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Unlinking Theorem for Symmetric Quasi-Convex Polynomials *

HONG Hejing

(*Department of Mathematics, Nanjing University, Nanjing, 210093, China; Clinchoice Inc. of Nanjing, Nanjing, 211100, China*)

HU Zechun⋆

(*College of Mathematics, Sichuan University, Chengdu, 610065, China*)

Abstract: Let μ_n be the standard Gaussian measure on $\mathbb{R}^n$ and X be a random vector on $\mathbb{R}^n$ with the law μ_n. U-conjecture states that if f and g are two polynomials on $\mathbb{R}^n$ such that $f(X)$ and $g(X)$ are independent, then there exist an orthogonal transformation $Y = LX$ on $\mathbb{R}^n$ and an integer k such that $f \circ L^{-1}$ is a function of $(y_1, y_2, \cdots, y_k)$ and $g \circ L^{-1}$ is a function of $(y_{k+1}, y_{k+2}, \cdots, y_n)$. In this case, f and g are said to be unlinked. In this note, we prove that two symmetric, quasi-convex polynomials f and g are unlinked if $f(X)$ and $g(X)$ are independent.

Keywords: U-conjecture; quasi-convex polynomial; Gaussian correlation conjecture

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§1. Introduction and Main Result

Let μ_n be the standard Gaussian measure on $\mathbb{R}^n$ $(n \geqslant 2)$ and X be a random vector on $\mathbb{R}^n$ with the law μ_n. In 1973, Kagan et al. [1] considered the following problem: if f and g are two polynomials on $\mathbb{R}^n$ such that $f(X)$ and $g(X)$ are independent, then is it possible to find an orthogonal transformation $Y = LX$ on $\mathbb{R}^n$ and an integer k such that $f \circ L^{-1}$ is a function of $(y_1, y_2, \cdots, y_k)$ and $g \circ L^{-1}$ is a function of $(y_{k+1}, y_{k+2}, \cdots, y_n)$? If the answer is positive, then f and g are said to be unlinked. This problem is called U-conjecture and is still open.

The U-conjecture is true for the case $n = 2$, and some special cases have been proved for larger number of variables (see Sections 11.4 − 11.6 of [1]). In 1994, Bhandari and

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⋆Corresponding author, E-mail: zchu@scu.edu.cn.

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DasGupta [2] proved that the U-conjecture holds for two symmetric convex functions f and g under an additional condition. The additional condition can be canceled since the Gaussian correlation conjecture has been proved (see [3] or [4]).

Bhandari and Basu [5] proved that the U-conjecture holds for two nonnegative convex polynomials f and g with $f(0)=0$. Hargé [6] proved that if $f,g:\mathbb{R}^n\to\mathbb{R}$ are two convex functions in $L^2(\mu_n)$, and f is a real analytic function satisfying $f(x)\geqslant f(0)$, $\forall\, x\in\mathbb{R}^n$, and f and g are independent with respect to μ_n, then they are unlinked.

Malicet et al. [7] proved that the U-conjecture is true when f,g belong to a class of polynomials, which is defined based on the infinitesimal generator of Ornstein-Uhlenbeck semigroup.

In Remark 2 of [5], the authors wish that their result could be extended to symmetric, quasi-convex polynomials. In this note, we will give an affirmative answer based on the first author's master thesis [8] and prove the following result.

Theorem 1 Two symmetric, quasi-convex polynomials f and g are unlinked if f and g are independent with respect to μ_n.

§2. Proof of Theorem 1

Before giving the proof of Theorem 1, we present some preliminaries.

A function $f:\mathbb{R}^n\to\mathbb{R}$ is called *quasi-convex* if for any $\alpha\in[0,1]$ and any $x,y\in\mathbb{R}^n$,

$$f(\alpha x+(1-\alpha)y)\leqslant\max\{f(x),f(y)\}.$$

It's easy to know that a convex function is quasi-convex. About the properties of quasi-convex functions, and the relations between convex and quasi-convex functions, refer to a survey paper [9].

Lemma 2 Suppose that $g:\mathbb{R}\to\mathbb{R}$ is a quasi-convex polynomial and there exist $\lambda_1,\lambda_2\in\mathbb{R}$ such that $g(\lambda_1)\neq g(\lambda_2)$. Then one of the following two claims holds.

(a) There exists λ_0 such that $g(u)<g(v)$ for any $\lambda_0\leqslant u<v$ and $\lim\limits_{\lambda\to\infty}g(\lambda)=\infty$.

(b) There exists λ_0 such that $g(u)<g(v)$ for any $v<u\leqslant\lambda_0$ and $\lim\limits_{\lambda\to-\infty}g(\lambda)=\infty$.

