应用概率统计　第 32 卷
第 4 期　2016 年 8 月

Chinese Journal of Applied Probability and Statistics
Aug., 2016, Vol. 32, No. 4, pp. 393-407
doi: 10.3969/j.issn.1001-4268.2016.04.006

Precise Large Deviations of Nonnegative, Non-Identical and Negatively Associated Random Variables*

YUAN Liangliang

(*Department of Science, Taiyuan Institute of Technology, Taiyuan, 030008, China*)

SONG Lixin　　FENG Jinghai

(*School of Mathematical Sciences, Dalian University of Technology, Dalian, 116024, China*)

Abstract: In this paper, precise large deviations of nonnegative, non-identical distributions and negatively associated random variables are investigated. Under certain conditions, the lower bound of the precise large deviations for the non-random sum is solved and the uniformly asymptotic results for the corresponding random sum are obtained. At the same time, we deeply discussed the compound renewal risk model, in which we found that the compound renewal risk model can be equivalent to renewal risk model under certain conditions. The relative research results of precise large deviations are applied to the more practical compound renewal risk model, and the theoretical and practical values are verified. In addition, this paper also shows that the impact of this dependency relationship between random variables to precise large deviations of the final result is not significant.

Keywords: precise large deviations; negative associated; non-random sums; random sums; compound renewal risk model

2010 Mathematics Subject Classification: 60F10; 60F05; 60G50

§1. Introduction

Mainstream research on precise large deviation probabilities has been concentrated on the study of the asymptotics $\mathsf{P}(S(t)-\mathsf{E}S(t)>x)\sim\lambda(t)\,\overline{F}(x)$, which holds uniformly for all $x\geqslant\gamma\lambda(t)$ for every fixed $\gamma>0$ as $t\to\infty$. Here $\{X_n, n\geqslant 1\}$ is a sequence of independent, identically distributed (i.i.d.) nonnegative heavy-tailed random variables with common distribution function F and finite expectation μ, independent of a process $\{N(t), t\geqslant 0\}$ driven by a sequence of nonnegative, integer-valued r.v.'s. Assume that $\lambda(t)=\mathsf{E}N(t)<\infty$ for all $t\geqslant 0$ but $\lambda(t)\to\infty$, as $t\to\infty$. All limit relations, unless

*The research was supported by the National Natural Sciences Foundation of China (11101061, 11371077 and 61175041).

Received March 19, 2015. Revised May 7, 2015.

explicitly stated, are for $t \to \infty$ or consequently for $\lambda(t) \to \infty$. $S(t) = \sum_{i=1}^{N(t)} X_i$, $t \geqslant 0$, denote random sum.

For classical works of precise large deviations with heavy tails, we refer the reader to [1–4], while for recent works, we refer to [5–8] among many others.

We say X (or its df F) is heavy-tailed if it has no exponential moments. An important subclass of heavy-tailed distributions is $\mathscr{D}$, which consists of all distributions with dominated variation in the sense that the relation $\limsup_{x\to\infty} \overline{F}(xy)/\overline{F}(x) < \infty$ holds for any $y \in (0,1)$ (or equivalently, for $y = 1/2$). Another slightly smaller subclass is $\mathscr{C}$, which consists of all distributions with consistent variation in the sense that $\lim_{y\searrow 1} \liminf_{x\to\infty} \overline{F}(xy)/\overline{F}(x) = 1$ or, equivalently, $\lim_{y\nearrow 1} \limsup_{x\to\infty} \overline{F}(xy)/\overline{F}(x) = 1$.

Strolling in past literature on precise large deviations, we find that most works were conducted only for independent r.v.'s, though several dealing with non-identically distributed r.v.'s, we refer the reader to [9].

It is their work that motivates our study. We will extend and improve their results in the following directions:

Firstly, we extend the relationship of r.v.'s from the independent case to NA structure (see Definition 1 as below);

Secondly, we do not require that r.v.'s $\{X_k,\ k \geqslant 1\}$ have the same distribution, which will play an important role in the study of a general compound renewal risk model (see Definition 18 in Section 4), where several types of claims may have potentially different distributions.

As an application of the above results, we will also discuss precise large deviations in general compound renewal risk model.

At the end of this section, we introduce corresponding concept of negative associated.

Definition 1 Random variables $X_1, X_2, \ldots, X_k$ are said to be negatively associated (NA) if for every pair of disjoint subsets A_1, A_2 of $\{1, 2, \ldots, k\}$,

$$\mathrm{Cov}\,\{f_1(X_i, i \in A_1), f_2(X_j, j \in A_2)\} \leqslant 0, \tag{1}$$

whenever f_1 and f_2 are increasing. This dependence structure was first introduced by [10] and [11].

The rest of paper is organized as follows. In Section 2 we introduce some useful lemmas in the paper. The main results are presented in Section 3. In Section 4, we apply our main results to a realistic example (General Compound Renewal Risk Model) and obtain a specific result. Finally the proofs of our results are given in Section 5.

§2. Preliminaries

We restate the following results that were obtained in the literature of the precise large deviations. We need the following lemmas to prove the main results behind. At first, we cite Theorem 3.1 of [9], which extends the related results of [4] from the identically distributed case to the non-identically distributed case under some extra conditions.

Lemma 2 Let X, $\{X_n,\, n \geqslant 1\}$ be independent nonnegative r.v.'s with distribution functions $F \in \mathscr{C}$, $\{F_n,\, n \geqslant 1\}$ and finite expectations $\overline{\mu}$, $\{\mu_n,\, n \geqslant 1\}$, respectively. Assume that

(i) the distribution functions $\{F_n,\, n \geqslant 1\}$ and F satisfy Assumption (A):

$$\lim_{n\to\infty} \sum_{1\leqslant k\leqslant n} \overline{F}_k(x)/(n\,\overline{F}(x)) = 1 \ \text{ holds uniformly for } \ x \geqslant X_0, \ \text{ for some } \ X_0 > 0;$$

(ii) the expectations $\overline{\mu}$ and $\{\mu_n,\, n \geqslant 1\}$ satisfy Assumption (B):

$$\lim_{n\to\infty} \sum_{1\leqslant k\leqslant n} \mu_k/n = \overline{\mu} < \infty, \quad \text{and} \quad \sup_{n\geqslant 1} \mu_n < \infty.$$

Then, for any fixed $\gamma > 0$, $\mathsf{P}(S_n - \mathsf{E}(S_n) > x) \sim n\,\overline{F}(x)$ holds uniformly for $x \geqslant \gamma n$, where $S_n = \sum\limits_{k=1}^{n} X_k$.

