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The Existence and Uniqueness of Solutions to Stochastic Age-Dependent Population Equations with Jumps *

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Abstract: In this paper, we introduce a class of stochastic age-dependent population equations with Poisson jumps. Existence and uniqueness of energy solutions for stochastic age-dependent population dynamic system are proved under local non-Lipschitz condition in Hilbert space.

Keywords: stochastic age-dependent population equations; energy solutions; local non-Lipschitz condition; Poisson jumps

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§1. Introduction

Stochastic differential equations can be found in many applications in such areas as economics, biology, finance, ecology and other sciences [1–3]. In recent years, stochastic partial differential equations in a separable Hilbert space have been studied by many authors and various results on the existence, uniqueness and the asymptotic behavior of the solutions have been established (e.g., [4–8]).

The purpose of this paper is to discuss by energy method (that is, the method by energy equality) the existence and uniqueness of the energy solutions to the stochastic age-dependent population equations:

$$\begin{cases} \mathrm{d}_t P = -\dfrac{\partial P}{\partial a}\mathrm{d}t - \mu(t,a)P\mathrm{d}t + f(t,P)\mathrm{d}t + g(t,P)\mathrm{d}W(t) + \displaystyle\int_Z h(t,P,z)\widetilde{N}(\mathrm{d}t,\mathrm{d}z), & \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{in } J=(0,A)\times(0,T); & \\ P(0,a) = P_0(a), & \text{in } [0,A]; \\ P(t,0) = \displaystyle\int_0^A \beta(t,a)P(t,a)\mathrm{d}a, & \text{in } [0,T], \end{cases} \tag{1}$$

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where $P(t,a)$ denotes the population density of age a at time t, $\beta(t,a)$ denotes the fertility rate of females of age a at time t, $\mu(t,a)$ denotes the mortality rate of age a at time t. $f(t,P)+g(t,P)\,\mathrm{d}W(t)/\mathrm{d}t+\int_Z h(t,P,z)\widetilde{N}(\mathrm{d}t,\mathrm{d}z)/\mathrm{d}t$ is the effects of external environment for population system, such as emigration, earthquake and so on. d_tP is the differential of P relative to t, i.e., $\mathrm{d}_tP=(\partial P/\partial t)\,\mathrm{d}t$.

Recently, the stochastic population equations have received a great deal of attention. For example, Wang and Wang [9] gave the convergence of the semi-implicit Euler method for stochastic age-dependent population equations with Poisson jumps. Li et al. [10] studied the convergence of numerical solutions to stochastic age-dependent population equations with Markovian switching. Ma et al. [11] investigated numerical analysis for stochastic age-dependent population equations with fractional Brownian motion and the asymptotic stability of stochastic age-dependent population equations with Markovian switching in [12]. Zhang et al. [13] showed the existence, uniqueness and exponential stability of the solutions for stochastic age-dependent population. For the case where f, g and h satisfy the global Lipschitz condition and the coercivity condition, many results are known. However this global Lipschitz condition is seemed to be considerably strong when one discusses variable applications in real world.

We are concerned with stochastic age-dependent population equations with Poisson jump for the case where f, g and h do not necessarily satisfy the global Lipschitz condition. Thus we discuss the existence and uniqueness of weak solutions to stochastic age-dependent population equations with Poisson jump (1) with the condition proposed by the author [6, 7]. This condition was investigated by [14], [15] and the others as non-Lipschitz condition. So in this paper we consider the existence and uniqueness of weak solutions for the case where f, g and h satisfy the local non-Lipschitz condition of this type for the end of wider applications.

The contents of this paper are as follows. In Section 2 the preliminaries are given. In Section 3 the existence and uniqueness of the local energy solutions are discussed.

