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A Model about Insurer with Dividend and Irregularly Checking Surplus *

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Abstract: A Markov observation model with dividend is defined and the interpretation of the practical significance is given. We try to use an irreducible and homogeneous discrete-time Markov chain to modulate the inter-observation times and embed a dividend strategy. In the Markov observation model with dividend, a system of liner equations for the expected discounted value of dividends until ruin time is derived. Moreover, an explicit expression is obtained and proved. Finally, some interesting properties are illustrated by numerical analysis and by comparing with the complete compound binomial model with dividend.

Keywords: Markov observation; Markov chain; constant dividend barrier; discounted value; ruin time

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§1. Introduction and Background

The compound binomial model (CBM) is a discrete time analogue of the compound Poisson model (CPM), and is more rich in practical significance. From it proposed, a number of papers and books have studied this model. See [1–4] and references therein. Moreover, the complete compound binomial model (CCBM) is a special case of the CBM.

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In recent years, risk models with an embedded dividend strategy have received considerable attention in ruin theory. Dividend strategy, which firstly proposed by De Finetti [5], reflected more realistically the surplus cash flows in an insurance portfolio, and he found that the optimal strategy must be a barrier strategy. From then on, many papers have studied the dividend barrier. From a shareholder's perspective, the amount of dividends is a most important factor. Hence, we usually discuss the expected value of dividends until ruin time.

In the CCBM, it is necessary to observe the current value of the surplus every time. But in reality, it may be more reasonable to assume that the company is only checked on a periodic basis (see e.g. Asmussen and Albrecher 2010 for a recent survey). Randomized observation period was firstly proposed by Albrecher and Cheung [7]. They discussed the compound Poisson risk model with randomized observation periods and obtained an expression for the discounted penalty function. Afterwards Albrecher and Cheung [8] discussed the compound Poisson risk model with randomized observation periods under a dividend strategy. Avanzi et al. [9] discussed a periodic dividend barrier strategy in a dual model inspired by it.

Inspired by [7] and [8], we will consider the CCBM with Markov observation. That is we can only observe the surplus process at some special time that is related to a Markov chain. It is reasonable to use an irreducible discrete-time Markov chain to modulate the observation times. For example, the board of directors usually will decide to the time to share out bonus to shareholders according to the present situation rather than the former. So affected by the Markov observation, the ruin probability and related quantities will change correspondingly.

This paper is structured as follows: a accurate definition of the Markov observation model with dividend is given and the explanation of this model is provided in Section 2. In Section 3, we derive a system of equations for the expected discounted value of dividends until the time of ruin. Moreover, an explicit expression is obtained and proved. Finally, we illustrate some numerical examples and compare Markov observation model with the CCBM to see the effect of Markov observation in Section 4.

§2. Markov Observation Model with Dividend

In the CCBM, $\{U(t), t = 0, 1, 2, \ldots\}$, which is denoted the surplus process of an insurer, is given by

$$U(t) = u + t - \sum_{i=1}^{t} X_i\xi_i, \qquad t = 0, 1, 2, \ldots, \tag{1}$$

where the initial surplus u is a non-negative integer, $\sum\limits_{i=1}^{t} X_i\xi_i$ is the aggregate claim up to time t. In any time period,the probability with only a claim occurring is θ $(0 < \theta < 1)$, and the probability with no claim occurring is $\lambda = 1 - \theta$. We denote by $\xi_t = 1$ the event where a claim occurs in the time period $(t-1, t]$ and denote by $\xi_t = 0$ the event where no claim occurs in the time period $(t-1, t]$. The occurrences of claims in different time periods are independent events. X_t denotes the claim amount that occurs at time t, and $X_1, X_2, X_3, \ldots$, are mutually independent, identically distributed (i.i.d.), positive integer valued random variables. Throughout this paper, $\mathsf{P}(A)$ denote the probability of event A occurring. The common discrete distribution of $X = \{X_t, t = 1, 2, \ldots\}$ is $\mathsf{P}(X = k) = p(k)$, $k = 1, 2, \ldots$. Denote $F(k) = \mathsf{P}(X \leqslant k) = \sum\limits_{j=1}^{k} p(j)$ with $F(0) = 0$. We assume the claim amounts $X = \{X_t, t = 1, 2, \ldots\}$ are independent of $\xi = \{\xi_t, t = 1, 2, \ldots\}$.

