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The Perturbed Compound Poisson Risk Model with Constant Interest *

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Abstract

This study has considered the compound Poisson risk model perturbed by diffusion with constant interest and obtained an integral-differential equation for the Gerber-Shiu discounted penalty function. Asymptotic expression for the ultimate ruin probability also derived across the study.

Keywords: Constant interest, compound Poisson model, Gerber-Shiu function, ruin probability.

AMS Subject Classification: 62P05, 62E20, 60F10.

§1. Introduction

This study investigates some recent work on perturbing compound Poisson risk model with constant interest. The surplus process of the insurer satisfies

$$U(t)=\mathrm{e}^{rt}x+c\int_0^t \mathrm{e}^{r(t-v)}\mathrm{d}v-\int_0^t \mathrm{e}^{r(t-v)}\mathrm{d}S_v+\sigma\int_0^t \mathrm{e}^{r(t-v)}\mathrm{d}B(v), \tag{1.1}$$

where $x\geq 0$ is the initial reserve, $c>0$ is the constant rate of premium, $\sigma\geq 0$ is a fixed constant and r is a nonnegative constant, which represents the interest rate. S_t is taken to be a compound Poisson process, i.e. $S_t=\sum\limits_{k=1}^{N(t)}X_k$, $t\geq 0$, where $N(t)=\#\{k=1,2,\ldots:\theta_k\leq t\}$ is a Poisson process with intensity $\lambda>0$ and $\{X_k,k=1,2,\ldots\}$ is a sequence of independent, identically distributed (i.i.d.), and nonnegative random variables, in which X_k represents the amount of the k-th claim, and θ_k is the arrival time of the k-th claim. $f(\cdot)$ denotes the probability density function of X_1, and $F(\cdot)$ denotes the distribution function of X_1. Furthermore, $\{B(t),t\geq 0\}$ denotes a standard Brownian motion. For

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$k = 1, 2, \ldots$, denote by τ_k the inter-time between the $(k-1)$-th claim and the k-th claim. Then $\{\tau_k, k \geq 1\}$ is a sequence of i.i.d. random variables with exponential distribution with the parameter λ, and $\theta_k = \sum\limits_{i=1}^{k} \tau_i$. $\{X_k, k = 1, 2, \ldots\}$, $\{N(t), t \geq 0\}$ and $\{B(t), t \geq 0\}$ are assumed as mutually independent. Then (1.1) can be rewrote as

$$U(t) = \mathrm{e}^{rt}x + \frac{c}{r}(\mathrm{e}^{rt} - 1) - \sum_{k=1}^{N(t)} \mathrm{e}^{r(t-\theta_k)}X_k + \sigma \int_0^t \mathrm{e}^{r(t-v)}\mathrm{d}B(v). \tag{1.2}$$

Obviously, the risk process is a homogeneous strong Markov process. And the risk model has been studied in many existing work, see Gao and Liu (2010), Wang and Wu (2008) and the references therein.

The time of ruin for risk process (1.2) is defined as $T = \inf\{t \geq 0 \,|\, U(t) < 0\}$. We define the corresponding ultimate ruin probability $\psi(x) = \mathsf{P}(T < \infty \,|\, U(0) = x)$. We consider the Gerber-Shiu discounted penalty function of the surplus immediately prior to ruin and the deficit at ruin when ruin occurs as a function of initial surplus x, namely,

$$\Phi_{r,\delta}(x) = \mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-), |U(T)|)I(T < \infty)],$$

where $\omega(y_1, y_2)$, $0 \leq y_1, y_2 < \infty$, is a nonnegative function and δ is a nonnegative parameter. We can interpret $\exp\{-\delta T\}$ as the "discount factor".

The financial explanations of $\omega(y_1, y_2)$ can be found in Gerber and Shiu (1998). The Gerber-Shiu discounted penalty function has been studied by many scholars including Zhao et al. (2014), Tang and Wei (2010), Sun (2005), Bao and Ye (2007), Cai and Dickson (2002), Lin and Pavlova (2006), and Cheung (2011). The ruin probability is also a central research topic in insurance mathematics and applied probability. Studies involve ruin probability can be found in Wang and Li (2014), Zhou and Zhu (2014), Yang and Wang (2010), Zhang and Wang (2012), among others.

