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Approximation of the Tail Probabilities of Randomly Weighted Sums in Presence of Dependence and Heavy Tails *

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Abstract: Let $\{X_k; k \geqslant 1\}$ be a sequence of real-valued random variables and $\{\theta_k; k \geqslant 1\}$ be other n random variables which are independent of the form sequence. Suppose that $\{X_k; k \geqslant 1\}$ are pairwise generalized negatively orthant dependent with heavy tails under the condition that $\{\theta_k; k \geqslant 1\}$ are independent or associated, some asymptotic formulas are established.

Keywords: randomly weighted sums; pairwise generalized negatively orthant dependent; heavy-tailed distribution; positively associated

2010 Mathematics Subject Classification: 62E20; 60F05

§1. Introduction

Let $\{X_k; k \geqslant 1\}$ be n real-valued random variables (rv's) with distribution functions $F_k = 1 - \overline{F}_k$, and $\{\theta_k; k \geqslant 1\}$ be other n rv's. In this paper, we study the tail probabilities of the randomly weighted sums S_n^θ and their maximum M_n^θ, defined, respectively, by

$$S_n^\theta = \sum_{k=1}^{n} \theta_k X_k, \tag{1}$$

and

$$M_n^\theta = \max_{1 \leqslant l \leqslant n} S_l^\theta. \tag{2}$$

The randomly weighted sums (1) can be found in many stochastic models. Consider a discrete time risk model in which the surplus of an insurance company is invested into a risky asset. X_k can be thought of as the net loss (the total claim amount minus total incoming premium) within the time period k and θ_k as the discount factor from time k

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to time 0 (the present). Then S_n^θ in (1) denotes the total net loss of the company at time n. The mainstream study of the tail behavior of S_n^θ has been restricted to the case that $X_1, X_2, \ldots, X_n$ are independent and identically distributed rv's with heavy tails. See [1] and [2], among many others.

A distribution function F on $(-\infty, +\infty)$ is long tailed (notation $F \in \mathscr{L}$) if the relation

$$\lim_{x\to\infty} \frac{\overline{F}(x+y)}{\overline{F}(x)} = 1$$

holds for some (or, equivalently, for all) $y \geqslant 0$. A famous subclass of long tailed distribution is subexponential. By definition, a distribution function F on $[0, +\infty)$ is said to be subexponential (notation $F \in \mathscr{S}$) if the relation

$$\lim_{x\to\infty} \frac{\overline{F^{*n}}(x)}{\overline{F}(x)} = n$$

holds for some (or, equivalently, for all) $n \geqslant 2$. More generally, a distribution function F on $(-\infty, +\infty)$ is said to be subexponential if the distribution $F(x)I\{0 \leqslant x < +\infty\}$ is subexponential, where $I\{A\}$ is the indictor function of the set A. Closely related is the class $\mathscr{D}$ of distributions with dominatedly-varying tails, if the relation

$$\limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} < \infty$$

holds for any $y \in (0, 1)$ (or, equivalently, for $y = 1/2$). Another important subclass of heavy-tailed distributions is the consistently varying class (denoted by $\mathscr{C}$). A distribution function F is in $\mathscr{C}$ if

$$\lim_{y\searrow 1} \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1, \quad \text{or, equivalently,} \quad \lim_{y\nearrow 1} \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} = 1.$$

A slight small class is the ERV (extended regularly varying) class if the relation

$$y^{-\beta} \leqslant \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \leqslant \limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \leqslant y^{-\alpha}$$

holds for some α, β with $0 < \alpha \leqslant \beta < \infty$ and any $y > 1$. Some related discussions on heavy-tailed distributions can be found in [3] and [4]. It is well known that these classes satisfy the following inclusions:

$$\mathrm{ERV}(-\alpha, -\beta) \subset \mathscr{C} \subset \mathscr{D} \cap \mathscr{L} \subset \mathscr{S} \subset \mathscr{L}.$$

Set

$$\overline{F}_*(y) = \liminf_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} \qquad \text{and} \qquad \mathbb{J}_F = \inf\Big\{ -\frac{\ln \overline{F}_*(y)}{\ln y} : y > 1 \Big\},$$

where $\mathbb{J}_F$ is called the upper Matuszewska index of the distribution function F. One can easily see that for $p > \mathbb{J}_F$, it holds that

$$x^{-p} = o(\overline{F}(x)), \qquad x \to \infty. \tag{3}$$

For more details of Matuszewska index see [3].