In this paper, we will modify that the CCBM (1) can only be observed at random times. Let the following essential factors be given:

1. a non-negative integer u and a positive integer n, denote $S = \{1, 2, \ldots, n\}$;
2. four stochastic processes M, ξ, X, η in the probability space $(\Omega, \mathscr{F}, \mathsf{P})$:
    a) $M = \{M_k, k = 0, 1, 2, \ldots\}$ is an irreducible and homogeneous discrete-time Markov chain with state space S and one step transition matrix $P = (p_{ij}, 1 \leqslant i, j \leqslant n)$;
    b) $\xi = \{\xi_i, i = 1, 2, \ldots\}$ is i.i.d.; the common distribution is the binomial distribution $B(\theta)$, $\theta \in (0, 1)$, $\lambda = 1 - \theta$;
    c) $X = \{X_t, t = 1, 2, \ldots\}$ is i.i.d., positive and integer-valued stochastic series with the common distribution $\mathsf{P}(X = k) = p(k)$, $k = 1, 2, \ldots$. Denote $F(k) = \mathsf{P}(X \leqslant k) = \sum\limits_{j=1}^{k} p(j)$ with $F(0) = 0$;
    d) $\eta = \{\eta(t), t \in S\}$ is a series of independent and positive integer valued random variables; the distribution is $\mathsf{P}(\eta(i) = k) = q_{ik}$, $\forall\, i \in S$, $k = 1, 2, 3, \ldots, m$, $G_i(k) = \mathsf{P}(\eta(i) \leqslant k) = \sum\limits_{j=1}^{k} q_{ij}$ with $G_i(0) = 0$;
    e) M, ξ, X, η are mutually independent.

Let

$$Z_k = \begin{cases} 0 & k = 0; \\ \sum\limits_{j=1}^{k} \eta(M_{j-1}) & k = 1, 2, \ldots. \end{cases}$$

Definition 1 The 4-tuple $C = (\eta, \xi, X, M)$ is called Markov observation in CCBM, for short, we call it Markov observation model. $\Re_C = (G, \theta, F, P)$ is called the numerical characteristic of the Markov observation model. And $\{C(k), k = 0, 1, 2, \ldots\}$ is called the surplus of Markov observation model, where $\{C(k), k = 0, 1, 2, \ldots\}$ can be described recursively by

$$\begin{cases} C(0) = u, \\ C(k) = C(k-1) + \eta(M_{k-1}) - \sum\limits_{i=Z_{k-1}}^{Z_k} X_i \xi_i, \qquad k = 1, 2, \ldots. \end{cases} \tag{2}$$

Remark 2 The Markov observation model can be interpreted on the basis of CCBM as follows. We use an irreducible discrete-time Markov chain M_k to modulate the observation times. Then $\eta(M_k)$ denote the kth inter-observation time of the Markov observation model. And Z_k is the kth observation time of the Markov observation model.

Definition 3 Given a Markov observation model $C = (\eta, \xi, X, M)$ and a positive integer b. The 5-tuple $V = (\eta, \xi, X, M, b)$ is called Markov observation model with a constant dividend barrier, for short, we call it Markov observation model with dividend. $\Re_V = (G, \theta, F, P, b)$ is called the numerical characteristic of the Markov observation model with dividend. And $\{V(k), k = 0, 1, 2, \ldots\}$ is called the surplus of Markov observation model with dividend, where $\{V(k), k = 0, 1, 2, \ldots\}$ can be described recursively by

$$\begin{cases} V(0) = u, \\ V(k) = \min\Big\{V(k-1) + \eta(M_{k-1}) - \sum\limits_{i=Z_{k-1}}^{Z_k} X_i \xi_i,\, b\Big\}, \qquad k = 1, 2, \ldots. \end{cases} \tag{3}$$

Remark 4 The Markov observation model with dividend can be interpreted as follows. We introduce a constant dividend barrier into the Markov observation model. Any surplus of the insurer be observed above the level b is immediately paid out to the shareholders so that the surplus is brought back to the level b. So, it is reasonable to assume that $u \leqslant b$ (if $u > b$, the part in excess would be paid out to the shareholders immediately). And we should point out the assumption that the dividend is paid out after the premium received and claims paid out.

Define the ruin time in the Markov observation model with dividend is

$$T = Z_{k^*}, \tag{4}$$

where $k^* = \min\{k \geqslant 1; V(k) < 0\}$. We have to emphasize that the ruin time is only determined by surplus of observation time, has nothing to do with the surplus of non-observation time.