The rest of the paper consists of two sections. Section 2 presents the integral-differential equations for the Gerber-Shiu discounted penalty function and the ultimate ruin probability. While the asymptotic expression for the ultimate ruin probability will be given after presenting a series of lemmas in Section 3.

§2. Integral-Differential Equation for the Gerber-Shiu Discounted Penalty Function

Let us first recall the definition of the class $\mathscr{S}^*$ (Schmidli, 2005). A distribution function $F(x)$ is in $\mathscr{S}^*$ if it has finite mean μ_F and

$$\lim_{x\to\infty} \int_0^x \frac{(1 - F(x-y))(1 - F(y))}{1 - F(x)}\mathrm{d}y = 2\mu_F.$$

This condition can also be written as

$$\lim_{x\to\infty}\int_0^{x/2}\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y=\mu_F.$$

If $F(x)$ has a regularly varying tail then $F(x)\in\mathscr{S}^*$. Moreover, the log-normal and the heavy-tailed Weibull distributions belong to $\mathscr{S}^*$. Thus, $\mathscr{S}^*$ contains all heavy-tailed distribution functions of interest.

For two positive functions $a(\cdot)$ and $b(\cdot)$, we write $a(x)\sim b(x)$ if $\lim\limits_{x\to\infty}a(x)/b(x)=1$.

For notational convenience, we introduce $\kappa=2\lambda/\sigma^2$ and $\gamma(x)=-\psi''(x)/\kappa(1-F(x))$ $-\,rx\psi'(x)/\lambda(1-F(x))$. We let $\ell(x)=f(x)/(1-F(x))$ be the hazard rate.

Theorem 2.1 Consider the insurance risk model introduced in Section 1. The function $\Phi_{r,\delta}(x)$ satisfies the following integral-differential equation

$$\begin{aligned}&\frac{1}{2}\sigma^2\Phi''_{r,\delta}(x)+(rx+c)\Phi'_{r,\delta}(x)-(\lambda+\delta)\Phi_{r,\delta}(x)\\&+\lambda\int_0^x\Phi_{r,\delta}(x-y)f(y)\mathrm{d}y+\lambda\int_x^\infty\omega(x,y-x)f(y)\mathrm{d}y=0.\end{aligned}\tag{2.1}$$

Proof For any $h>0$, define $m(s)=\mathrm{e}^{rs}x+c(\mathrm{e}^{rs}-1)/r+\sigma\int_0^s\mathrm{e}^{r(s-v)}\mathrm{d}B(v)$ and $\widetilde{T}_h=\inf\{s>0|m(s)<0\}\wedge h$. By considering the occurrence time θ_1 of the first claim, we have

$$\begin{aligned}\Phi_{r,\delta}(x)&=\mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-),|U(T)|)I(T<\infty)I(\theta_1>h)]\\&\quad+\mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-),|U(T)|)I(T<\infty)I(\theta_1\le h)]\\&=E_1+E_2.\end{aligned}\tag{2.2}$$

By the strong Markov property, we get

$$\begin{aligned}E_1&=\mathsf{E}\big[\mathrm{e}^{-\delta\widetilde{T}_h}\mathsf{E}_{U(\widetilde{T}_h)}[\mathrm{e}^{-\delta T}\omega(U(T-),|U(T)|)I(T<\infty)]I(\theta_1>h)\big]\\&=\int_h^\infty\mathsf{E}\big[\mathrm{e}^{-\delta\widetilde{T}_h}\Phi_{r,\delta}(U(\widetilde{T}_h))\big]\lambda\mathrm{e}^{-\lambda t}\mathrm{d}t\\&=\mathrm{e}^{-\lambda h}\mathsf{E}\big[\mathrm{e}^{-\delta\widetilde{T}_h}\Phi_{r,\delta}(U(\widetilde{T}_h))\big].\end{aligned}\tag{2.3}$$