In the next Lemma we establish an important asymptotical relation for the tail probabilities of sums of NA r.v.'s.

Lemma 3 Let $\{X_n,\, n \geqslant 1\}$ be nonnegative NA r.v.'s with common distribution function $F \in \mathscr{C}$ and finite expectation μ. Then, for any fixed $n \geqslant 1$, the relation

$$\mathsf{P}(S_n > x) \sim n\,\overline{F}(x) \ \text{ holds as } \ x \to \infty. \tag{2}$$

Remark 4 The relation (2) is the popularization of Theorem 3.1 and Theorem 3.2 of [6]. Compared with the later, our result is focus on any fixed n. Using the same approach as used in the proof of Theorem 4.1 of [4] we can easily obtain the conclusion.

Lemma 5 Let $\{X_k,\, k \geqslant 1\}$ be NA r.v.'s with distribution functions $\{F_k,\, k \geqslant 1\}$ and mean vector be 0, satisfying $\sup\limits_{k\geqslant 1} \mathsf{E}(X_k^+)^r < \infty$ for some $r > 1$. Then for each fixed $\gamma > 0$ and $p > 0$, there exist positive numbers υ and $C = C(\upsilon, \gamma)$ irrespective to x and n such that for all $x \geqslant \gamma n$ and $n = 1, 2, \ldots$,

$$\mathsf{P}\Big(\sum_{1\leqslant k\leqslant n} X_k \geqslant x\Big) \leqslant \sum_{1\leqslant k\leqslant n} \overline{F}_k(\upsilon x) + C x^{-p}.$$

Remark 6 In fact, Lemma 5 is a modification of Lemma 2.3 of [6]. We just need give some modifications as following:

(i) $n\overline{F}(\upsilon x)$ in [6] is replaced by $\sum\limits_{k=1}^{n}\overline{F}_k(\upsilon x)$;

(ii) $h'=(\upsilon x)^{-1}\log\big(\upsilon^{q-1}x^q/[n\,\mathsf{E}(X_1^+)^q]+1\big)$ in [6] is replaced by

$$h=(\upsilon x)^{-1}\log\Big(\upsilon^{q-1}x^q\Big/\sum_{k=1}^{n}\mathsf{E}(X_k^+)^q+1\Big);$$

(iii) $C'=\sup\limits_{x\geqslant 0}\exp\big\{1/\upsilon+\upsilon^{q-1}x^q\,\overline{F}(\upsilon x)/\mathsf{E}(X_1^+)^q\big\}\big(\upsilon^{q-1}\gamma/\mathsf{E}(X_1^+)^q\big)^{-1/(2\upsilon)}<\infty$ in [6] is replaced by

$$C=\sup_{x\geqslant 0}\exp\Big\{\frac{1}{\upsilon}+\upsilon^{q-1}x^q\sum_{k=1}^{n}\overline{F}_k(\upsilon x)\Big/\sum_{k=1}^{n}\mathsf{E}(X_k^+)^q\Big\}\Big(\upsilon^{q-1}\gamma\Big/\max_{1\leqslant k\leqslant n}\mathsf{E}(X_k^+)^q\Big)^{-1/(2\upsilon)}<\infty.$$

This lemma will be used in deriving the lower bound of the large-deviation probabilities in the proof of Theorem 15.

Lemma 7 and Lemma 8 can be found in [4]. These inequalities will play a key role in the proof of Theorem 15.

Lemma 7 For a distribution function $F\in\mathscr{D}$ with a finite expectation, $1\leqslant\gamma_F<\infty$ and as $x\to\infty$, $x^{-p}=o(\overline{F}(x))$ for any $p>\gamma_F$.

Lemma 8 For a distribution function $F\in\mathscr{D}$ and every $\rho>\gamma_F$, there exist positive x_0 and B such that, for all $\theta\in(0,1]$ and all $x\geqslant\theta^{-1}x_0$, $\overline{F}(\theta x)/\overline{F}(x)\leqslant B\theta^{-\rho}$.

Lemmas 9 and 10 are reformulations of Lemmas 3.3 and 3.5 of [3]. We will need these two lemmas in the later part of this paper.

Lemma 9 Let $\{\zeta(t),\,t\geqslant 0\}$ be a stochastic process with a common expectation $\mathsf{E}\zeta(t)=1$. If for any fixed $\delta>0$, $\mathsf{E}\zeta(t)I_{\{\zeta(t)>1+\delta\}}=o(1)$, then $\zeta(t)\xrightarrow{\mathsf{P}}1$.

Lemma 10 Suppose $\{Y_n,\,n\geqslant 1\}$ is a sequence of i.i.d. non-negative r.v.'s with a common mean $\mathsf{E}Y_1=1/\lambda$, constituting a renewal counting process $\{N(t),\,t\geqslant 0\}$. We have for any positive constants δ and m, $\sum\limits_{k>(1+\delta)\lambda(t)}k^m\mathsf{P}(N(t)=k)=o(1)$.

Next, we give three useful lemmas, which are the popularization and application of Theorem 1 in [12], Lemma 2.3 in [13] and Lemma 3.2 of [14] respectively. These inequalities will play a key role in the proof of Theorem 15 and Theorem 16.

Lemma 11 Let $\{X_n,\,n\geqslant 1\}$ be NA r.v.'s with distribution functions $\{F_n,\,n\geqslant 1\}$, $x>0$ be any positive constant, and let $(y_1,y_2,\ldots,y_n)$ be any set of positive numbers. Then for $y > \max\limits_{1\leqslant k\leqslant n}\{y_k\}$ and $0 < t \leqslant 1$, we have

$$\mathsf{P}(S_n > x) \leqslant \sum_{k=1}^{n} \mathsf{P}(X_k \geqslant y_k) + P_1,$$

where

$$P_1 = \exp\Big\{\frac{x}{y} - \frac{x}{y}\ln\Big[xy^{t-1}\Big/\Big(\sum_{k=1}^{n}\int_0^{y_k} u^t\,\mathrm{d}F_k(u)\Big) + 1\Big]\Big\}.$$

Lemma 12 Let $\{X_n, n \geqslant 1\}$ be NA r.v.'s with distribution functions $\{F_n, n \geqslant 1\}$ and finite expectations $\{\mu_n, n \geqslant 1\}$, and let $\{N(t), t \geqslant 0\}$ be a stochastic process generated by non-negative integer-valued r.v.'s independent of the sequence $\{X_n, n \geqslant 1\}$. Assume that

(i) the expectations $\{\mu_n, n \geqslant 1\}$ satisfy that for some $\overline{\mu} < \infty$, $\lim\limits_{n\to\infty} n^{-1}\sum\limits_{k=1}^{n}\mu_k = \overline{\mu}$;

(ii) the stochastic process $N(t)$ satisfies that $N(t)/\lambda(t) \xrightarrow{\mathsf{P}} 1$, as $t \to \infty$.