A sample path of the surplus of Markov observation model with dividend is depicted in Figure 1. We should note that: 1) if the surplus of company becomes negative, but is again positive at the next observation time, the insurer is not ruined. In Figure 1, $t = 9$ is the time of ruin for CCBM but z_5 is the ruin time for Markov observation model with dividend; 2) if the surplus of company goes over b, but is again below b at the observation time, the shareholders will not receive any dividend. Comprehensive Figure 1 and Figure 2, we can see that $t = 5$ is not the time of dividend but $t = 6$ is the time of dividend.

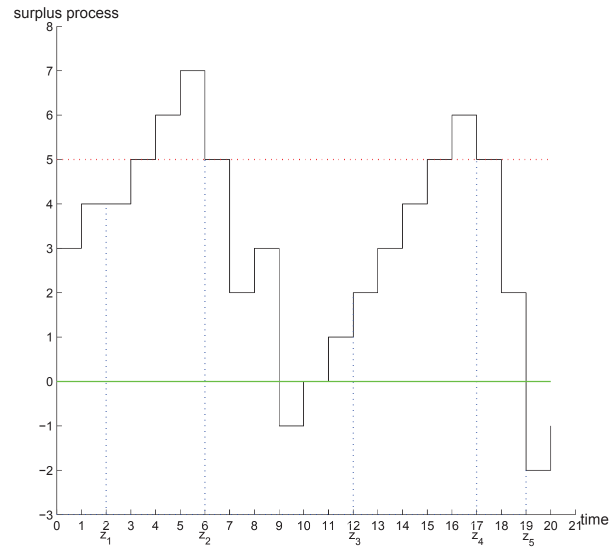

Figure 1 Sample path of the surplus of Markov observation model with dividend

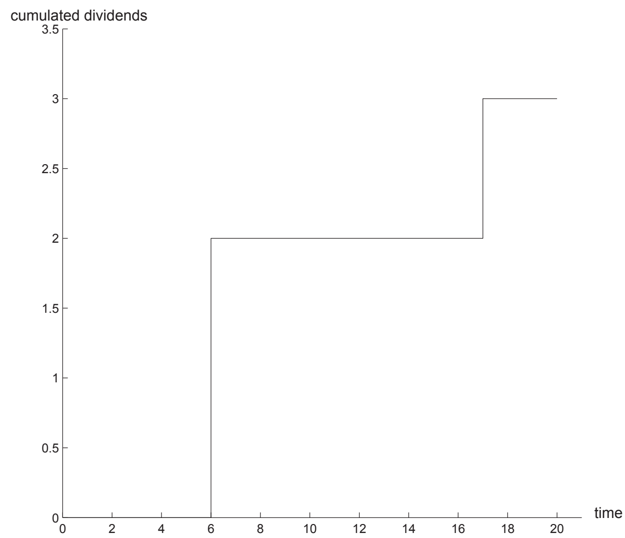

Figure 2 Sample path of the dividends of Markov observation model with dividend

The safety loading condition for this model is $\theta\mathsf{E}[X] < 1$. From the perspective of the shareholders, their first concern is the amount of dividends received from the insurer. So, in this article, we will mainly study the expected discounted value of the accumulated dividends up to ruin time in Markov observation model with dividend. Let v $(0 < v \leqslant 1)$ denote the discounted factor, $D_1^s(u)$ denote the expected discounted value of the first dividend under the conditions $V(0) = u$ and $M_0 = s\,(\in S)$, and $D^s(u)$ denote the expected discounted value of all dividends up to the ruin time under the conditions $V(0) = u$ and $M_0 = s\,(\in S)$.

§3. The Expected Discounted Value of Dividends

Lemma 5 $\{Z_k, k = 1, 2, \ldots\}$ is a homogeneous Markov process and $\{V_k, k = 1, 2, \ldots\}$ is a homogeneous piecewise-deterministic Markov process (PDMP).

The proof is obvious. It is omitted here.

Theorem 6 Given the Markov observation model with dividend $V = (\eta, \xi, X, M, b)$. For all $u \geqslant 0$ and $s \in S$, we have

$$D_1^s(u) = \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} \sum_{x=0}^{u+k} f_k(x)[D_1^j(u+k-x)I_{\{u+k-x\leqslant b\}} + (u+k-x-b)I_{\{u+k-x>b\}}], \quad (5)$$

and

$$D^s(u) = \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} \sum_{x=0}^{u+k} f_k(x)[D^j(u+k-x)I_{\{u+k-x\leqslant b\}} + [(u+k-x-b) + D^j(b)]I_{\{u+k-x>b\}}], \quad (6)$$

where

$$f_k(x) = \begin{cases} 0, & x < 0; \\ v^k\lambda^k, & x = 0; \\ v^k \sum_{i=1}^{k} \binom{k}{i} \theta^i \lambda^{k-i} p^{*(i)}(x), & x = 1, 2, \ldots, \end{cases}$$

and $p^{*(i)}(x)$ denotes the i order convolution of $p(x)$.