§2. Preliminaries

Let

$$\begin{aligned}V&=H^1([0,A])\\&\equiv\Big\{\varphi\,\Big|\,\varphi\in L^2([0,A]),\ \frac{\partial\varphi}{\partial a}\in L^2([0,A]),\ \text{where }\frac{\partial\varphi}{\partial a}\text{ is generalized partial derivative}\Big\}.\end{aligned}$$

V is a Sobolev space. $H=L^2([0,A])$ such that $V\hookrightarrow H\equiv H^*\hookrightarrow V^*$. V^* is the dual space of V. We denote by $\|\cdot\|$, $|\cdot|$ and $\|\cdot\|_*$ the norms in V, H and V^* respectively; by $\langle\cdot,\cdot\rangle$

the duality product between V, V^*, and by $(\cdot,\cdot)$ the scalar product in H.

Let $W(t)$ be a Wiener process defined on complete probability space $(\Omega, \mathscr{F}, \mathsf{P})$ and taking its values in the separable Hilbert space K, with increment covariance operator Q. Let $\{\mathscr{F}_t\}_{t\geqslant 0}$ be the σ-algebra generated by $\{W(s), 0 \leqslant s \leqslant t\}$, then $W(t)$ is a martingale relative to $(\mathscr{F}_t)_{t\geqslant 0}$ and we have the following representation of $W(t)$: $W(t) = \sum\limits_{i=1}^{\infty} \sqrt{\lambda_i}\beta_i(t)e_i$, where $\{e_i\}_{i\geqslant 1}$ is an orthonormal set of eigenvectors of Q, $\beta_i(t)$ are mutually independent real Wiener processes with incremental covariance $\lambda_i > 0$, $Qe_i = \lambda_i e_i$ and $\operatorname{tr} Q = \sum\limits_{i=1}^{\infty} \lambda_i$ (tr denotes the trace of an operator). For an operator $G \in \mathscr{L}(K, H)$ be the space of all bounded linear operators from K into H, we denote by $\|G\|_2$ its denotes the Hilbert-Schmidt norm, i.e. $\|G\|_2^2 = \operatorname{tr}(GQG^{\mathsf{T}})$.

Let $C = C([0, T]; H)$ be the space of all continuous function from $[0, T]$ into H with sup-norm $\|\psi\|_C = \sup\limits_{0\leqslant s\leqslant T} |\psi(s)|$, $L^p_V = L^p([0, T]; V)$ and $L^p_H = L^p([0, T]; H)$.

Let $p = (p(t))$, $t \in D_p$ be a stationary $\mathscr{F}_t$-Poisson point process with characteristic measure λ. Denote by $N(\mathrm{d}t, \mathrm{d}z)$ the Poisson counting measure associated with p, i.e., $N(t, Z) = \sum\limits_{s\in D_p, s\leqslant t} I_Z(p(s))$ with measurable set $Z \in \mathscr{B}(Z - \{0\})$ which denotes the Borel σ-field of $Z - \{0\}$. Let $\widetilde{N}(\mathrm{d}t, \mathrm{d}z) := N(\mathrm{d}t, \mathrm{d}z) - \mathrm{d}t\lambda(\mathrm{d}z)$ be the compensated Poisson measure which is independent of $W(t)$.

Let $f(t, \cdot) : L^2_H \to H$, $g(t, \cdot) : L^2_H \to \mathscr{L}(K, H)$ and $h(t, \cdot, \cdot) : L^2_H \times Z \to H$ be a family of nonlinear operators, $\mathscr{F}_t$-measurable almost surely in t.

Moreover, we impose the following conditions:

(H1) $\mu(t, a)$, $\beta(t, a)$ are nonnegative measurable, and

$$\begin{cases} 0 \leqslant \mu_0 \leqslant \mu(t, a) < \infty & \text{in } J, \\ 0 \leqslant \beta(t, a) \leqslant \overline{\beta} < \infty & \text{in } J. \end{cases}$$

(H2) (i) (The growth condition) there exists a function $H(t, r) : R^+ \times R^+ \to R^+$ such that $H(t, r)$ is locally integrable in $t \geqslant 0$ for any fixed $r \geqslant 0$, and is continuous monotone nondecreasing and concave in r for any fixed $t \in [0, T]$. Furthermore, for any fixed $t \in [0, T]$ and $u \in C$, the following inequality is satisfied:

$$|f(t, u)|^2 + \|g(t, u)\|_2^2 + \int_Z |h(t, u, z)|^2 \lambda(\mathrm{d}z) \leqslant H(t, \|u\|_C^2), \qquad t \in [0, T];$$

(ii) for any constant $\gamma > 0$, the differential equation, $\mathrm{d}\theta/\mathrm{d}t = \gamma H(t, \theta)$, $t \in [0, T]$, has a solution $\theta(t) = \theta(t; 0, \theta_0)$ on $[0, T]$ for any initial value θ_0.