It should remark that the independence among the underlying rv's limits the usefulness of obtained results. Zhang et al.[5] obtained the asymptotic of the tail probabilities of S_n^θ in (1) under the condition that $\{X_k; k \geqslant 1\}$ are bivariate upper independence with ERV tails. Later, Gao and Wang[6] extended the results in [5] to $\mathscr{C}$ class. Zhang[7] studied the tail probability for weighted sums of $\sum\limits_{k=1}^{n} c_k X_k$ with the assumptions that the X_k's are either independent or pairwise-asymptotical independent with heavy tails. Some more works on this topic can be found in [8], [9] and [10], among many others.

This article extends the study to the case where the underlying random variables are pairwise generalized negatively orthant dependent (GNOD). A sequence of random variables $X_1, X_2, \ldots, X_n$ with distributions F_i, $1 \leqslant i \leqslant n$ are pairwise GNOD if both

$$\mathsf{P}(X_i > x_i, X_j > x_j) \leqslant g_U(x_i, x_j)\overline{F}_i(x_i)\overline{F}_j(x_j) \tag{4}$$

and

$$\mathsf{P}(X_i \leqslant x_i, X_j \leqslant x_j) \leqslant g_L(x_i, x_j)F_i(x_i)F_j(x_j) \tag{5}$$

hold for all positive integers $i \neq j$, $0 < g_L(x_i, x_j)$, $g_U(x_i, x_j) < \infty$ for $\max\{x_i, x_j\} < \infty$. Recall that if $g_U(\cdot) = 1$ and $g_L(\cdot) = 1$ in (4) and (5), $\{X_k; k \geqslant 1\}$ are called pairwise negatively orthant dependent (NOD), see [11] and [12]; if $g_U(\cdot) = g_L(\cdot) = M > 0$ and (4), (5) are allowed not only for two distinct random variables but more, $\{X_k; k \geqslant 1\}$ are extended negatively dependent (END), see [13] and [14]. Obviously, the pairwise GNOD structure allows a wide range of negative dependence structures among random variables, such as NOD, END, even some positive dependence. It should be remarked that the pairwise GNOD structure is different from the widely orthant dependent (WOD) structure mentioned in [15]. As pointed out in [15], the dominating coefficients $g_U(n)$ and $g_L(n)$ are functions of n, if $n = 2$ the dominating coefficients $g_U(2)$ and $g_L(2)$ are constants while $g_L(x_i, x_j)$ and $g_U(x_i, x_j)$ can go infinity as $\max\{x_i, x_j\} \to \infty$. Furthermore, an example in which the underlying random variables are pairwise GUNOD but not pairwise-asymptotical independent can be constructed in terms of the bivariate Pareto distribution (see in [16]). Let X_k, $k \geqslant 1$ be nonnegative random variables whose joint survival function is $\overline{H}_\theta(x_i, x_j) = (1 + x_i + x_j)^{-\theta}$, where $x_i > 0$, $x_j > 0$, $i \neq j$, $\theta > 0$. Then the marginal survival functions $\overline{F}_i$ and $\overline{F}_j$ are $\overline{F}_i(x_i) = (1+x_i)^{-\theta}$ and $\overline{F}_j(x_j) = (1+x_j)^{-\theta}$ for all $x_i > 0$, $x_j > 0$. If

$$\lim_{\min\{x_i,x_j\}\to\infty} \frac{x_i}{x_j} < \infty,$$

then

$$\lim_{\min\{x_i,x_j\}\to\infty} \frac{\overline{H}_\theta(x_i,x_j)}{\overline{F}_j(x_j)} \neq 0,$$

obviously, X_k, $k \geqslant 1$ are not pairwise-asymptotical independent while we can take

$$g_U(x_i,x_j) = \frac{(1+x_i)^\theta(1+x_j)^\theta}{(x_i+x_j)^\theta}$$

such that (4) holds.