Then $\mathsf{E}S(t) \sim \overline{\mu}\lambda(t)$, i.e. $\mathsf{E}S(t) = \overline{\mu}\lambda(t)(1 + o(1))$.

Lemma 13 Let $\{X_n, n \geqslant 1\}$ be NA non-negative r.v.'s with common distribution function and finite expectation μ. Then, for all $\upsilon > 0$, $x > 0$ and $n \geqslant 1$, $\mathsf{P}(S_n > x) \leqslant n\,\overline{F}(x/\upsilon) + (\mathrm{e}\mu n/x)^{\upsilon}$.

Remark 14 In fact, all the work that we need to do is just changing the independence property among the r.v.'s $\{X_n, n \geqslant 1\}$ which appears in Theorem 1 of [12], Lemma 2.3 of [13] and Lemma 3.2 of [14] into negative associated structure. However, this kind of change of relationships will not effect the final result at all.

§3. Main Results

Based on Lemma 2, we hope to obtain more results under relatively relaxed conditions. For nonnegative r.v.'s $\{X_n, n \geqslant 1\}$ with distributions $\{F_n, n \geqslant 1\}$, we need the following condition: there exists a proper distribution F on $[0, \infty)$ and $0 < \alpha < \beta < \infty$ such that

$$\alpha = \liminf_{x\to\infty}\inf_{n\geqslant 1}\overline{F}_n(x)/\overline{F}(x) \leqslant \limsup_{x\to\infty}\sup_{n\geqslant 1}\overline{F}_n(x)/\overline{F}(x) = \beta. \tag{3}$$

The following theorem is a result about precise large deviations of nonrandom sum:

Theorem 15 Let $\{X_n, n \geqslant 1\}$ be a sequence of nonnegative NA r.v.'s with distribution functions $\{F_n, n \geqslant 1\}$ and finite expectations $\{\mu_n, n \geqslant 1\}$; X be a nonnegative random variable with a distribution function $F \in \mathscr{C}$ and a finite expectation $\overline{\mu}$. Assume that

(i) the distribution functions $\{F_n, n \geqslant 1\}$ and F satisfy (3);

(ii) the expectations $\overline{\mu}$ and $\{\mu_n, n \geqslant 1\}$ satisfy Assumption (B) in Lemma 2.

Then, for any fixed $\gamma > 0$,

$$\alpha \leqslant \liminf_{n\to\infty} \inf_{x\geqslant\gamma n} \frac{\mathsf{P}(S_n - \mathsf{E}(S_n) > x)}{n\,\overline{F}(x)} \leqslant \limsup_{n\to\infty} \sup_{x\geqslant\gamma n} \frac{\mathsf{P}(S_n - \mathsf{E}(S_n) > x)}{n\,\overline{F}(x)} \leqslant \beta, \tag{4}$$

where $S_n = \sum\limits_{k=1}^{n} X_k$.

Based on Theorem 15, we have the asymptotic results for random sum as follows:

Theorem 16 Let $\{X_n, n \geqslant 1\}$ be a sequence of nonnegative NA r.v.'s with distribution functions $\{F_n, n \geqslant 1\}$ and finite expectations $\{\mu_n, n \geqslant 1\}$; X be a nonnegative random variable with a distribution function $F \in \mathscr{C}$ and a finite expectation $\overline{\mu}$. Assume that

(i) the distribution functions $\{F_n, n \geqslant 1\}$ and F satisfy (3);

(ii) the expectations $\overline{\mu}$ and $\{\mu_n, n \geqslant 1\}$ satisfy Assumption (B) in Lemma 2;

(iii) $\{N(t), t \geqslant 0\}$ is a non-negative and integer-valued process independent of $\{X_n, n \geqslant 1\}$, and satisfies Assumption I: $\mathsf{E}N^p(t)I_{\{N(t)>(1+\delta)\lambda(t)\}} = O(\lambda(t))$.

Then, for any fixed $\gamma > 0$,

$$\alpha \leqslant \liminf_{t\to\infty} \inf_{x\geqslant\gamma\lambda(t)} \frac{\mathsf{P}(S(t) - \mathsf{E}(S(t)) > x)}{\lambda(t)\,\overline{F}(x)} \leqslant \limsup_{t\to\infty} \sup_{x\geqslant\gamma\lambda(t)} \frac{\mathsf{P}(S(t) - \mathsf{E}(S(t)) > x)}{\lambda(t)\,\overline{F}(x)} \leqslant \beta. \tag{5}$$

§4. Application to General Compound Renewal Risk Model

In this section, we provide a realistic application (Compound Renewal Risk Model) of Theorem 16. Tang et al.[3] studied the precise large deviations in the compound renewed model, the model is as follows:

Definition 17 The Compound Renewal Risk Model

(a) the individual claim sizes $\{X_n, n \geqslant 1\}$ are i.i.d. nonnegative r.v.'s with a common distribution function F and a finite mean $\mu = \mathsf{E}X_1$;

(b) the accident inter-arrival times $\{Y_n, n \geqslant 1\}$ are i.i.d. non-negative r.v.'s with a finite mean $\mathsf{E}Y_1 = 1/\lambda$, independent of $\{X_n, n \geqslant 1\}$;

(c) the number of accidents in the interval $[0, t]$ is denoted by $\tau(t) = \sup\{n \geqslant 1 : T_n \leqslant t, t \geqslant 0\}$, where $T_n = \sum\limits_{i=1}^{n} Y_i$, $n \geqslant 1$, denote the arrival time of the nth accident; the number of individual claims caused by the nth accident is a non-negative, integer-valued r.v. Z_n, and $\{Z_n, n \geqslant 1\}$ constitutes a process of i.i.d. r.v.'s with a common df W, independent of $\{X_n, n \geqslant 1\}$ and $\{Y_n, n \geqslant 1\}$;

(d) the total number of claims up to time t is given by $N'(t) = \sum_{i=1}^{\tau(t)} Z_i$, $t \geqslant 0$; the total claim amount process $\{S'(t), t \geqslant 0\}$ is defined by

$$S'(t) = \sum_{i=1}^{N'(t)} X_i. \tag{6}$$

For more details in compound renewal risk model, Tang et al.[3] proved the precise large deviations results, while Kaas and Tang[5] proved again the precise large deviations results when the number of individual claims $\{Z_n, n \geqslant 1\}$ in Definition 17 are ND structure. Based on Definition 17 and Equation (3), we introduce the following more realistic model in the context of insurance.