(H3) (i) (The local condition) for any integer $N > 0$ there exists a function $G_N : R^+ \times R^+ \to R^+$ such that $G_N(t, r)$ is locally integrable in $t \in [0, T]$ for any fixed $r \geqslant 0$

and is continuous, monotone nondecreasing and concave in r with $G_N(t,0) = 0$. Furthermore, the following inequality is satisfied: for any $u, v \in C$ with $|u|, |v| \leqslant N$,

$$|f(t,u) - f(t,v)|^2 + \|g(t,u) - g(t,v)\|_2^2 + \int_Z |h(t,u,z) - h(t,v,z)|^2 \lambda(\mathrm{d}z)$$
$$\leqslant G_N(t, \|u - v\|_C^2), \qquad t \in [0,T];$$

(ii) for any constant $\gamma > 0$, if a nonnegative function $z(t)$ satisfies that

$$z(t) \leqslant \gamma \int_0^t G_N(s, z(s))\mathrm{d}s,$$

for all $t \in [0,T]$, then $z(t) \equiv 0$ holds for any $t \in [0,T]$.

(H4) (i) (The global condition) there exists a function $G(t,r): R^+ \times R^+ \to R^+$ such that $G(t,r)$ is locally integrable in $t \in [0,T]$ for any fixed $r \geqslant 0$ and is continuous, monotone nondecreasing and concave in r for any fixed $t \in [0,T]$. $G(t,0) = 0$ for any fixed $t \in [0,T]$. Furthermore, the following inequality is satisfied: for any $u, v \in C$,

$$|f(t,u) - f(t,v)|^2 + \|g(t,u) - g(t,v)\|_2^2 + \int_Z |h(t,u,z) - h(t,v,z)|^2 \lambda(\mathrm{d}z)$$
$$\leqslant G(t, \|u - v\|_C^2), \qquad t \in [0,T];$$

(ii) for any constant $\gamma > 0$, if a nonnegative function $z(t)$ satisfies that

$$z(t) \leqslant \gamma \int_0^t G(s, z(s))\mathrm{d}s,$$

for all $t \in [0,T]$, then $z(t) \equiv 0$ holds for any $t \in [0,T]$.

Remark 1 (H3) (i) is a generalization of the following condition:

(i) (The local Lipschitz condition) for any fixed integer $N > 0$, there exists an $L_N > 0$ such that for any $u, v \in C$ with $|u| \leqslant N$ and $|v| \leqslant N$,

$$|f(t,u) - f(t,v)|^2 + \|g(t,u) - g(t,v)\|_2^2 + \int_Z |h(t,u,z) - h(t,v,z)|^2 \lambda(\mathrm{d}z)$$
$$\leqslant L_N \|u - v\|_C^2, \qquad t \in [0,T].$$

(H4) (i) is a generalization of the following condition:

(i) (The global Lipschitz condition) there exists a constant $L > 0$ such that $u, v \in C$,

$$|f(t,u) - f(t,v)|^2 + \|g(t,u) - g(t,v)\|_2^2 + \int_Z |h(t,u,z) - h(t,v,z)|^2 \lambda(\mathrm{d}z)$$
$$\leqslant L \|u - v\|_C^2, \qquad t \in [0,T].$$

Lemma 2 (See [8]) For any $t \geqslant 0$, there exists a constant $c > 0$ such that

$$\begin{aligned}
&\mathsf{E}\sup_{0\leqslant s\leqslant t}\Big|\int_0^s\int_Z[|h(l,P(l),z)\Big|^2+2(P(l),h(l,P(l),z))]\widetilde{N}(\mathrm{d}l,\mathrm{d}z)\\
\leqslant\ &\frac{1}{4}\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t}|P(s)|^2\Big]+c\mathsf{E}\int_0^t\int_Z|h(s,P(s),z)|^2\lambda(\mathrm{d}z)\mathrm{d}s.
\end{aligned}$$

§3. Existence and Uniqueness of Energy Solutions

In this section we discuss the existence and uniqueness of energy solutions to the stochastic age-dependent population equations with jumps (1) in a Hilbert space. First we give the definition of the energy solution to (1).