Hereafter, we will use the assumptions that for $\nu > 0$, $c > 0$ and some real function $h(x) \nearrow \infty$ as $x \to \infty$,

$$\lim_{x\to\infty} g_U(x, xh(x))\overline{F}_k(h(x)x) = 0. \tag{6}$$

$$\lim_{x\to\infty} |g_U(x,\nu)| \leqslant c. \tag{7}$$

Throughout this paper, all limit relationships are for $x \to \infty$ unless otherwise stated. Write $x^+ = \max\{x, 0\}$ for a real number x. The relation $a(x) \sim b(x)$ stands for $\lim a(x)/b(x) = 1$ while the relations $a(x) \gtrsim b(x)$ and $b(x) \lesssim a(x)$ stand for $\liminf a(x)/b(x) \geqslant 1$.

The remaining part of this paper is organized as follows. Section 2 presents our main results. Section 3 proves the main results, after showing some necessary lemmas.

§2. Main Results

Our first main result is the following:

Theorem 1 Relations

$$\mathsf{P}(S_n^\theta > x) \sim \mathsf{P}(M_n^\theta > x) \sim \mathsf{P}\Big(\max_{1\leqslant k\leqslant n} \theta_k X_k > x\Big) \sim \sum_{k=1}^n \mathsf{P}(\theta_k X_k > x)$$

hold under the following three assumptions:

(A$_1$) $\{X_k; k \geqslant 1\}$ are pairwise GNOD with distribution functions $F_k \in \mathscr{D}\cap\mathscr{L}$ and $\overline{F}_k(x) > 0$ for all x and $k = 1, 2, \ldots$, and the conditions (6) and (7) hold for some real function $h(x)$ and positive constants ν and c;

(A$_2$) $\{\theta_k; k \geqslant 1\}$ are independent and satisfy $\mathsf{P}(a \leqslant \theta_k \leqslant b) = 1$ for some $0 < a \leqslant b < \infty$;

(A$_3$) The sequences $\{X_k; k \geqslant 1\}$ and $\{\theta_k; k \geqslant 1\}$ are mutually independent.

Next we are going to weaken the two-side boundedness condition of $\{\theta_k; k \geqslant 1\}$. To obtain the expected results, a dependence structure on $\{\theta_k; k \geqslant 1\}$ is required. Recall that rv's $\{\theta_k; k \geqslant 1\}$ are said to be (positively) associated if the inequality

$$\mathsf{E}f_1(\theta_1,\theta_2,\ldots,\theta_n)f_2(\theta_1,\theta_2,\ldots,\theta_n) \geqslant \mathsf{E}f_1(\theta_1,\theta_2,\ldots,\theta_n)\mathsf{E}f_2(\theta_1,\theta_2,\ldots,\theta_n) \tag{8}$$

holds for all coordinatewise (not necessarily strictly) increasing functions f_1 and f_2 for which the moments involved exist. Trivially, if f_1 is coordinatewise increasing but f_2 is coordinatewise decreasing, then inequality (8) is changed to

$$\mathsf{E}f_1(\theta_1,\theta_2,\ldots,\theta_n)f_2(\theta_1,\theta_2,\ldots,\theta_n) \leqslant \mathsf{E}f_1(\theta_1,\theta_2,\ldots,\theta_n)\mathsf{E}f_2(\theta_1,\theta_2,\ldots,\theta_n). \tag{9}$$

Related discussions can be found in [17].

Our second main result is:

Theorem 2 Relations

$$\mathsf{P}(S_n^\theta > x) \sim \mathsf{P}(M_n^\theta > x) \sim \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x)$$

hold under the following three assumptions:

(B_1) $\{X_k; k \geqslant 1\}$ are pairwise GNOD with distribution functions $F_k \in \mathscr{D} \cap \mathscr{L}$ and $\overline{F}_k(x) > 0$ for all x and $k = 1, 2, \ldots$, and the conditions (6) and (7) hold for some real function $h(x)$ and positive constants ν and c;

(B_2) $\{\theta_k; k \geqslant 1\}$ are associated and satisfy $\mathsf{P}(0 \leqslant \theta_k \leqslant b) = 1$, but, $\mathsf{P}(\theta_k = 0) < 1$ for some $0 < b < \infty$;

(B_3) The sequences $\{X_k; k \geqslant 1\}$ and $\{\theta_k; k \geqslant 1\}$ are mutually independent.