Definition 18 The General Compound Renewal Risk Model is given by conditions (b)–(d) in Definition 17 and

(a′) the individual claim sizes $\{X_n, n \geqslant 1\}$ are NA non-negative r.v.'s with a finite mean vector $\boldsymbol{\mu} = (\mathsf{E}X_1, \mathsf{E}X_2, \ldots, \mathsf{E}X_n, \ldots)$;

(e) the individual claim sizes $\{X_{nk}, 1 \leqslant k \leqslant Z_n\}$ caused by the nth accident with common distribution F_n and finite expectation μ_n for every fixed n, $n \geqslant 1$.

Generally speaking, however, it describes a more realistic risk model since the random sum (6) is equal to

$$S'(t) = \sum_{n=1}^{\tau(t)} \sum_{k=1}^{Z_n} X_{nk} = \sum_{k=1}^{Z_1} X_{1k} + \sum_{k=1}^{Z_2} X_{2k} + \cdots + \sum_{k=1}^{Z_{\tau(t)}} X_{\tau(t)k} = \sum_{n=1}^{\tau(t)} A_n, \tag{7}$$

where $A_n = \sum_{k=1}^{Z_n} X_{nk}$, $1 \leqslant n \leqslant \tau(t)$. From [11], the increasing functions defined on disjoint subsets of a set of NA r.v.'s are NA, we know that $\{A_n, n \geqslant 1\}$ are NA non-negative r.v.'s with distribution functions $\{G_n, n \geqslant 1\}$. We have the asymptotic result for random sum (7) as follows:

Theorem 19 In the general compound renewal risk model, let F, $F_n \in \mathscr{C}$, $n \geqslant 1$ satisfy relation (3), and $\mathsf{E}Z_1^p < \infty$ for some $p > 1$. Then, for any fixed $\gamma > 0$,

$$\alpha \leqslant \liminf_{t\to\infty} \inf_{x\geqslant\gamma\lambda'(t)} \frac{\mathsf{P}(S'(t) - \mathsf{E}(S'(t)) > x)}{\lambda'(t)\,\overline{F}(x)} \leqslant \limsup_{t\to\infty} \sup_{x\geqslant\gamma\lambda'(t)} \frac{\mathsf{P}(S'(t) - \mathsf{E}(S'(t)) > x)}{\lambda'(t)\,\overline{F}(x)} \leqslant \beta, \tag{8}$$

where $\lambda'(t) = \mathsf{E}N'(t)$ and $S'(t) = \sum_{i=1}^{\tau(t)} A_i$.

Proof Since $\{A_n, n \geqslant 1\}$ are NA non-negative r.v.'s with distribution functions $\{G_n, n \geqslant 1\}$, we know that $S'(t)$ is random sum of nonnegative NA r.v.s $\{A_n, n \geqslant 1\}$. Combining $\mathsf{E}A_n = \mu_n \mathsf{E}Z_1$ and Assumption (B) we can easily see that Assumption (B) in Theorem 16 are satisfied for $\{A_n, n \geqslant 1\}$. Using Lemma 10 we know that renewal counting process $\tau(t)$ satisfies Assumption I in Theorem 16. To complete the proof of Theorem 19, we need to verify that G_n, $n \geqslant 1$ satisfy relation (3). So we only need to verify that for all $i \geqslant 1$, there exist some $C_i > 0$, such that $\overline{G}_i(x) \sim C_i \overline{F}_i(x)$ as $x \to \infty$.

For all $\upsilon > 0$, $x > 0$ and $n \geqslant 1$, using Lemma 13, we know that

$$\mathsf{P}\Big(\sum_{1 \leqslant k \leqslant n} X_{ik} > x\Big) \leqslant n\overline{F}_i(x/\upsilon) + (\mathrm{e}\mu n/x)^{\upsilon}, \qquad \text{for } n \geqslant 1.$$

Let $\upsilon = p$, where $p > \gamma_F \geqslant 1$, for all large n, we have

$$\begin{aligned}\mathsf{P}\Big(\sum_{1 \leqslant k \leqslant n} X_{ik} > x\Big) &\leqslant n\overline{F}_i(x/\upsilon) + (\mathrm{e}\mu n/x)^{\upsilon} \leqslant nB_i p^{\rho}\,\overline{F}_i(x) + (\mathrm{e}\mu)^p n^p\,\overline{F}_i(x) \\ &\leqslant 2C_{i0} n^p\,\overline{F}_i(x),\end{aligned}$$

where C_{i0} is a positive number irrespective to x and n, we use Lemma 7 and Lemma 8 in the second inequality.

Since $\mathsf{E}Z_1^p < \infty$, by the dominated convergence theorem, we have

$$\begin{aligned}\lim_{x\to\infty} \frac{\overline{G}_i(x)}{\overline{F}_i(x)} &= \lim_{x\to\infty} \Big\{\Big[\sum_{n=1}^{\infty} \mathsf{P}\Big(\sum_{k=1}^{n} X_{ik} > x\Big)\Big] \Big/ \overline{F}_i(x)\Big\} \mathsf{P}(Z_i = n) \\ &= \sum_{n \geqslant 1} \lim_{x\to\infty} \Big\{\Big[\mathsf{P}\Big(\sum_{k=1}^{n} X_{ik} > x\Big)\Big] \Big/ \overline{F}_i(x)\Big\} \mathsf{P}(Z_1 = n) \\ &= \sum_{n \geqslant 1} n\mathsf{P}(Z_1 = n) = \mathsf{E}Z_1. \end{aligned} \tag{9}$$

Using (9) we have

$$\alpha \mathsf{E}Z_1 = \liminf_{x\to\infty} \inf_{n \geqslant 1} \overline{G}_n(x)/\overline{F}(x) \leqslant \limsup_{x\to\infty} \sup_{n \geqslant 1} \overline{G}_n(x)/\overline{F}(x) = \beta \mathsf{E}Z_1. \tag{10}$$

So, using Theorem 16 we have that, for any fixed $\gamma > 0$,

$$\begin{aligned}\alpha \mathsf{E}Z_1 &\leqslant \liminf_{t\to\infty} \inf_{x \geqslant \gamma \mathsf{E}\tau(t)} \frac{\mathsf{P}(S'(t) - \mathsf{E}(S'(t)) > x)}{\mathsf{E}\tau(t)\,\overline{F}(x)} \\ &\leqslant \limsup_{t\to\infty} \sup_{x \geqslant \gamma \mathsf{E}\tau(t)} \frac{\mathsf{P}(S'(t) - \mathsf{E}(S'(t)) > x)}{\mathsf{E}\tau(t)\,\overline{F}(x)} \leqslant \beta \mathsf{E}Z_1. \end{aligned} \tag{11}$$