Proof By conditioning on the first observation time Z_1, we can consider $D_1^s(u)$ and $D^s(u)$ in the first period $(0, Z_1]$ and separate the four possible cases as following:

(i) no claim occurs in $(0, Z_1]$ and no dividends occurs in $(0, Z_1]$;

(ii) no claim occurs in $(0, Z_1]$ and a dividends occurs in $(0, Z_1]$;

(iii) at least one claim occurs in $(0, Z_1]$ and no dividends occurs in $(0, Z_1]$;

(iv) at least one claim occurs in $(0, Z_1]$ and a dividends occurs in $(0, Z_1]$.

We can easily derive the following equations with the formula of full probability and the the Markov property.

$$\begin{aligned}
D_1^s(u) = & \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \lambda^k D_1^k(u+k) I_{\{u+\eta(j)\leqslant b\}} \\
& + \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \lambda^k (u+k-b) I_{\{u+\eta(j)>b\}} \\
& + \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \sum_{x=1}^{u+k} \sum_{i=1}^{k} \binom{k}{i} \theta^i \lambda^{k-i} p^{*(i)}(x) D_1^j(u+k-x) I_{\{u+k-x\leqslant b\}} \\
& + \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \sum_{x=1}^{u+k} \sum_{i=1}^{k} \binom{k}{i} \theta^i \lambda^{k-i} p^{*(i)}(x) (u+k-x-b) I_{\{u+k-x>b\}},
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
D^s(u) = & \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \lambda^k D^j(u+k) I_{\{u+k\leqslant b\}} \\
& + \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \lambda^k [(u+k-b) + D^j(b)] I_{\{u+k)>b\}} \\
& + \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \sum_{x=1}^{u+k} \sum_{i=1}^{k} \binom{k}{i} \theta^i \lambda^{k-i} p^{*(i)}(x) D^j(u+k-x) I_{\{u+\eta(j)-x\leqslant b\}} \\
& + \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} v^k \sum_{x=1}^{u+k} \sum_{i=1}^{k} \binom{k}{i} \theta^i \lambda^{k-i} p^{*(i)}(x) [(u+k-x-b) + D^j(b)] I_{\{u+k-x>b\}}.
\end{aligned} \tag{8}$$

We denote

$$f_k(x) = \begin{cases} 0, & x < 0; \\ v^k (1-\theta)^k, & x = 0; \\ v^k \sum_{i=1}^{k} \binom{k}{i} \theta^i (1-\theta)^{k-i} p^{*(i)}(x), & x = 1, 2, \ldots. \end{cases}$$

Then Equation (7), Equation (8) can be respectively rewritten as

$$D_1^s(u) = \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} \sum_{x=0}^{u+k} f_k(x) [D_1^j(u+k-x) I_{\{u+k-x\leqslant b\}} + (u+k-x-b) I_{\{u+k-x>b\}}], \tag{9}$$

and

$$D^s(u) = \sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} \sum_{x=0}^{u+k} f_k(x) [D^j(u+k-x) I_{\{u+k-x\leqslant b\}} + [(u+k-x-b) + D^j(b)] I_{\{u+k-x>b\}}]. \tag{10}$$

This completes the proof. $\square$

To drive the solution of the Equation (5), we change it as

$$\begin{aligned}&\sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} \sum_{x=0}^{u+k} f_k(x) D_1^j(u+k-x) I_{\{u+k-x\leqslant b\}} - D_1^s(u)\\ =&-\sum_{j=1}^{n} p_{sj} \sum_{k=1}^{m} q_{jk} \sum_{x=0}^{u+k} f_k(x)(u+k-x-b) I_{\{u+k-x>b\}}.\end{aligned} \tag{11}$$