Definition 3 An $\mathscr{F}_t$-adapted stochastic process $P(t)$ on the probability space $(\Omega, \mathscr{F}, \mathsf{P})$ is called the energy solution to (1) if the following conditions are satisfied:

(i) $P(t) \in I^2(0,T;V) \cap L^2(\Omega; C(0,T;H))$;

(ii) the following equation holds in V^* almost surely,

$$\begin{cases}
P(t)=P_0-\displaystyle\int_0^t\frac{\partial P(s)}{\partial a}\mathrm{d}s-\int_0^t\mu(s,a)P(s)\mathrm{d}s+\int_0^t f(s,P(s))\mathrm{d}s\\
\qquad\qquad+\displaystyle\int_0^t g(s,P(s))\mathrm{d}W(s)+\int_0^t\int_Z h(s,P,z)\widetilde{N}(\mathrm{d}s,\mathrm{d}z), & \text{in } J;\\
P(0,a)=P_0(a), & \text{in } [0,A];\\
P(t,0)=\displaystyle\int_0^A\beta(t,a)P(s)\mathrm{d}a, & \text{in } [0,T],
\end{cases}$$

where $P(t) := P(t,a)$, $P_0 := P(0,a)$;

(iii) the following stochastic energy equality holds: for $t \in [0,T]$,

$$\begin{aligned}
|P(t)|^2=&|P_0|^2+2\int_0^t\Big\langle-\frac{\partial P(s)}{\partial a}-\mu(s,a)P(s),P(s)\Big\rangle\mathrm{d}s+2\int_0^t(P(s),f(s,P(s)))\mathrm{d}s\\
&+2\int_0^t(P(s),g(s,P(s))\mathrm{d}W(s))+\int_0^t\|g(s,P(s))\|_2^2\mathrm{d}s\\
&+\int_0^t\int_Z[|h(s,P(s),z)|^2+2(P(s),h(s,P(s),z))]\widetilde{N}(\mathrm{d}s,\mathrm{d}z). \qquad (2)
\end{aligned}$$

When the coefficients f and g (i.e. $h \equiv 0$) of (1) satisfy the global Lipschitz condition, Zhang et al.[13] proved the existence and uniqueness of the energy solution to (1). So using the similar method as by [13], we have the following theorem for (1) of which the coefficients f, g and h satisfy global non-Lipschitz condition.

Theorem 4 Assume that conditions (H1) and (H4) are satisfied. Then there exists a unique energy solution $P(t)$ to (1).

We are in position to prove the main theorem in this paper.

Theorem 5 Assume that conditions (H1)–(H3) are satisfied. Then there exists a unique energy solution $P(t)$ to (1).

Proof Let N be a natural integer and let $T_0 \in (0, (1/4) \wedge (1/(4|A\overline{\beta}^2 - 2\mu_0|)) \wedge T)$. We define the sequence of the functions $\{f_N(t,u)\}$, $\{g_N(t,u)\}$ and $\{h_N(t,u,z)\}$ for $(t,u) \in [0,T_0] \times C$ as follows:

$$f_N(t,u) = \begin{cases} f(t,u), & \text{if } |u| \leqslant N; \\ f(t, Nu/|u|), & \text{if } |u| > N, \end{cases}$$

$$g_N(t,u) = \begin{cases} g(t,u), & \text{if } |u| \leqslant N; \\ g(t, Nu/|u|), & \text{if } |u| > N, \end{cases}$$

$$h_N(t,u,z) = \begin{cases} h(t,u,z), & \text{if } |u| \leqslant N; \\ h(t, Nu/|u|, z), & \text{if } |u| > N. \end{cases}$$