§3. Proofs of Main Results

3.1 Several Lemmas

We start with a lemma below as a direct consequence of [1].

Lemma 3 If $F \in \mathscr{D}$, then for each $p > \mathbb{J}_F$, there exist positive constants x_0 and B such that, for all $\theta \in (0, 1]$ and $x \geqslant \theta^{-1}x_0$,

$$\frac{\overline{F}(\theta x)}{\overline{F}(x)} \leqslant B\theta^{-p}.$$

Lemma 4 Under the conditions in Theorem 1, for all $n \geqslant 1$, it holds that

$$\mathsf{P}\Big(\max_{1\leqslant k\leqslant n} \theta_k X_k > x\Big) \sim \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x).$$

Proof It is obviously that

$$\mathsf{P}\Big(\max_{1\leqslant k\leqslant n} \theta_k X_k > x\Big) \leqslant \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x).$$

On the other hand, for any $0 < \varepsilon < 1$, there exists some $r > 0$ such that $r(1-\varepsilon) > p$, by (3), it holds that

$$\begin{aligned}
&\mathsf{P}\Big(\max_{1\leqslant k\leqslant n} \theta_k X_k > x\Big) \\
\geqslant\ & \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x) - \sum_{1\leqslant k<l\leqslant n} \mathsf{P}(\theta_k X_k > x, \theta_l X_l > x) \\
\geqslant\ & \sum_{k=1}^{n} \mathsf{P}\big(\theta_k X_k > x\big) - \sum_{1\leqslant k<l\leqslant n} \big(\mathsf{P}(\theta_k > x^{1-\varepsilon}) + \mathsf{P}(\theta_k X_k > x, \theta_l X_l > x, \theta_k \leqslant x^{1-\varepsilon})\big) \\
\geqslant\ & \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x) - \sum_{1\leqslant k<l\leqslant n} \big(x^{-r(1-\varepsilon)}\mathsf{E}\theta_k^r + \mathsf{P}(X_k > x^{\varepsilon}, \theta_l X_l > x)\big) \\
\geqslant\ & \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x) - \sum_{1\leqslant k<l\leqslant n} \big(x^{-r(1-\varepsilon)}\mathsf{E}\theta_k^r + g_U(x, x^{\varepsilon})\mathsf{P}(bX_l > x)\overline{F}_k(x^{\varepsilon})\big) \\
\sim\ & \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x),
\end{aligned}$$

where the last step holds by (5) and the arbitrariness of ε. The proof is accomplished. $\square$

At last, we establish a result not only at the core of the proof of our main results but also of independent interest in its own right:

Lemma 5 In addition to condition (7), let $\{X_k; k \geqslant 1\}$ be pairwise GNOD and nonnegative rv's with distribution functions $F_k \in \mathscr{D} \cap \mathscr{L}$. $\{\theta_k; k \geqslant 1\}$ is another sequence of random variables which is independent of $\{X_k; k \geqslant 1\}$ satisfying $\mathsf{P}(a \leqslant \theta_k \leqslant b) = 1$ for some $0 < a \leqslant b < \infty$. Then

$$\mathsf{P}\Big(\sum_{k=1}^{n} \theta_k X_k > x\Big) \sim \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x).$$

Proof Since $\sum\limits_{k=1}^{n} \theta_k X_k \geqslant \max \theta_k X_k$, by Lemma 4, it suffices to show that

$$\mathsf{P}\Big(\sum_{k=1}^{n} \theta_k X_k > x\Big) \lesssim \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x). \tag{10}$$