For E_2, we have

$$\begin{aligned}E_2&=\mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-),|U(T)|)I(T<\infty)I(\theta_1\le h)I(\widetilde{T}_{\theta_1}<\theta_1)]\\&\quad+\mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-),|U(T)|)I(T<\infty)I(\theta_1\le h)I(\widetilde{T}_{\theta_1}=\theta_1)]\\&=E_3+E_4.\end{aligned}\tag{2.4}$$

If $\widetilde{T}_{\theta_1} < \theta_1$, then $T = \widetilde{T}_{\theta_1}$, and thus

$$\begin{aligned} E_3 &= \mathsf{E}_x\big[\mathrm{e}^{-\delta \widetilde{T}_{\theta_1}}\omega(U(\widetilde{T}_{\theta_1}-), |U(\widetilde{T}_{\theta_1})|)I(\theta_1 \le h)I(\widetilde{T}_{\theta_1} < \theta_1)\big] \\ &= \int_0^h \mathsf{E}_x\big[\mathrm{e}^{-\delta \widetilde{T}_t}\omega(U(\widetilde{T}_t-), |U(\widetilde{T}_t)|)I(\widetilde{T}_t < t)\big]\lambda \mathrm{e}^{-\lambda t}\mathrm{d}t. \end{aligned} \tag{2.5}$$

For E_4, by conditioning on the amount of the first claim, whether the first claim causes ruin, we get

$$\begin{aligned} E_4 &= \mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-), |U(T)|)I(T < \infty)I(\theta_1 \le h)I(\widetilde{T}_{\theta_1} = \theta_1)I(X_1 \le m(\theta_1))] \\ &\quad + \mathsf{E}_x[\mathrm{e}^{-\delta T}\omega(U(T-), |U(T)|)I(T < \infty)I(\theta_1 \le h)I(\widetilde{T}_{\theta_1} = \theta_1)I(X_1 > m(\theta_1))] \\ &= E_5 + E_6. \end{aligned} \tag{2.6}$$

By the strong Markov property, we obtain

$$\begin{aligned} E_5 &= \mathsf{E}\big[\mathrm{e}^{-\delta\theta_1}\mathsf{E}_{U(\theta_1)}[\mathrm{e}^{-\delta T}\omega(U(T-), |U(T)|)I(T < \infty)]I(\theta_1 \le h)I(\widetilde{T}_{\theta_1} = \theta_1) \\ &\qquad \times I(X_1 \le m(\theta_1))\big] \\ &= \int_0^h \int_0^{m(t)} \mathrm{e}^{-\delta t}\Phi_{r,\delta}(m(t) - y)\mathsf{P}(\widetilde{T}_t = t)f(y)\lambda \mathrm{e}^{-\lambda t}\mathrm{d}y\mathrm{d}t. \end{aligned} \tag{2.7}$$

On the other hand, if $X_1 > m(\theta_1)$, then $T = \theta_1$, and thus

$$\begin{aligned} E_6 &= \mathsf{E}_x[\mathrm{e}^{-\delta\theta_1}\omega(U(\theta_1-), |U(\theta_1)|)I(\theta_1 \le h)I(\widetilde{T}_{\theta_1} = \theta_1)I(X_1 > m(\theta_1))] \\ &= \int_0^h \int_{m(t)}^{\infty} \mathrm{e}^{-\delta t}\omega(m(t), y - m(t))\mathsf{P}(\widetilde{T}_t = t)f(y)\lambda \mathrm{e}^{-\lambda t}\mathrm{d}y\mathrm{d}t. \end{aligned} \tag{2.8}$$

From equalities (2.2)-(2.8), we see that

$$\Phi_{r,\delta}(x) = E_1 + E_3 + E_5 + E_6.$$

Using Itô's formula, we have

$$\lim_{h\downarrow 0}\frac{E_1 - \Phi_{r,\delta}(x)}{h} = \frac{1}{2}\sigma^2\Phi''_{r,\delta}(x) + (rx + c)\Phi'_{r,\delta}(x) - (\lambda + \delta)\Phi_{r,\delta}(x).$$