Then the functions $\{f_N(t,u)\}$, $\{g_N(t,u)\}$ and $h_N(t,u,z)$ for $u, v \in C$, $t \in [0,T_0]$ satisfy (H2) and the following inequality

$$\begin{aligned} & |f_N(t,u) - f_N(t,v)|^2 + \|g_N(t,u) - g_N(t,v)\|_2^2 + \int_Z |h_N(t,u,z) - h_N(t,v,z)|^2 \lambda(\mathrm{d}z) \\ \leqslant{}& G_N(t, \|u-v\|_C^2). \end{aligned}$$

Thus by Theorem 4 there exist the unique energy solutions $P_N(t)$ and $P_{N+1}(t)$, respectively to the following stochastic age-dependent population equations:

$$\begin{aligned} P(t) = P_0 &- \int_0^t \frac{\partial P(s)}{\partial a} \mathrm{d}s - \int_0^t \mu(s,a) P(s) \mathrm{d}s + \int_0^t f_N(s, P(s)) \mathrm{d}s \\ &+ \int_0^t g_N(s, P(s)) \mathrm{d}W(s) + \int_0^t \int_Z h_N(s, P(s), z) \widetilde{N}(\mathrm{d}s, \mathrm{d}z), \\ P(t) = P_0 &- \int_0^t \frac{\partial P(s)}{\partial a} \mathrm{d}s - \int_0^t \mu(s,a) P(s) \mathrm{d}s + \int_0^t f_{N+1}(s, P(s)) \mathrm{d}s \\ &+ \int_0^t g_{N+1}(s, P(s)) \mathrm{d}W(s) + \int_0^t \int_Z h_{N+1}(s, P(s), z) \widetilde{N}(\mathrm{d}s, \mathrm{d}z). \end{aligned}$$

Define the stopping times $\sigma_N := T_0 \wedge \inf\{t \in [0,T] : |P_N(t)| \geqslant N\}$, $\sigma_{N+1} := T_0 \wedge \inf\{t \in [0,T] : |P_{N+1}(t)| \geqslant N+1\}$, $\tau_N := \sigma_N \wedge \sigma_{N+1}$. By the energy equality

$$|P_{N+1}(t) - P_N(t)|^2$$

$$
\begin{aligned}
&= 2\int_0^t \Big\langle P_{N+1}(s)-P_N(s), -\frac{\partial P_{N+1}(s)}{\partial a}+\frac{\partial P_N(s)}{\partial a}-\mu(s,a)(P_{N+1}(s)-P_N(s))\Big\rangle \mathrm{d}s\\
&\quad + 2\int_0^t \langle P_{N+1}(s)-P_N(s), f_{N+1}(s,P_{N+1}(s))-f_N(s,P_N(s))\rangle \mathrm{d}s\\
&\quad + 2\int_0^t \langle P_{N+1}(s)-P_N(s), g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\rangle \mathrm{d}W(s)\\
&\quad + \int_0^t \|g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\|_2^2 \mathrm{d}s\\
&\quad + \int_0^t\int_Z \big[|h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z)|^2\\
&\quad + 2(P_{N+1}(s)-P_N(s), h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z))\big]\widetilde{N}(\mathrm{d}s,\mathrm{d}z)\\
&\leqslant -2\int_0^t \Big\langle P_{N+1}(s)-P_N(s), \frac{\partial(P_{N+1}(s)-P_N(s))}{\partial a}\Big\rangle \mathrm{d}s - 2\mu_0\int_0^t |P_{N+1}(s)-P_N(s)|^2 \mathrm{d}s\\
&\quad + 2\int_0^t \langle P_{N+1}(s)-P_N(s), f_{N+1}(s,P_{N+1}(s))-f_N(s,P_N(s))\rangle \mathrm{d}s\\
&\quad + \int_0^t \|g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\|_2^2 \mathrm{d}s\\
&\quad + 2\int_0^t \langle P_{N+1}(s)-P_N(s), g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\rangle \mathrm{d}W(s)\\
&\quad + \int_0^t\int_Z \big[|h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z)|^2\\
&\quad + 2(P_{N+1}(s)-P_N(s), h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z))\big]\widetilde{N}(\mathrm{d}s,\mathrm{d}z).
\end{aligned}
$$

