z_{n,k})| > \lambda - \varepsilon \Big) \\
&\le l_n \max_{1\le k\le l_n} \{\exp\{-na_n^d(\lambda-\varepsilon)t\}\}\Psi_{z_{n,k}}^n(t). \qquad (2.10)
\end{aligned}$$

Now, Combining (2.9) and (2.10) we get

$$\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}(\|f_n(\cdot) - f_n(-\cdot)\|_\infty > \lambda) \le -\Big\{ (\lambda-\varepsilon)t - \sup_{x\in\mathbb{R}^d} \Psi_x(t) \Big\}.$$

Then, by Lemma 2.4,

$$\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}(\|f_n(\cdot) - f_n(-\cdot)\|_\infty > \lambda) \le -J(\lambda-\varepsilon).$$

Therefore, by the continuity of J,

$$\limsup_{n\to\infty} \frac{1}{na_n^d} \log \mathsf{P}(\|f_n(\cdot) - f_n(-\cdot)\|_\infty > \lambda) \le -J(\lambda). \qquad \square$$

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基于$\mathbb{R}^d$上核密度估计的对称检验的大偏差

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设f_n是基于一个核函数K和取值于$\mathbb{R}^d$的独立同分布随机变量列的一个非参数核密度估计. 本文推广了在He和Gao (2008)中相应大偏差的结果, 即证明统计量$\sup\limits_{x\in\mathbb{R}^d}|f_n(x)-f_n(-x)|$的大偏差.

关 键 词: 对称检验, 核密度估计, 大偏差.

学科分类号: O211.4.