Noting that $u=0,1,\ldots,b$, and $s=1,2,\ldots,n$, we can rewrite Equation (11) as

$$\boldsymbol{AD}=\boldsymbol{v} \tag{12}$$

or

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1(b+1)} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2(b+1)} \\ A_{31} & A_{32} & A_{33} & \cdots & A_{3(b+1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{(b+1)1} & A_{(b+1)2} & A_{(b+1)3} & \cdots & A_{(b+1)(b+1)} \end{pmatrix} \begin{pmatrix} D_1(0) \\ D_1(1) \\ D_1(2) \\ \vdots \\ D_1(b) \end{pmatrix} = \begin{pmatrix} v_1(0) \\ v_1(1) \\ v_1(2) \\ \vdots \\ v_1(b) \end{pmatrix}, \tag{13}$$

where

1. $s\neq k$, $A_{sk}=(a_{ij})_{n\times n}$
$$a_{ij}=p_{ij}\sum_{l=1}^{m} q_{jl} f_l(s-k+l),$$
2. $s=k$, $A_{sk}=(b_{ij})_{n\times n}$
   - $i=j$, $b_{ij}=p_{ij}\sum\limits_{l=1}^{m} q_{jl} f_l(l)-1$,
   - $i\neq j$, $b_{ij}=p_{ij}\sum\limits_{l=1}^{m} q_{jl} f_l(l)$,

and

$$D_1(i)=\begin{pmatrix} D_1^1(i) & D_1^2(i) & D_1^3(i) & \cdots & D_1^n(i)\end{pmatrix}^{\mathsf{T}};$$

$$v_1(i)=\begin{pmatrix} -\sum\limits_{j=1}^{n} p_{1j} \sum\limits_{k=1}^{m} q_{jk} \sum\limits_{x=0}^{i+k} f_k(x)(i+k-x-b) I_{\{i+k-x>b\}} \\ -\sum\limits_{j=1}^{n} p_{2j} \sum\limits_{k=1}^{m} q_{jk} \sum\limits_{x=0}^{i+k} f_k(x)(i+k-x-b) I_{\{i+k-x>b\}} \\ -\sum\limits_{j=1}^{n} p_{3j} \sum\limits_{k=1}^{m} q_{jk} \sum\limits_{x=0}^{i+k} f_k(x)(i+k-x-b) I_{\{i+k-x>b\}} \\ \vdots \\ -\sum\limits_{j=1}^{n} p_{nj} \sum\limits_{k=1}^{m} q_{jk} \sum\limits_{x=0}^{i+k} f_k(x)(i+k-x-b) I_{\{i+k-x>b\}} \end{pmatrix}.$$

Theorem 7 Given the Markov observation model with dividend $V=(\eta,\xi,X,M,b)$. Then the matrix $\boldsymbol{A}$ is nonsingular; the system of linear Equation (12) admits a unique solution given by

$$\boldsymbol{D}=\boldsymbol{A}^{-1}\boldsymbol{v} \tag{14}$$

or

$$\begin{pmatrix} D_1(0)\\ D_1(1)\\ D_1(2)\\ \vdots\\ D_1(b)\end{pmatrix}=\begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1(b+1)}\\ A_{21} & A_{22} & A_{23} & \cdots & A_{2(b+1)}\\ A_{31} & A_{32} & A_{33} & \cdots & A_{3(b+1)}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ A_{(b+1)1} & A_{(b+1)2} & A_{(b+1)3} & \cdots & A_{(b+1)(b+1)}\end{pmatrix}^{-1}\begin{pmatrix} v_1(0)\\ v_1(1)\\ v_1(2)\\ \vdots\\ v_1(b)\end{pmatrix}. \tag{15}$$

Proof To prove this theorem, it is sufficient to prove that $\boldsymbol{A}$ is nonsingular.

It can be easily seen that $\forall\, l=1,2,\ldots,m$,

$$f_l(s-1+l)+f_l(s-2+l)+\cdots+f_l(s-b+1+l)\leqslant v^k\sum_{i=1}^{k}\binom{k}{i}p^iq^{k-i}\leqslant 1.$$

In each r row, we can write $r=sn+k$. Then we can see

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{l=1}^{m}p_{ki}q_{il}f_l(s-1+l)+\sum_{i=1}^{n}\sum_{l=1}^{m}p_{ki}q_{il}f_l(s-2+l)\\
&\quad+\cdots+\sum_{i=1}^{n}\sum_{l=1}^{m}p_{ki}q_{il}f_l(s-b+1+l)\\
&=p_{k1}\sum_{l=1}^{m}q_{1l}[f_l(s-1+l)+f_l(s-2+l)+\cdots+f_l(s-b+1+l)]\\
&\quad+p_{k2}\sum_{l=1}^{m}q_{2l}[f_l(s-1+l)+f_l(s-2+l)+\cdots+f_l(s-b+1+l)]\\
&\quad+\cdots+p_{kn}\sum_{l=1}^{m}q_{nl}[f_l(s-1+l)+f_l(s-2+l)+\cdots+f_l(s-b+1+l)]\\
&\leqslant p_{k1}\sum_{l=1}^{m}q_{1l}+p_{k2}\sum_{l=1}^{m}q_{2l}+\cdots+p_{kn}\sum_{l=1}^{m}q_{nl}\\
&=p_{k1}+p_{k2}+\cdots+p_{kn}\\
&=1.
\end{aligned} \tag{16}$$