Since

$$\lim_{h\downarrow 0}\frac{E_3}{h} = 0,$$

$$\lim_{h\downarrow 0}\frac{E_5}{h} = \lambda\int_0^x \Phi_{r,\delta}(x - y)f(y)\mathrm{d}y,$$

and

$$\lim_{h\downarrow 0}\frac{E_6}{h} = \lambda\int_x^\infty \omega(x, y - x)f(y)\mathrm{d}y,$$

we can get the equation (2.1). $\square$

Corollary 2.1 Consider the insurance risk model introduced in Section 1. The ultimate ruin probability $\psi(x)$ satisfies the following integral-differential equation

$$\frac{1}{2}\sigma^2\psi''(x)+(rx+c)\psi'(x)-\lambda\psi(x)+\lambda\int_0^x\psi(x-y)f(y)\mathrm{d}y+\lambda(1-F(x))=0. \tag{2.9}$$

Proof When $\delta=0$ and $\omega(y_1,y_2)\equiv 1$, $\Phi_{r,\delta}(x)=\psi(x)$. Thus, by the equation (2.1) we can get the equation (2.9). $\square$

§3. Asymptotic Expression for the Ultimate Ruin Probability

Lemma 3.1 The ultimate ruin probability

$$\psi(x)=\kappa\int_x^\infty\Big[a\mathrm{e}^{-rz^2/\sigma^2}+\mathrm{e}^{-rz^2/\sigma^2}\int_0^z\mathrm{e}^{ry^2/\sigma^2}(1-F(y))\gamma(y)\mathrm{d}y\Big]\mathrm{d}z,$$

where $a=-\psi'(0)/\kappa$.

Proof The definition of $\gamma(x)$ can be expressed as

$$(\psi'(x)\mathrm{e}^{rx^2/\sigma^2})'=-\kappa\mathrm{e}^{rx^2/\sigma^2}\gamma(x)(1-F(x)).$$

Integration yields

$$\psi'(x)=-\kappa\Big[a\mathrm{e}^{-rx^2/\sigma^2}+\mathrm{e}^{-rx^2/\sigma^2}\int_0^x\mathrm{e}^{ry^2/\sigma^2}(1-F(y))\gamma(y)\mathrm{d}y\Big].$$

Integrating the above equation from x to ∞, we get

$$\psi(x)=\kappa\int_x^\infty\Big[a\mathrm{e}^{-rz^2/\sigma^2}+\mathrm{e}^{-rz^2/\sigma^2}\int_0^z\mathrm{e}^{ry^2/\sigma^2}(1-F(y))\gamma(y)\mathrm{d}y\Big]\mathrm{d}z. \qquad \square$$

Lemma 3.2 Suppose $F\in\mathscr{S}^*$ and $\lim\limits_{x\to\infty}\ell(x)=0$. Let $g(x)=-\psi'(x)/(1-F(x))$, then $\lim\limits_{x\to\infty}g(x)=0$.

Proof By the equation (2.9) of Corollary 2.1, we obtain

$$\frac{1}{2}\sigma^2\psi''(x)+(rx+c)\psi'(x)-\lambda\int_0^x\psi'(x-y)(1-F(y))\mathrm{d}y+\lambda\delta(0)(1-F(x))=0,$$

where $\delta(0)=1-\psi(0)\in(0,1)$. Substituting $\psi'(x)=-g(x)(1-F(x))$ and $\psi''(x)=g(x)f(x)-g'(x)(1-F(x))$ in the above equation, we see that,

$$\begin{aligned}&\frac{1}{2}\sigma^2g(x)f(x)-\frac{1}{2}\sigma^2g'(x)(1-F(x))-(rx+c)g(x)(1-F(x))\\&+\lambda\int_0^x g(x-y)(1-F(x-y))(1-F(y))\mathrm{d}y+\lambda\delta(0)(1-F(x))=0.\end{aligned}$$

Dividing by $1-F(x)$, we get

$$\frac{1}{2}\sigma^2 g(x)\ell(x)-\frac{1}{2}\sigma^2 g'(x)-(rx+c)g(x) \\ +\lambda\int_0^x g(x-y)\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y+\lambda\delta(0)=0. \tag{3.1}$$