Combining (11) and $\lambda'(t) = \mathsf{E}Z_1 \mathsf{E}\,\tau(t)$ we have (8), then the proof of Theorem 19 is completed. $\square$

§5. Proofs

5.1 The Proof of Theorem 15

Proof We modify the proof of Theorem 15 in [4]. At first, we estimate the lower bound. For any $\lambda > 1$,

$$
\begin{aligned}
&\mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big) \\
\geqslant\, &\mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x,\ \max_{1\leqslant j\leqslant n} X_j > \lambda x\Big) \\
\geqslant\, &\sum_{1\leqslant j\leqslant n} \mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x, X_j > \lambda x\Big) - \sum_{1\leqslant j<l\leqslant n} \mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x, X_j > \lambda x, X_l > \lambda x\Big) \\
\geqslant\, &\sum_{1\leqslant j\leqslant n} \mathsf{P}\Big(S_n - X_j - \sum_{1\leqslant k\leqslant n} \mu_k > (1-\lambda)x, X_j > \lambda x\Big) - \Big(\sum_{1\leqslant j\leqslant n} \overline{F}_j(\lambda x)\Big)^2 \\
\geqslant\, &\sum_{1\leqslant j\leqslant n} \overline{F}_j(\lambda x)\Big(1 - \sum_{j\leqslant k\leqslant n} \overline{F}_k(\lambda x)\Big) - \sum_{1\leqslant j\leqslant n} \mathsf{P}\Big(S_n^{(j)} - \sum_{1\leqslant k\leqslant n} \mu_k \leqslant (1-\lambda)x\Big),
\end{aligned} \tag{12}
$$

where $S_n^{(j)} = \sum_{1\leqslant k\neq j\leqslant n} X_k$. Here we use the NA r.v.'s property (see Property P_1 and P_2 of [11]) in the second inequality, use an elementary inequality $\mathsf{P}(AB) \geqslant \mathsf{P}(B) - \mathsf{P}(A^c)$ for all events A and B in the third inequality. For any $\delta_1 > 0$, using Assumption (A), for all large x and for all k, $k \geqslant 1$, we have that

$$
(1-\delta_1)\alpha\,\overline{F}(x) \leqslant \overline{F}_k(x) \leqslant (1+\delta_1)\beta\,\overline{F}(x). \tag{13}
$$

We estimate the second term in (12), for all large x, $x \geqslant X_0$, we have

$$
\mathsf{P}\Big(S_n^{(j)} - \sum_{1\leqslant k\leqslant n} \mu_k \leqslant (1-\lambda)x\Big) \leqslant \mathsf{P}\Big(\sum_{1\leqslant k\neq j\leqslant n} (\mu_k - X_k) \geqslant (\lambda-1)x/2\Big).
$$

By NA r.v.'s property (see [14] and Definition 2.3 of [11]), the r.v.'s $\{\mu_k - X_k,\ k \geqslant 1\}$ are still NA. Then for arbitrarily fixed $\gamma > 0$ and $p > \gamma_F$, by Lemma 5 there exist positive constants v_0 and C irrespective to x and n such that the inequality

$$
\begin{aligned}
\mathsf{P}\Big(\sum_{1\leqslant k\neq j\leqslant n} (\mu_k - X_k) \geqslant (\lambda-1)x/2\Big) &\leqslant \sum_{1\leqslant k\neq j\leqslant n} \mathsf{P}(\mu_k - X_k \geqslant (\lambda-1)x/(2v_0)) + Cx^{-p} \\
&\leqslant \sum_{1\leqslant k\leqslant n} F_k(-(\lambda-1)x/(4v_0)) + Cx^{-p}
\end{aligned}
$$

holds for all $x \geqslant \gamma n$ and $n \geqslant 1$. Using the fact that $\{X_n,\ n \geqslant 1\}$ be non-negative r.v.'s and Lemma 7, we know that

$$
\mathsf{P}\Big(S_n^{(j)} - \sum_{1\leqslant k\leqslant n} \mu_k \leqslant (1-\lambda)x\Big) = o(\overline{F}(\lambda x)). \tag{14}
$$

Plugging (13) and (14) into (12) yields that

$$\mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big) \geqslant (1-\delta_1)\alpha n\,\overline{F}(\lambda x)(1-(1+\delta_1)\beta n\,\overline{F}(\lambda x)) - \delta_1(n\,\overline{F}(\lambda x)).$$

Let $\delta_1 \downarrow 0$, we have

$$\liminf_{n\to\infty} \inf_{x\geqslant\gamma n} \mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big)/(n\,\overline{F}(\lambda x)) \geqslant \liminf_{n\to\infty} \inf_{x\geqslant\gamma n} \alpha(1-\beta n\,\overline{F}(\lambda x)).$$

Hence, we have

$$\begin{aligned}&\liminf_{n\to\infty} \inf_{x\geqslant\gamma n} \mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big)/(n\,\overline{F}(x))\\ \geqslant\ &\Big(\liminf_{n\to\infty} \inf_{x\geqslant\gamma n} \alpha(1-\beta n\,\overline{F}(\lambda x))\Big) \liminf_{x\to\infty} \overline{F}(\lambda x)/\,\overline{F}(x) = \alpha \liminf_{x\to\infty} \overline{F}(\lambda x)/\,\overline{F}(x).\end{aligned}$$

Here, we use $n\,\overline{F}(\lambda x) \to 0$, as $n\to\infty$, holds uniformly for $x \geqslant \gamma n$ in the inequality. Since $F \in \mathscr{C}$ and $\lambda > 1$ is arbitrary, we can conclude that

$$\liminf_{n\to\infty} \inf_{x\geqslant\gamma n} \mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big)/(n\,\overline{F}(x)) \geqslant \alpha \lim_{\lambda\searrow 1} \liminf_{x\to\infty} \overline{F}(\lambda x)/\,\overline{F}(x) = \alpha. \tag{15}$$

Now we start to estimate the upper bound. For any $\theta \in (0,1)$, we define

$$\widetilde{X}_k := X_k I_{(X_k\leqslant\theta x)} \ \text{ for } \ k\geqslant 1, \quad \widetilde{S}_n := \sum_{1\leqslant k\leqslant n} \widetilde{X}_k \quad \text{and} \quad \widetilde{x} := x + \sum_{1\leqslant k\leqslant n} \mu_k.$$