If $v>0$, then $f_k(x)\geqslant 0$, and hence the l.h.s. of the Equation 16 is positive. From Equation (16), we can see

$$0\leqslant\sum_{l=1}^{m}p_{kk}q_{kl}f_l(l)<1.$$

Equation (16) can be written as

$$\begin{aligned}
&\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ki}q_{il}f_l(s-1+l)+\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ki}q_{il}f_l(s-2+l)+\cdots+\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ki}q_{il}f_l(l-1)\\
&\quad+\sum_{i=1,i\neq k}^{n}\sum_{l=1}^{m} p_{ki}q_{il}f_l(l)+\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ki}q_{il}f_l(l+1)+\cdots+\sum_{i=1}^{n}\sum_{l=1}^{m} p_{ki}q_{il}f_l(s-b+1+l)\\
&\leqslant 1-\sum_{l=1}^{m} p_{kk}q_{kl}f_l(l)\\
&=\Big|\sum_{l=1}^{m} p_{kk}q_{kl}f_l(l)-1\Big|. \qquad (17)
\end{aligned}$$

Equation (17) leads to that the absolute value of diagonal (the rth element in the rth row) is greater than the sum of others in this row. Owing to r is a arbitrary, $\boldsymbol{A}$ is a (row) strictly diagonally dominant matrix. Hence, $\boldsymbol{A}$ is nonsingular, which leads to the result. □

Similarly, we can get the following equations from Equation (6).

$$\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{D}}=\boldsymbol{v} \qquad (18)$$

or

$$\begin{pmatrix}
\widetilde{A}_{11} & \widetilde{A}_{12} & \widetilde{A}_{13} & \cdots & \widetilde{A}_{1(b+1)}\\
\widetilde{A}_{21} & \widetilde{A}_{22} & \widetilde{A}_{23} & \cdots & \widetilde{A}_{2(b+1)}\\
\widetilde{A}_{31} & \widetilde{A}_{32} & \widetilde{A}_{33} & \cdots & \widetilde{A}_{3(b+1)}\\
\vdots & \vdots & \vdots & \ddots & \vdots\\
\widetilde{A}_{(b+1)1} & \widetilde{A}_{(b+1)2} & \widetilde{A}_{(b+1)3} & \cdots & \widetilde{A}_{(b+1)(b+1)}
\end{pmatrix}
\begin{pmatrix}\widetilde{D}(0)\\ \widetilde{D}(1)\\ \widetilde{D}(2)\\ \vdots\\ \widetilde{D}(b)\end{pmatrix}
=\begin{pmatrix}v(0)\\ v(1)\\ v(2)\\ \vdots\\ v(b)\end{pmatrix}, \qquad (19)$$

where

1. when $k\neq b+1$,

   - $s\neq k$, $\widetilde{A}_{sk}=(\widetilde{a}_{ij})_{n\times n}$

     $$\widetilde{a}_{ij}=p_{ij}\sum_{l=1}^{m} q_{jl}f_l(s-k+l),$$

   - $s=k$, $\widetilde{A}_{sk}=(\widetilde{b}_{ij})_{n\times n}$

     $$\begin{cases} i=j, & \widetilde{b}_{ij}=p_{ij}\sum\limits_{l=1}^{m} q_{jl}f_l(l)-1;\\ i\neq j, & \widetilde{b}_{ij}=p_{ij}\sum\limits_{l=1}^{m} q_{jl}f_l(l),\end{cases}$$

2. when $k=b+1$, $\widetilde{A}_{sk}=(\widetilde{c}_{ij})_{n\times n}$

$$\begin{cases} i=j, & \widetilde{c}_{ij}=p_{ij}\sum\limits_{l=1}^{m}\sum\limits_{h=1}^{s-b+l} q_{jl}f_l(s-b+l-h)-1; \\ i\neq j, & \widetilde{c}_{ij}=p_{ij}\sum\limits_{l=1}^{m}\sum\limits_{h=1}^{s-b+l} q_{jl}f_l(s-b+l-h), \end{cases}$$

and

$$\widetilde{D}(i)=\begin{pmatrix} D^1(i) & D^2(i) & D^3(i) & \cdots & D^n(i) \end{pmatrix}^{\mathsf{T}}.$$