We first show that $g(x)$ is bounded. Suppose the opposite. Let $\{x_n\}$ be a sequence tending to infinity such that $g(x_n)\ge g(z)$ for all $z\le x_n$. Such a sequence exists if $g(x)$ is unbounded. Clearly, $g'(x_n)\ge 0$. By the equation (3.1), we can obtain

$$g(x_n)\Big[\frac{1}{2}\sigma^2\ell(x_n)-(rx_n+c)+\lambda\int_0^{x_n}\frac{(1-F(x_n-y))(1-F(y))}{1-F(x_n)}\mathrm{d}y\Big]+\lambda\delta(0) \\ \ge\frac{1}{2}\sigma^2 g'(x_n)\ge 0.$$

Because $F\in\mathscr{S}^*$ the integral is bounded. Because the hazard rate tends to zero the left hand side can be made arbitrarily small, in particular much smaller than zero. Thus, $g(x)$ must be bounded. By the equation (3.1), it follows that $\sigma^2 g'(x)/2+rxg(x)$ is bounded. If $g(x)$ would not converge then there must be a sequence $\{x_n\}$ tending to infinity such that $g(x_n)\to\overline{\lim_{x\to\infty}}\,g(x)>0$ and $g'(x_n)=0$. But then $\sigma^2 g'(x_n)/2+rx_n g(x_n)$ would be unbounded. Thus, $g(x)$ converges. Suppose that the limit is not zero. Because $\sigma^2 g'(x)/2+rxg(x)$ is bounded this is only possible if $g'(x)/g(x)\le-\epsilon$ for x large enough. Integration over (x_0,z) yields $g(z)\le g(x_0)\mathrm{e}^{-\epsilon(z-x_0)}$. Thus, $\lim_{x\to\infty}g(x)=0$. $\square$

Lemma 3.3 Under the conditions of Lemma 3.2, we have $\lim_{x\to\infty}\gamma(x)=1$.

Proof From Lemma 3.2 we know that $\lim_{x\to\infty}g(x)=0$. Choose $\epsilon>0$. There is x_0 such that $g(x)<\epsilon$ for all $x>x_0$. Thus,

$$\int_0^{x/2}g(x-y)\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y<\epsilon\int_0^{x/2}\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y,$$

for $x>2x_0$. Using the arbitrariness of ϵ, we see that,

$$\lim_{x\to\infty}\int_0^{x/2}g(x-y)\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y=0.$$

Similarly, we can obtain

$$\lim_{x\to\infty}\int_{x_0}^{x/2}g(y)\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y=0.$$

Moreover,

$$\lim_{x\to\infty}\int_0^{x_0}g(y)\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y=\int_0^{x_0}g(y)(1-F(y))\mathrm{d}y,$$

it follows that

$$\begin{aligned}
&\lim_{x\to\infty}\int_0^x g(x-y)\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y\\
=&\lim_{x\to\infty}\int_0^{x/2}(g(y)+g(x-y))\frac{(1-F(x-y))(1-F(y))}{1-F(x)}\mathrm{d}y\\
=&\int_0^\infty g(y)(1-F(y))\mathrm{d}y\\
=&\,\psi(0).
\end{aligned}$$

From (3.1), we can get

$$\lim_{x\to\infty}\Big[\frac{\sigma^2\psi''(x)}{2(1-F(x))}+\frac{rx\psi'(x)}{1-F(x)}\Big]=-\lambda.$$

Thus,

$$\lim_{x\to\infty}\gamma(x)=1. \qquad \square$$

Theorem 3.1 Consider the insurance risk model introduced in Section 1 in which the claim-size distribution F belongs to the class $\mathscr{S}^*$. Then

$$\psi(x)\sim\frac{2\lambda}{\sigma^2}\int_x^\infty \mathrm{e}^{-rz^2/\sigma^2}\int_0^z \mathrm{e}^{ry^2/\sigma^2}(1-F(y))\mathrm{d}y\mathrm{d}z.$$