For an arbitrarily fixed number $l > 0$, we have

$$\begin{aligned}
&\mathsf{P}\Big(\sum_{k=1}^{n} \theta_k X_k > x\Big) \\
&\leqslant \mathsf{P}\Big(\max_{1\leqslant k\leqslant n} \theta_k X_k > x - l\Big) + \mathsf{P}\Big(\sum_{k=1}^{n} \theta_k X_k > x, \max_{1\leqslant k\leqslant n} \theta_k X_k \leqslant x - l\Big) \\
&= J_1 + J_2.
\end{aligned} \tag{11}$$

For J_1. Recall that $F_k \in \mathscr{D} \cap \mathscr{L}$, thus

$$J_1 \sim \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x). \tag{12}$$

Now, turn to J_2. With the pairwise GNOD assumption, we have

$$\begin{aligned}
J_2 &= \mathsf{P}\Big(\sum_{k=1}^{n} \theta_k X_k > x, \bigcup_{k=1}^{n} \Big\{\theta_k X_k > \frac{x}{n}\Big\}, \max_{1\leqslant k\leqslant n} \theta_k X_k \leqslant x - l\Big) \\
&\leqslant \sum_{k=1}^{n} \mathsf{P}\Big(\sum_{j\neq k} \theta_j X_j > l, \theta_k X_k > \frac{x}{n}\Big) \\
&\leqslant \sum_{k=1}^{n} \sum_{j\neq k} \mathsf{P}\Big(\theta_j X_j > \frac{l}{n-1}, \theta_k X_k > \frac{x}{n}\Big) \\
&\leqslant \sum_{k=1}^{n} \sum_{j\neq k} g_U\Big(\frac{l}{b(n-1)}, \frac{x}{bn}\Big) \mathsf{P}\Big(\theta_j X_j > \frac{l}{n-1}\Big) \mathsf{P}\Big(\theta_k X_k > \frac{x}{n}\Big) \\
&\leqslant \sum_{k=1}^{n} \sum_{j\neq k} C(n,x) \mathsf{P}(\theta_k X_k > x) = o\Big(\sum_{k=1}^{n} \mathsf{P}(\theta_k X_k > x)\Big),
\end{aligned}$$

where the last but one step holds by Lemma 3 and the fact of

$$\sup_{x\to\infty} |C(n,x)| = \Big| g_U\Big(\frac{l}{b(n-1)}, \frac{x}{bn}\Big) B n^p \mathsf{P}\Big(\theta_j X_j > \frac{l}{n-1}\Big)\Big| < \infty.$$

This ends the proof of Lemma 5. □

3.2 Proof Theorem 1

Proof Notice that

$$\max \theta_k X_k \leqslant S_n \leqslant M_n \leqslant \sum_{k=1}^{n} \theta_k X_k^+. \tag{13}$$

Since

$$\mathsf{P}(\theta_k X_k > x) = \mathsf{P}(\theta_k X_k^+ > x) \qquad \text{for } x > 0. \tag{14}$$

By Lemma 4 and Lemma 5, we only need to verify

$$\mathsf{P}\Big(\sum_{k=1}^{n} \theta_k X_k^+ > x\Big) \sim \sum_{k=1}^{n} \mathsf{P}(\theta_k X_k^+ > x). \tag{15}$$

It follows from the dominated convergence theorem that

$$\begin{aligned}\mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x\Big) &= \mathsf{E}\Big[\mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x \,|\, \theta_1,\theta_2,\ldots,\theta_n\Big)\Big] \\ &\sim \mathsf{E}\Big[\sum_{k=1}^{n}\mathsf{P}(\theta_k X_k^+ > x \,|\, \theta_1,\theta_2,\ldots,\theta_n)\Big] \\ &= \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x\Big). \end{aligned} \tag{16}$$

In view of (13) – (16), the proof of Theorem 1 is accomplished. □

3.3 Proof Theorem 2

Proof In this step, the proof is motivated by the idea of [18] in proving Theorem 2. Recall that

$$S_n \leqslant M_n \leqslant \sum_{k=1}^{n}\theta_k X_k^+.$$

To obtain the desired result, we formulate the proofs into two steps.