Theorem 8 Given the Markov observation model with dividend $V=(\eta,\xi,X,M,b)$. Then the matrix $\widetilde{\boldsymbol{A}}$ is nonsingular; the system of linear Equation (18) admits a unique solution, which is given by

$$\widetilde{\boldsymbol{D}}=\widetilde{\boldsymbol{A}}^{-1}\boldsymbol{v} \tag{20}$$

or

$$\begin{pmatrix} \widetilde{D}(0) \\ \widetilde{D}(1) \\ \widetilde{D}(2) \\ \vdots \\ \widetilde{D}(b) \end{pmatrix} = \begin{pmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} & \widetilde{A}_{13} & \cdots & \widetilde{A}_{1(b+1)} \\ \widetilde{A}_{21} & \widetilde{A}_{22} & \widetilde{A}_{23} & \cdots & \widetilde{A}_{2(b+1)} \\ \widetilde{A}_{31} & \widetilde{A}_{32} & \widetilde{A}_{33} & \cdots & \widetilde{A}_{3(b+1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \widetilde{A}_{(b+1)1} & \widetilde{A}_{(b+1)2} & \widetilde{A}_{(b+1)3} & \cdots & \widetilde{A}_{(b+1)(b+1)} \end{pmatrix}^{-1} \begin{pmatrix} v(0) \\ v(1) \\ v(2) \\ \vdots \\ v(b) \end{pmatrix}. \tag{21}$$

§4. Numerical Illustrations

Example 9 When $\forall\, i\in S$, $\eta(i)=r$ (r does not relate with i). The Markov observation model becomes the CCBM with observing periodically, and r is the observation period. Moreover $r=1$, the Markov observation model becomes the CCBM. The CCBM with a constant dividend barrier is discussed by Wu and Tan [4]. Here we use a different method to derive the equations and expressions. However, we obtain the explicit expressions, while Wu and Tan [4] just obtain the approximate solutions.

Remark 10 The Markov observation model is a extent both of the CCBM with observing periodically and the CCBM. The Markov observation model with dividend is a extent of the CCBM under a consist dividend barrier.

Example 11 Let $v=1$, $b=5$. When $\theta=1/4$, then $1-\theta=3/4$. Assume that $\mathsf{P}(x=k)=1/3$, $k=1,2,3$. And $P=\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$, $q_{ij}=1/2$, $i,j=1,2$. Then we can obtain the consequence of the expected discounted value of all dividends up to the ruin time of the Markov observation model with dividend. Table 1 below gives the data.

Table 1 Values of the expected discounted value of all dividends

$D^i(u)$	$u=0$	$u=1$	$u=2$	$u=3$	$u=4$	$u=5$
$i=1$	224.76	268.93	294.78	305.91	311.58	314.11
$i=2$	224.76	268.93	294.78	305.91	311.58	314.11

From the Table 1, we can see: 1) the initial state does not influence the expected discounted value of all dividends. This is because that the probability from any one state to another state is equal, which can be proved easily. 2) With other conditions being same, the expected discounted value of all dividends, with initial surplus increasing, is increasing. This conclusion show that the stronger the funds of insurance company, the more the dividends of their shareholders.

Now using this properties, we can only choose $D^1(u)$ to be a representative. Let us firstly compare the effect of random observation times on ruin related quantities, in particular in comparison with the CCBM. We consider a CCBM with a constant dividend barrier. When $\theta=1/4$, then $1-\theta=3/4$. Assume that $v=1$ and $\mathsf{P}(X=1)=\mathsf{P}(X=2)=\mathsf{P}(X=1)=1/3$. Let $M(u)$ denote the expected discounted value of all dividends up to the ruin time when initial surplus is u in the CCBM. Then we can get the result of the expected discounted value of all dividends up to the ruin time.

$$\begin{aligned} &(\ M(0)\quad M(1)\quad M(2)\quad M(3)\quad M(4)\quad M(5)\ )^{\mathsf{T}} \\ =&(\ 153.73\quad 187.89\quad 212.56\quad 221.84\quad 226.65\quad 228.74\ )^{\mathsf{T}}. \end{aligned}$$

From the result, we can see that so different are the data of $D^1(u)$ and $M(u)$. Moreover, the graphic is concave, i.e. with increase of the initial surplus, the increment of $D^1(u)$ and $M(u)$ are reduced. But we do not know the relation and the reasons. Further to derive the relationship, in case of other conditions are same, we assume $p=1/3$, $p=2/5$ and $p=12/25$ respectively. Using the result of our paper, we can obtain the following data and figures.