Since

$$
\begin{aligned}
&-\Big\langle P_{N+1}(s)-P_N(s), \frac{\partial(P_{N+1}(s)-P_N(s))}{\partial a}\Big\rangle\\
&= -\int_0^A (P_{N+1}(s)-P_N(s))\mathrm{d}_a(P_{N+1}(s)-P_N(s)) = \frac{1}{2}\Big(\int_0^A \beta(t,a)(P_{N+1}(s)-P_N(s))\mathrm{d}a\Big)^2\\
&\leqslant \frac{1}{2}\int_0^A \beta^2(s,a)\mathrm{d}a\int_0^A (P_{N+1}(s)-P_N(s))^2\mathrm{d}a \leqslant \frac{1}{2}A\overline{\beta}^2|P_{N+1}(s)-P_N(s)|^2.
\end{aligned}
$$

Therefore, we get that

$$
\begin{aligned}
&|P_{N+1}(t)-P_N(t)|^2\\
&\leqslant |A\overline{\beta}^2-2\mu_0|\int_0^t |P_{N+1}(s)-P_N(s)|^2\mathrm{d}s + \int_0^t \|g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\|_2^2\mathrm{d}s\\
&\quad + 2\int_0^t \langle P_{N+1}(s)-P_N(s), f_{N+1}(s,P_{N+1}(s))-f_N(s,P_N(s))\rangle \mathrm{d}s\\
&\quad + 2\int_0^t \langle P_{N+1}(s)-P_N(s), g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\rangle \mathrm{d}W(s)
\end{aligned}
$$

$$+\int_0^t\int_Z\big[|h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z)|^2$$
$$+2(P_{N+1}(s)-P_N(s),h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z))\big]\widetilde{N}(\mathrm{d}s,\mathrm{d}z).$$

It holds that

$$\begin{aligned}
&2\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}\int_0^s\langle P_{N+1}(r)-P_N(r),f_{N+1}(r,P_{N+1}(r))-f_N(r,P_N(r))\rangle\mathrm{d}r\Big]\\
\leqslant{}&\mathsf{E}\int_0^{t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\mathrm{d}s+\mathsf{E}\int_0^{t\wedge\tau_N}|f_{N+1}(s,P_{N+1}(s))-f_N(s,P_N(s))|^2\mathrm{d}s\\
\leqslant{}&T\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\Big]+\mathsf{E}\int_0^{t\wedge\tau_N}|f_{N+1}(s,P_{N+1}(s))-f_N(s,P_N(s))|^2\mathrm{d}s.
\end{aligned}$$

By the Burkholder-Davis-Gundy's inequality, we have

$$\begin{aligned}
&2\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}\Big|\int_0^s\langle P_{N+1}(r)-P_N(r),g_{N+1}(r,P_{N+1}(r))-g_N(r,P_N(r))\rangle\mathrm{d}W(r)\Big|\Big]\\
\leqslant{}&\frac{1}{4}\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\Big]+k\mathsf{E}\int_0^{t\wedge\tau_N}\|g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\|_2^2\mathrm{d}s,
\end{aligned}$$

for some positive constant $k>0$.

Applying Lemma 2, for any $t\geqslant 0$ and certain positive constant c we can yield that

$$\begin{aligned}
&\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}\Big|\int_0^s\int_Z\big[|h_{N+1}(r,P_{N+1}(r),z)-h_N(r,P_N(r),z)|^2\\
&\quad+2(P_{N+1}(r)-P_N(r),h_{N+1}(r,P_{N+1}(r),z)-h_N(r,P_N(r),z))\big]\widetilde{N}(\mathrm{d}r,\mathrm{d}z)\Big|\Big]\\
\leqslant{}&\frac{1}{4}\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\Big]\\
&\quad+c\mathsf{E}\int_0^{t\wedge\tau_N}\int_Z|h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z)|^2\lambda(\mathrm{d}z)\mathrm{d}s.
\end{aligned}$$