By a standard truncation argument, we can show that

$$\begin{aligned}\mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big) &\leqslant \mathsf{P}\Big(\max_{1\leqslant k\leqslant n} X_k > \theta x\Big) + \mathsf{P}\Big(\max_{1\leqslant k\leqslant n} X_k \leqslant \theta x, S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big)\\ &\leqslant \sum_{1\leqslant k\leqslant n} \mathsf{P}(X_k > \theta x) + \mathsf{P}(\widetilde{S}_n > \widetilde{x}).\end{aligned} \tag{16}$$

Applying (13) to the first term in (16), we can conclude that, for any $\delta_2 > 0$,

$$\begin{aligned}\mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n} \mu_k > x\Big) &\leqslant \sum_{1\leqslant k\leqslant n} \overline{F}_k(\theta x) + \mathsf{P}(\widetilde{S}_n > \widetilde{x})\\ &\leqslant (1+\delta_2)\beta n\,\overline{F}(\theta x) + \mathsf{P}(\widetilde{S}_n > \widetilde{x}).\end{aligned} \tag{17}$$

We estimate the second term in (17). Let $a = \{-\log(n\,\overline{F}(\theta x)), 1\}$, which tends to ∞ holds uniformly for $x \geqslant \gamma n$. For arbitrarily fixed $h = h(x,n) > 0$, we have

$$\mathsf{P}(\widetilde{S}_n > \widetilde{x})/(n\,\overline{F}(\theta x)) \leqslant \mathrm{e}^{-h\widetilde{x}+a}\,\mathsf{E}\,\mathrm{e}^{h\widetilde{S}_n} \leqslant \exp\Big\{\sum_{1\leqslant k\leqslant n} \int_0^{\theta x} (\mathrm{e}^{ht}-1)\,\mathrm{d}F_k(t) - h\widetilde{x} + a\Big\}. \tag{18}$$

Here we use the property of NA r.v.'s (see Property P_2 and P_6 of [11]) in the second inequality. The value of h above will be specified later. We split the integral on the right-hand side of (18) into two terms, and applying an inequality $e^x - 1 \leqslant xe^x$ for all x, we obtain that for every $k \geqslant 1$,

$$\begin{aligned}\int_0^{\theta x}(e^{ht}-1)\,dF_k(t) &= \int_0^{\theta x/a^2}(e^{ht}-1)\,dF_k(t) + \int_{\theta x/a^2}^{\theta x}(e^{ht}-1)\,dF_k(t)\\ &\leqslant e^{h\theta x/a^2}\int_0^{\theta x/a^2} ht\,dF_k(t) + e^{h\theta x}\,\overline{F}_k(\theta x/a^2)\\ &\leqslant h\mu_k e^{h\theta x/a^2} + e^{h\theta x}\,\overline{F}_k(\theta x/a^2).\end{aligned} \tag{19}$$

Plugging (19) into (18) yields that, for all large n, for any $\delta_3 > 0$ and for any $\delta_4 > 0$, we have

$$\begin{aligned}\frac{\mathsf{P}(\widetilde{S}_n > \widetilde{x})}{n\,\overline{F}(\theta x)} &\leqslant \exp\Big\{h\sum_{1\leqslant k\leqslant n}\mu_k e^{h\theta x/a^2} + e^{h\theta x}\sum_{1\leqslant k\leqslant n}\overline{F}_k(\theta x/a^2) - h\widetilde{x} + a\Big\}\\ &\leqslant \exp\Big\{h\sum_{1\leqslant k\leqslant n}\mu_k(e^{h\theta x/a^2}-1) + (1+\delta_3)\beta e^{h\theta x} n\,\overline{F}(\theta x/a^2) - hx + a\Big\}\\ &\leqslant \exp\big\{(1+\delta_4)hn\overline{\mu}(e^{h\theta x/a^2}-1) + (1+\delta_3)\beta Ba^{2\rho}e^{h\theta x}n\,\overline{F}(\theta x) - hx + a\big\}.\end{aligned} \tag{20}$$

Here we use (3) in the second inequality, and use Assumption (B) and Lemma 8 in the third inequality. Let $h = (a - 2\rho\log a)/(\theta x)$ in (20), we obtain that, for all large n, for any $\delta_5 > 0$,

$$\begin{aligned}\mathsf{P}(\widetilde{S}_n > \widetilde{x})/(n\,\overline{F}(\theta x)) &\leqslant \exp\big\{(1+\delta_4)nh\overline{\mu}(e^{a^{-1}}-1) + (1+\delta_3)\beta B - (a-2\rho\log a)\theta^{-1} + a\big\}\\ &\leqslant e^{(1+\delta_3)B\beta}\exp\{(1-\theta^{-1} + (1+\delta_4)\delta_5)a\}.\end{aligned} \tag{21}$$

Let $\delta_2 \downarrow 0$, $\delta_3 \downarrow 0$, $\delta_4 \downarrow 0$, $\delta_5 \downarrow 0$, combining (21) with (17) we have

$$\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\mathsf{P}\Big(S_n - \sum_{1\leqslant k\leqslant n}\mu_k > x\Big)/(n\,\overline{F}(\theta x)) \leqslant \beta + \limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\mathsf{P}(\widetilde{S}_n > \widetilde{x})/(n\,\overline{F}(\theta x)) = \beta.$$

Since $F \in \mathscr{C}$ and the arbitrariness of $\theta \in (0,1)$ we obtain that

$$\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\frac{\mathsf{P}\Big(S_n - \sum\limits_{k=1}^n \mu_k > x\Big)}{n\,\overline{F}(x)} = \lim_{\theta\nearrow 1}\limsup_{n\to\infty}\sup_{x\geqslant\gamma n}\left(\frac{\mathsf{P}\Big(S_n - \sum\limits_{k=1}^n \mu_k > x\Big)}{n\,\overline{F}(\theta x)}\,\frac{\overline{F}(\theta x)}{\overline{F}(x)}\right) \leqslant \beta. \tag{22}$$

The result (4) follows from (15) and (22). $\square$

5.2 The Proof of Theorem 16

Proof By Lemma 9 with $\zeta(t) = N(t)/\lambda(t)$, we can easily see that Assumption I implies

$$N(t)/\lambda(t) \xrightarrow{\mathsf{P}} 1. \tag{23}$$

By the same approach as used in the proof of Lemma 4.2 of [2] and Theorem 4.1 of [4] we know that, for any $\delta > 0$, we have

$$\begin{aligned} & \mathsf{P}(S(t) - \mathsf{E}(S(t)) > x) \\ = & \sum_{n\geqslant 0} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S(t)) > x) \\ = & \Big(\sum_{n\leqslant(1+\delta)\lambda(t)} + \sum_{n>(1+\delta)\lambda(t)}\Big)\mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S(t)) > x). \end{aligned} \tag{24}$$