Step 1: Firstly, we deal with the case when θ_k is strictly positive, for small constant $\varepsilon > 0$, we have

$$\begin{aligned}&\mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x\Big) \\ &= \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x,\ \bigcup_{k=1}^{n}(\theta_k < \varepsilon)\Big) + \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x,\ \bigcap_{k=1}^{n}(\varepsilon \leqslant \theta_k \leqslant b)\Big) \\ &= \mathsf{E}I\Big\{\sum_{k=1}^{n}\theta_k X_k^+ > x\Big\}I\Big\{\bigcup_{k=1}^{n}(\theta_k < \varepsilon)\Big\} + \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x,\ \bigcap_{k=1}^{n}(\varepsilon \leqslant \theta_k \leqslant b)\Big) \\ &\leqslant \mathsf{E}I\Big\{\sum_{k=1}^{n}\theta_k X_k^+ > x\Big\}\mathsf{E}I\Big\{\bigcup_{k=1}^{n}(\theta_k < \varepsilon)\Big\} + \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x,\ \bigcap_{k=1}^{n}(\varepsilon \leqslant \theta_k \leqslant b)\Big) \\ &= \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x\Big)\mathsf{P}\Big(\bigcup_{k=1}^{n}(\theta_k < \varepsilon)\Big) + J_3, \end{aligned} \tag{17}$$

where

$$J_3 = \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x,\ \bigcap_{j=1}^{n}(\varepsilon \leqslant \theta_k \leqslant b)\Big).$$

With the arbitrariness of ε and the condition that θ_k is strictly positive here, it follows from Lemma 5 that

$$J_3 \geqslant \Big(1 - \mathsf{P}\Big(\bigcup_{k=1}^{n}(\theta_k < \varepsilon)\Big)\Big)\mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x\Big) \sim \sum_{k=1}^{n}\mathsf{P}(\theta_k X_k^+ > x). \tag{18}$$

Combining with Theorem 1,

$$\mathsf{P}(S_n^\theta > x) \geqslant \mathsf{P}\Big(S_n^\theta > x,\ \bigcap_{k=1}^{n}(\varepsilon \leqslant \theta_k \leqslant b)\Big)$$

$$\begin{aligned}&\sim \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x,\ \bigcap_{k=1}^{n}(\varepsilon\leqslant\theta_k\leqslant b)\Big)\\&\sim \sum_{k=1}^{n}\mathsf{P}(\theta_k X_k^+ > x).\end{aligned}\tag{19}$$

Step 2: Now consider the case where θ_k possibly assign a mass at value zero. We partition the whole space Ω as the union of A_K with $A_K=\{\theta_k>0$ for all $k\in K$ and $\theta_l=0$, for $l\notin K\}$ for $\emptyset\neq K\subset\{1,2,\ldots,n\}$. Then,

$$\begin{aligned}\mathsf{P}(S_n^\theta > x) &= \mathsf{P}\Big(S_n^\theta > x, \bigcup_K A_K\Big)\\&= \sum_K \mathsf{P}(S_n^\theta > x, A_K) = \sum_K \mathsf{P}\Big(\sum_{k=1}^{n}\theta_k X_k^+ > x, A_K\Big)\\&\geqslant \sum_{k=1}^{n}\mathsf{P}\Big(\theta_k X_k^+ > x, \bigcup_K A_K\Big) \sim \sum_{k=1}^{n}\mathsf{P}(\theta_k X_k^+ > x).\end{aligned}\tag{20}$$

In view of (17) – (20), the proof of Theorem 2 is accomplished. □

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重尾相依随机变量的随机加权和尾概率的渐近估计

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摘　要: 令$\{X_k; k \geqslant 1\}$为一列实值随机变量, $\{\theta_k; k \geqslant 1\}$为另一列与之独立的随机变量序列. 假设$\{X_k; k \geqslant 1\}$为两两广义负象限相依且服从重尾分布, 在$\{\theta_k; k \geqslant 1\}$独立和相依条件下, 本文得到了一些渐近估计.

关键词: 随机加权和; 两两广义负象限相依; 重尾分布; 正相依

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