Hence

$$\begin{aligned}
&\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\Big]\\
\leqslant{}&|A\overline{\beta}^2-2\mu_0|\mathsf{E}\int_0^{t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\mathrm{d}s+\frac{1}{2}\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N}|P_{N+1}(s)-P_N(s)|^2\Big]\\
&+(1+k)\mathsf{E}\int_0^{t\wedge\tau_N}\|g_{N+1}(s,P_{N+1}(s))-g_N(s,P_N(s))\|_2^2\mathrm{d}s\\
&+\mathsf{E}\int_0^{t\wedge\tau_N}|f_{N+1}(s,P_{N+1}(s))-f_N(s,P_N(s))|^2\mathrm{d}s\\
&+c\mathsf{E}\int_0^{t\wedge\tau_N}\int_Z|h_{N+1}(s,P_{N+1}(s),z)-h_N(s,P_N(s),z)|^2\lambda(\mathrm{d}z)\mathrm{d}s.
\end{aligned}$$

Then since for $0 \leqslant s \leqslant \tau_N$, $f_{N+1}(s, P_N(s)) = f_N(s, P_N(s))$, $g_{N+1}(s, P_N(s)) = g_N(s, P_N(s))$, $h_{N+1}(s, P_N(s), z) = h_N(s, P_N(s), z)$, we have that

$$
\begin{aligned}
&\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N} |P_{N+1}(s) - P_N(s)|^2\Big] \\
\leqslant\ & |A\overline{\beta}^2 - 2\mu_0|\mathsf{E}\int_0^{t\wedge\tau_N} |P_{N+1}(s) - P_N(s)|^2 \mathrm{d}s + \frac{1}{2}\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N} |P_{N+1}(s) - P_N(s)|^2\Big] \\
&+ (1+k)\mathsf{E}\int_0^{t\wedge\tau_N} \|g_{N+1}(s, P_{N+1}(s)) - g_{N+1}(s, P_N(s))\|_2^2 \mathrm{d}s \\
&+ \mathsf{E}\int_0^{t\wedge\tau_N} |f_{N+1}(s, P_{N+1}(s)) - f_{N+1}(s, P_N(s))|^2 \mathrm{d}s \\
&+ c\mathsf{E}\int_0^{t\wedge\tau_N}\int_Z |h_{N+1}(s, P_{N+1}(s), z) - h_{N+1}(s, P_N(s), z)|^2 \lambda(\mathrm{d}z)\mathrm{d}s.
\end{aligned}
$$

Since $T_0 > 0$ is given as a sufficiently small time as $|A\overline{\beta}^2 - 2\mu_0|T < 1/4$ and

$$
|A\overline{\beta}^2 - 2\mu_0|\mathsf{E}\int_0^{t\wedge\tau_N} |P_{N+1}(s) - P_N(s)|^2 \mathrm{d}s \leqslant \frac{1}{4}\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t\wedge\tau_N} |P_{N+1}(s) - P_N(s)|^2\Big],
$$

we get that

$$
\begin{aligned}
&\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t} |P_{N+1}(s\wedge\tau_N) - P_N(s\wedge\tau_N)|^2\Big] \\
\leqslant\ & 4(1+k)\mathsf{E}\int_0^t \|g_{N+1}(s\wedge\tau_N, P_{N+1}(s\wedge\tau_N)) - g_{N+1}(s\wedge\tau_N, P_N(s\wedge\tau_N))\|_2^2 \mathrm{d}s \\
&+ 4\mathsf{E}\int_0^t |f_{N+1}(s\wedge\tau_N, P_{N+1}(s\wedge\tau_N)) - f_{N+1}(s\wedge\tau_N, P_N(s\wedge\tau_N))|^2 \mathrm{d}s \\
&+ 4c\mathsf{E}\int_0^t\int_Z |h_{N+1}(s\wedge\tau_N, P_{N+1}(s\wedge\tau_N), z) - h_{N+1}(s\wedge\tau_N, P_N(s\wedge\tau_N), z)|^2 \lambda(\mathrm{d}z)\mathrm{d}s \\
\leqslant\ & 4(2+k+c)\mathsf{E}\int_0^t G_{N+1}(s\wedge\tau_N, \|P_{N+1}(s\wedge\tau_N) - P_N(s\wedge\tau_N)\|_C^2)\mathrm{d}s.
\end{aligned}
$$