Proof Let us first assume that $\lim\limits_{x\to\infty}\ell(x)=0$. Applying Lemma 3.1, we obtain

$$\psi(x)=\kappa\int_x^\infty\Big[a\mathrm{e}^{-rz^2/\sigma^2}+\mathrm{e}^{-rz^2/\sigma^2}\int_0^z \mathrm{e}^{ry^2/\sigma^2}(1-F(y))\gamma(y)\mathrm{d}y\Big]\mathrm{d}z.$$

Because $\int_0^z \mathrm{e}^{ry^2/\sigma^2}(1-F(y))\mathrm{d}y/\mathrm{e}^{rz^2/\sigma^2}$ tends to zero as $z\to\infty$, we can use L'Hospital's rule and Lemma 3.3 to obtain

$$\begin{aligned}
&\lim_{x\to\infty}\frac{\psi(x)}{\kappa\displaystyle\int_x^\infty \mathrm{e}^{-rz^2/\sigma^2}\int_0^z \mathrm{e}^{ry^2/\sigma^2}(1-F(y))\mathrm{d}y\mathrm{d}z}\\
=&\lim_{x\to\infty}\frac{a+\displaystyle\int_0^x \mathrm{e}^{ry^2/\sigma^2}\gamma(y)(1-F(y))\mathrm{d}y}{\displaystyle\int_0^x \mathrm{e}^{ry^2/\sigma^2}(1-F(y))\mathrm{d}y}\\
=&\lim_{x\to\infty}\gamma(x)\\
=&\,1.
\end{aligned}$$

This proves the result if $\ell(x)$ tends to zero as $x\to\infty$.

On the other hand, if $\ell(x)$ would not tend to zero as $x\to\infty$. As proved in Rolski et al. (1999) there is always a tail equivalent differentiable distribution function $\widetilde{F}(x)$ whose hazard rate $\widetilde{\ell}(x)$ converges to zero. Tail equivalent means that $1-\widetilde{F}(x)\sim 1-F(x)$. Construct $F_1(x)$ such that $1-F(x)\leq 1-F_1(x)$, $1-F(x)\sim(1-\epsilon)(1-F_1(x))$ and $\ell_1(x)\to 0$. The corresponding ultimate ruin probability of the claims with distribution function $F_1(x)$ is defined by $\psi_1(x)$. Because the claims with distribution function $F(x)$ are smaller than the claims with distribution function $F_1(x)$ we have $\psi(x)\leq\psi_1(x)$. Therefore,

$$\begin{aligned}&\overline{\lim_{x\to\infty}}\frac{\psi(x)}{\kappa\displaystyle\int_x^{\infty}\mathrm{e}^{-rz^2/\sigma^2}\int_0^z\mathrm{e}^{ry^2/\sigma^2}(1-F(y))\mathrm{d}y\mathrm{d}z}\\ \leq&\lim_{x\to\infty}\frac{\psi_1(x)}{(1-\epsilon)\kappa\displaystyle\int_x^{\infty}\mathrm{e}^{-rz^2/\sigma^2}\int_0^z\mathrm{e}^{ry^2/\sigma^2}(1-F_1(y))\mathrm{d}y\mathrm{d}z}\\ =&\frac{1}{1-\epsilon}.\end{aligned}$$

Similarly, we can obtain

$$\underline{\lim_{x\to\infty}}\frac{\psi(x)}{\kappa\displaystyle\int_x^{\infty}\mathrm{e}^{-rz^2/\sigma^2}\int_0^z\mathrm{e}^{ry^2/\sigma^2}(1-F(y))\mathrm{d}y\mathrm{d}z}\geq\frac{1}{1+\epsilon}.$$

Using the arbitrariness of ϵ, we can get the result. $\square$

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带常利率的扰动复合泊松风险模型

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本文考虑带常利率的扰动复合泊松风险模型, 得到了Gerber-Shiu折现罚金函数所满足的积分-微分方程, 并且得到了最终破产概率的精确渐进表达式.

关键词: 常利率, 复合泊松模型, Gerber-Shiu函数, 破产概率.

学科分类号: O211.4.