From Figure 3–6, we can observe that 1) when the probability θ is becoming greater, $M(u)$ and $D^1(u)$ are becoming smaller. This conclusion shows that in each time period, the greater the probability of compensation, the less dividends; 2) when the probability θ is approaching a limit $\theta=1/2$, $M(u)$ is becoming almost identical with $D^1(u)$. This conclusion shows when the probability of claim equal to the probability of no claim, the influence of Markov observation is very weak.

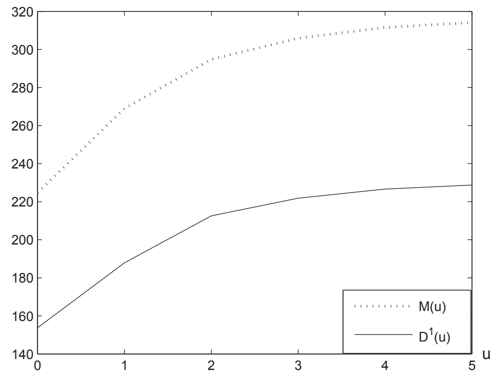

Figure 3 $\theta = 1/4$

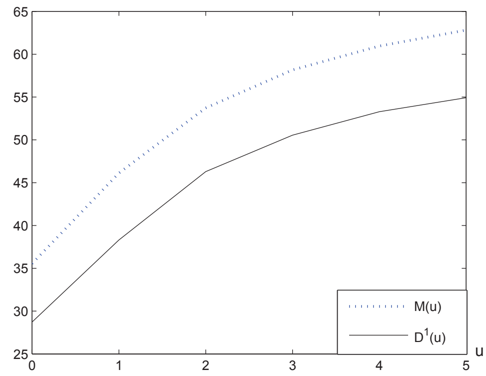

Figure 4 $\theta = 1/3$

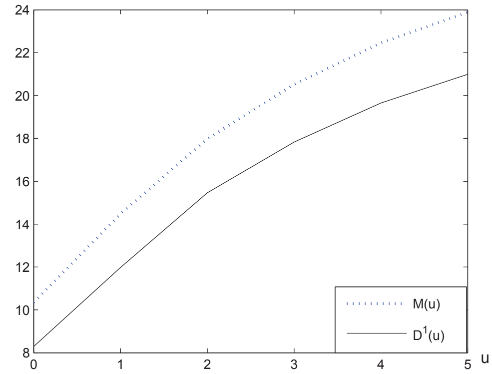

Figure 5 $\theta = 2/5$

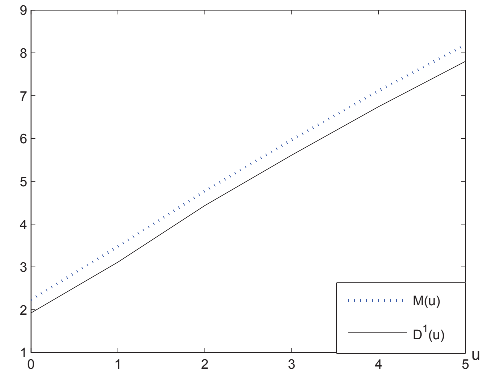

Figure 6 $\theta = 12/25$

Acknowledgements This research has investigated a Markov observation risk model with dividend, where the inter-observation time is modulated by a Markov chain. We study the expected discounted value of dividends until ruin time and obtain the explicit expression. Moreover, the expression of bankruptcy related amount (including the Gerber-Shiu function, ruin probability and survival probability, etc.) can be derived by similarly method.

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带分红和不定期观察的保险公司的建模研究

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摘 要: 文中用不可约的齐次离散时间马氏链来调控保险公司的观察时间间隔, 在此基础上引入门槛分红因素, 给出带分红的马氏观察模型的数学定义和实际意义和解释. 在带分红的马氏观察模型里, 首先得到了破产前的折现分红总量所满足的一系列方程, 然后计算出了破产前的折现分红总量的精确表达式并给出证明. 最后, 通过数值模型和与带分红的复合二项风险模型的对比分析, 总结出一些带分红的马氏观察模型的性质特点.

关键词: 马氏观察; 马氏链; 常数分红边界; 折现值; 破产时

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