First, we estimate the first term in (24), clearly,

$$\begin{aligned} & \sum_{n\leqslant(1+\delta)\lambda(t)} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S(t)) > x) \\ = & \sum_{|n-\lambda(t)|<\epsilon(t)\lambda(t)} + \sum_{n-\lambda(t)<-\epsilon(t)\lambda(t)} + \sum_{\epsilon(t)\lambda(t)<n-\lambda(t)<\delta\lambda(t)} \\ =: & K_1 + K_2 + K_3. \end{aligned} \tag{25}$$

Here $\epsilon(t)$ is a positive function, such that $\epsilon(t) \to 0$ as $t \to \infty$. By Lemma 12 and Assumption (B), we know that, for any $\delta_6 > 0$, $t \to \infty$,

$$(1-\delta_6)\lambda(t)\overline{\mu} \leqslant \mathsf{E}S(t) \leqslant (1+\delta_6)\lambda(t)\overline{\mu}.$$

Start with the estimation of K_1, for any $\delta_7 > 0$,

$$\begin{aligned} K_1 \leqslant & \sum_{|n-\lambda(t)|<\epsilon(t)\lambda(t)} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S_n) > x - (1+\delta_6)n\overline{\mu} + (1-\delta_6)\overline{\mu}\lambda(t)) \\ \leqslant & \sum_{|n-\lambda(t)|<\epsilon(t)\lambda(t)} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S_n) > x - \epsilon(t)\lambda(t)\overline{\mu} - \delta_6(n+\lambda(t))\overline{\mu}) \\ \leqslant & \beta(1+\delta_7)(1+\epsilon(t))\lambda(t)\,\overline{F}(x) \sum_{|n-\lambda(t)|<\epsilon(t)\lambda(t)} \mathsf{P}(N(t) = n). \end{aligned}$$

Here, in the last step, we use Theorem 15 and the fact

$$\overline{F}(x - \epsilon(t)\lambda(t)\overline{\mu} - \delta_6(n+\lambda(t))\overline{\mu}) \leqslant \beta(1+\delta_7)\overline{F}(x).$$

for any fixed $\gamma > 0$, holds uniformly for $x > \gamma\lambda(t)$, as $t \to \infty$, since $F \in \mathscr{C}$. For any $\delta_8 > 0$, by the same treatment we obtain the corresponding asymptotic lower bound as

$$K_1 \geqslant (1-\delta_8)\alpha(1-\epsilon(t))\lambda(t)\,\overline{F}(x)(1-o(1)) \sum_{|n-\lambda(t)|<\epsilon(t)\lambda(t)} \mathsf{P}(N(t) = n),$$

for any fixed $\gamma > 0$, $x > \gamma\lambda(t)$, as $t \to \infty$. Furthermore, according to (23), we have

$$\sum_{|n-\lambda(t)|<\epsilon(t)\lambda(t)} \mathsf{P}(N(t) = n) = \mathsf{P}(|N(t) - \lambda(t)| < \epsilon(t)\lambda(t)) \to 1, \qquad \text{as } t \to \infty.$$

Thus, we can obtain

$$\alpha \leqslant \liminf_{t\to\infty} \inf_{x\geqslant\gamma\lambda(t)} \frac{K_1}{\lambda(t)\,\overline{F}(x)} \leqslant \limsup_{t\to\infty} \sup_{x\geqslant\gamma\lambda(t)} \frac{K_1}{\lambda(t)\,\overline{F}(x)} \leqslant \beta. \tag{26}$$

Next, we estimate K_2, for any δ_9, $\delta_{10} > 0$,

$$\begin{aligned} K_2 &\leqslant \mathsf{P}\big(S_{[(1-\epsilon(t))\lambda(t)]} - \mathsf{E}(S(t)) > x\big) \sum_{n-\lambda(t)<-\epsilon(t)\lambda(t)} \mathsf{P}(N(t) = n) \\ &\leqslant \mathsf{P}\big(S_{[(1-\epsilon(t))\lambda(t)]} - \mathsf{E}S_{[(1-\epsilon(t))\lambda(t)]} > x + (1-\delta_9)\lambda(t)\overline{\mu} - (1+\delta_9)[(1-\epsilon(t))\lambda(t)]\overline{\mu}\big) \\ &\quad \times \mathsf{P}(N(t) - \lambda(t) < -\epsilon(t)\lambda(t)) \\ &\leqslant \delta_{10}(1+\delta_{10})^2\beta[(1-\epsilon(t))\lambda(t)]\,\overline{F}(x) = o(\lambda(t)\,\overline{F}(x)), \end{aligned} \tag{27}$$

where δ_9, and δ_{10} are small enough. By the same approach as used in the proof of K_2, we know that

$$K_3 = o(\lambda(t)\,\overline{F}(x)). \tag{28}$$

Plugging (26), (27) and (28) into (25) we can obtain for any fixed $\gamma > 0$,

$$\begin{aligned} \alpha &\leqslant \liminf_{t\to\infty} \inf_{x\geqslant\gamma\lambda(t)} \frac{\sum\limits_{n\leqslant(1+\delta)\lambda(t)} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S(t)) > x)}{\lambda(t)\,\overline{F}(x)} \\ &\leqslant \limsup_{t\to\infty} \sup_{x\geqslant\gamma\lambda(t)} \frac{\sum\limits_{n\leqslant(1+\delta)\lambda(t)} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S(t)) > x)}{\lambda(t)\,\overline{F}(x)} \leqslant \beta. \end{aligned} \tag{29}$$

for any fixed $\gamma > 0$, holds uniformly for $x > \gamma\lambda(t)$, as $t \to \infty$.