Thus, there exists a $\gamma > 0$ such that

$$
\begin{aligned}
&\mathsf{E}\Big[\sup_{0\leqslant s\leqslant t} |P_{N+1}(s\wedge\tau_N) - P_N(s\wedge\tau_N)|^2\Big] \\
\leqslant\ & \gamma\int_0^t G_{N+1}\Big(r\wedge\tau_N, \mathsf{E}\Big[\sup_{0\leqslant r\leqslant s} |P_{N+1}(r\wedge\tau_N) - P_N(r\wedge\tau_N)|^2\Big]\Big)\mathrm{d}r
\end{aligned}
$$

for all $t \in [0, T_0]$, by (H3),

$$
\mathsf{E}\Big[\sup_{0\leqslant s\leqslant T\wedge\tau_N} |P_{N+1}(s) - P_N(s)|^2\Big] = 0.
$$

Therefore we obtain that $P_{N+1}(t) = P_N(t)$ for $0 \leqslant t \leqslant T_0 \wedge \tau_N$, a.e. ω. For each $\omega \in \Omega$ there exists an $N_0(\omega) > 0$ such that $0 < T_0 \leqslant \tau_{N_0}$. Define $P(t)$ by $P(t) = P_{N_0}(t)$ for

$t \in [0, T_0]$. Since $P(t \wedge \tau_N) = P_N(t \wedge \tau_N)$, in V^* it holds that

$$\begin{aligned} P(t \wedge \tau_N) &= P(0) - \int_0^{t\wedge\tau_N} \frac{\partial P_N(s)}{\partial a}\mathrm{d}s - \int_0^{t\wedge\tau_N} \mu(s,a)P_N(s)\mathrm{d}s + \int_0^{t\wedge\tau_N} f_N(s, P_N(s))\mathrm{d}s \\ &\quad + \int_0^{t\wedge\tau_N} g_N(s, P_N(s))\mathrm{d}W(s) + \int_0^{t\wedge\tau_N}\int_Z h_N(s, P_N(s), z)\widetilde{N}(\mathrm{d}z)\mathrm{d}s \\ &= P_0 - \int_0^{t\wedge\tau_N} \frac{\partial P(s)}{\partial a}\mathrm{d}s - \int_0^{t\wedge\tau_N} \mu(s,a)P(s)\mathrm{d}s + \int_0^{t\wedge\tau_N} f(s, P(s))\mathrm{d}s \\ &\quad + \int_0^{t\wedge\tau_N} g(s, P(s))\mathrm{d}W(s) + \int_0^{t\wedge\tau_N}\int_Z h(s, P(s), z)\widetilde{N}(\mathrm{d}z)\mathrm{d}s. \end{aligned}$$

Letting $N \to \infty$, we have that in V^*

$$\begin{aligned} P(t) &= P_0 - \int_0^t \frac{\partial P(s)}{\partial a}\mathrm{d}s - \int_0^t \mu(s,a)P(s)\mathrm{d}s + \int_0^t f(s, P(s))\mathrm{d}s \\ &\quad + \int_0^t g(s, P(s))\mathrm{d}W(s) + \int_0^t\int_Z h(s, P(s), z)\widetilde{N}(\mathrm{d}z)\mathrm{d}s, \qquad t \in [0, T_0]. \end{aligned}$$

The energy equation for $P(t)$ holds as P_N satisfies the energy equation. Thus we have that $P(t)$ is an energy solution to (1), which completes the proof of the theorem. □

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带跳的随机年龄相关种群方程解的存在唯一性

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摘 要: 本文给出具有Poisson跳的随机年龄相关种群方程. 在局部非Lipschitz条件下, 证明了Hilbert空间中随机年龄相关种群方程解的存在唯一性.

关键词: 随机年龄相关种群方程; 能量解; 局部非Lipschitz条件; Poisson跳

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