To complete the proof, it remains to estimate the second term in (24). We use Lemma 11 and set $t = 1$, $y_k = x/(2v)$, $v > 1$, $y = x/v$, $v > 1$ $\big(y > \max\limits_{1\leqslant k\leqslant n}\{y_k\}$ for large $x\big)$, for any $\delta_{11}, \delta_{12} > 0$, we obtain

$$\begin{aligned} \mathsf{P}(S_n \geqslant x) &\leqslant \sum_{1\leqslant k\leqslant n} \mathsf{P}(X_k \geqslant y_k) + \exp\Big\{xy^{-1} - xy^{-1}\ln\Big(x\Big/\Big[\sum_{k=1}^{n}\int_0^{y_k} u\,\mathrm{d}F_k(u)\Big] + 1\Big)\Big\} \\ &\leqslant \sum_{1\leqslant k\leqslant n} \mathsf{P}(X_k \geqslant x/(2v)) + \exp\Big\{v - v\ln\Big(\frac{x}{n\overline{\mu}(1+\delta_{11})}\Big)\Big\} \\ &\leqslant (1+\delta_{12})\beta n\,\overline{F}(x/(2v)) + \mathrm{e}^v(n\overline{\mu}(1+\delta_{11}))^v x^{-v}. \end{aligned}$$

Hence, we have

$$\sum_{n>(1+\delta)\lambda(t)} \mathsf{P}(N(t) = n)\mathsf{P}(S_n - \mathsf{E}(S(t)) > x)$$

$$
\begin{aligned}
&\leqslant \sum_{n>(1+\delta)\lambda(t)} \mathsf{P}(N(t)=n)\mathsf{P}(S_n>x) \\
&\leqslant (1+\delta_{12})\beta\,\overline{F}(x/(2v)) \sum_{n>(1+\delta)\lambda(t)} n\mathsf{P}(N(t)=n) \\
&\quad + (\mathrm{e}\overline{\mu}(1+\delta_{11}))^v x^{-v} \sum_{n>(1+\delta)\lambda(t)} n^v\mathsf{P}(N(t)=n) \\
&=: J_1+J_2. \qquad (30)
\end{aligned}
$$

Firstly, we estimate J_1. From Assumption I, we know that

$$\sum_{n>(1+\delta)\lambda(t)} n\mathsf{P}(N(t)=n)=o(\lambda(t)).$$

So, we have

$$J_1 \leqslant (1+\delta_{12})B\beta(2v)^\rho\,\overline{F}(x)\,o(\lambda(t))=o(\lambda(t)\,\overline{F}(x)). \qquad (31)$$

Here we have used the Lemma 8 in the first inequality. Where B is a positive number and $\rho>\gamma_{_F}$. Next, we estimate J_2. Setting v in J_2 equal to p, where $p>\gamma_{_F}\geqslant 1$, we obtain

$$
\begin{aligned}
J_2 &= (\mathrm{e}\overline{\mu}(1+\delta_{11}))^p x^{-p} \sum_{n>(1+\delta)\lambda(t)} n^p\mathsf{P}(N(t)=n) \\
&= O(\lambda(t))(\mathrm{e}\overline{\mu}(1+\delta_1 1))^p x^{-p} = o(\lambda(t)\,\overline{F}(x)), \qquad (32)
\end{aligned}
$$

where we use Assumption I in the second equality, and use Lemma 7 in the last equality. Plugging (31), (32) into (30), we know that

$$\sum_{n>(1+\delta)\lambda(t)} \mathsf{P}(N(t)=n)\mathsf{P}(S_n-\mathsf{E}(S(t))>x)=o(\lambda(t)\,\overline{F}(x)). \qquad (33)$$

Plugging (29) and (33) into (24), we know that (5) holds. □

Acknowledgements We thank the referees for their valuable comments which help the authors improve the exposition of this paper significantly.

References

[1] Cline D B H, Hsing T. Large deviation probabilities for sums and maxima of random variables with heavy or subexponential tails [R]. Preprint. Texas: Texas A&M University, 1991.

[2] Klüppelberg C, Mikosch T. Large deviations of heavy-tailed random sums with applications in insurance and finance [J]. *J. Appl. Probab.*, 1997, **34(2)**: 293–308.

[3] Tang Q H, Su C, Jiang T, et al. Large deviations for heavy-tailed random sums in compound renewal model [J]. *Statist. Probab. Lett.*, 2001, **52(1)**: 91–100.

[4] Ng K W, Tang Q H, Yan J A, et al. Precise large deviations for sums of random variables with consistently varying tails [J]. *J. Appl. Probab.*, 2004, **41(1)**: 93–107.

[5] Kaas R, Tang Q H. A large deviation result for aggregate claims with dependent claim occurrences [J]. *Insurance Math. Econom.*, 2005, **36(3)**: 251–259.

[6] Tang Q H. Insensitivity to negative dependence of the asymptotic behavior of precise large deviations [J]. *Electron. J. Probab.*, 2006, **11(4)**: 107–120.

[7] Tang Q H. Insensitivity to negative dependence of asymptotic tail probabilities of sums and maxima of sums [J]. *Stoch. Anal. Appl.*, 2008, **26(3)**: 435–450.

[8] He W, Cheng D Y, Wang Y B. Asymptotic lower bounds of precise large deviations with nonnegative and dependent random variables [J]. *Statist. Probab. Lett.*, 2013, **83(1)**: 331–338.

[9] Lu D W, Song L X, Xu Y. Precise large deviations for sums of independent random variables with consistently varying tails [J]. *Comm. Statist. Theory Methods*, 2014, **43(1)**: 28–43.

[10] Ebrahimi N, Ghosh M. Multivariate negative dependence [J]. *Comm. Statist. Theory Methods*, 1981, **10(4)**: 307–337.

[11] Joag-Dev K, Proschan F. Negative association of random variables with applications [J]. *Ann. Statist.*, 1983, **11(1)**: 286–295.

[12] Fuk D k, Nagaev S V. Probability inequalities for sums of independent random variables [J]. *Theory Probab. Appl.*, 1971, **16(4)**: 643–660.

[13] Skučaitė A. Large deviations for sums of independent heavy-tailed random variables [J]. *Lithuanian Math. J.*, 2004, **44(2)**: 198–208.

[14] Block H W, Savits T H, Shaked M. Some concepts of negative dependence [J]. *Ann. Probab.*, 1982, **10(3)**: 765–772.

非负不同分布负相协随机变量下的精细大偏差

袁亮亮 宋立新 冯敬海

(太原工业学院理学系, 太原, 030008) (大连理工大学数学科学学院, 大连, 116024)

摘 要: 本文研究非负, 不同分布, 负相协随机变量的精细大偏差问题. 在相对较弱的条件下, 重点解决了非随机和的精细大偏差的下限问题, 得到相对应的随机和的一致渐近结论. 同时, 对复合更新风险模型进行了深入的讨论, 在一定的条件之下将其简化为一般的更新模型, 并将所得相关的精细大偏差的结论应用到更为实际的复合更新风险模型中, 验证了其理论与实际价值. 除此之外, 本文的研究还表明, 随机变量间的这种相依关系对精细大偏差的最终结果的影响并不大.

关键词: 精细大偏差; 负相协; 非随机和; 随机和; 复合更新风险模型

中图分类号: O211.4