Proof Since g is a polynomial on $\mathbb{R}$, we can write it as

$$g(\lambda)=a_n\lambda^n+a_{n-1}\lambda^{n-1}+\cdots+a_1\lambda+a_0. \tag{1}$$

By the assumption, g is not a constant, so $n\geqslant1$ and $a_n\neq0$. By (1), we obtain

$$g'(\lambda)=na_n\lambda^{n-1}+(n-1)a_{n-1}\lambda^{n-2}+\cdots+a_1. \tag{2}$$

Without loss of generality, we can assume that $\lambda_1 < \lambda_2$. We have the following two cases:

Case 1: $g(\lambda_1) < g(\lambda_2)$. Define $h(\lambda) := g(\lambda) - g(\lambda_1)$. Then $h(\lambda_1) = 0$. By the definition of quasi-convex function, we know that h is quasi-convex, and for any $\lambda > \lambda_2$, we have

$$h(\lambda_2) = h\Big(\frac{\lambda-\lambda_2}{\lambda-\lambda_1}\lambda_1 + \frac{\lambda_2-\lambda_1}{\lambda-\lambda_1}\lambda\Big) \leqslant \max\{h(\lambda_1), h(\lambda)\} = \max\{0, h(\lambda)\}. \tag{3}$$

Since $h(\lambda_2) = g(\lambda_2) - g(\lambda_1) > 0$, by (3), we get that for any $\lambda > \lambda_2$, $h(\lambda_2) \leqslant h(\lambda)$, i.e.

$$g(\lambda_2) \leqslant g(\lambda), \qquad \forall\, \lambda > \lambda_2. \tag{4}$$

By (1) and (4), we get that $a_n > 0$, and thus

$$\lim_{\lambda\to\infty} g(\lambda) = \lim_{\lambda\to\infty}(a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0) = \infty. \tag{5}$$

If $n = 1$, then $g(\lambda) = a_1\lambda + a_0$ with $a_1 > 0$, and thus (a) holds in this case. If $n \geqslant 2$, then

$$\lim_{\lambda\to\infty} g'(\lambda) = \lim_{\lambda\to\infty}[na_n\lambda^{n-1} + (n-1)a_{n-1}\lambda^{n-2} + \cdots + a_1] = \infty. \tag{6}$$

By (6), there exists λ_0 such that for any $\lambda > \lambda_0$, $g'(\lambda) > 0$, which together with (5) implies that (a) holds in this case.

Case 2: $g(\lambda_1) > g(\lambda_2)$. Define $\overline{h}(\lambda) := g(\lambda) - g(\lambda_2)$. Then $\overline{h}(\lambda_2) = 0$, and as in Case 1, $\overline{h}$ is a quasi-convex function and for any $\lambda < \lambda_1$, we have

$$\overline{h}(\lambda_1) = \overline{h}\Big(\frac{\lambda_1-\lambda}{\lambda_2-\lambda}\lambda_2 + \frac{\lambda_2-\lambda_1}{\lambda_2-\lambda}\lambda\Big) \leqslant \max\{\overline{h}(\lambda_2), \overline{h}(\lambda)\} = \max\{0, \overline{h}(\lambda)\}. \tag{7}$$

Since $\overline{h}(\lambda_1) = g(\lambda_1) - g(\lambda_2) > 0$, by (7), we obtain that for any $\lambda < \lambda_1$, $\overline{h}(\lambda_1) \leqslant \overline{h}(\lambda)$, i.e.

$$g(\lambda_1) \leqslant g(\lambda), \qquad \forall\, \lambda < \lambda_1. \tag{8}$$

By (8) and (1), we know that one of the following two claims must hold:

(i) n is even and $a_n > 0$;

(ii) n is odd and $a_n < 0$.

If (i) holds, then by the proof of Case 1 above, we know that (a) is true.

If (ii) holds, then

$$\lim_{\lambda\to-\infty} g(\lambda) = \lim_{\lambda\to-\infty}(a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0) = \infty. \tag{9}$$

If $n = 1$, then $g(\lambda) = a_1\lambda + a_0$ with $a_1 < 0$, and thus (b) holds in this case. If $n \geqslant 3$, then

$$\lim_{\lambda\to-\infty} g'(\lambda) = \lim_{\lambda\to-\infty}[na_n\lambda^{n-1} + (n-1)a_{n-1}\lambda^{n-2} + \cdots + a_1] = -\infty. \tag{10}$$

By (10), there exists λ_0 such that for any $\lambda < \lambda_0$, $g'(\lambda) < 0$, which together with (9) implies that (b) holds in this case. $\Box$

Corollary 3 Let $g : \mathbb{R} \to \mathbb{R}$ be a quasi-convex polynomial. If g has an upper bound, then g is a constant function.

Corollary 4 Let $U : \mathbb{R}^n \to \mathbb{R}$ be a quasi-convex polynomial. Suppose that for two fixed vectors $\beta_1, \beta_2 \in \mathbb{R}^n$, $U(\beta_1 + \lambda\beta_2)$ is a constant function of $\lambda \in \mathbb{R}$. Then for any fixed vector $b \in \mathbb{R}^n$, $U(b + \lambda\beta_2)$ is a constant function of λ.

Proof For any fixed vector $b \in \mathbb{R}^n$, define $g(\lambda) = U(b + \lambda\beta_2)$, $\lambda \in \mathbb{R}$. Then $g(\lambda)$ is a polynomial of λ. By the quasi-convexity of U, we know that for any $\alpha \in [0, 1]$ and $\lambda_1, \lambda_2 \in \mathbb{R}$, we have

$$\begin{aligned} g(\alpha\lambda_1 + (1-\alpha)\lambda_2) &= U(b + [\alpha\lambda_1 + (1-\alpha)\lambda_2]\beta_2) \\ &= U(\alpha(b + \lambda_1\beta_2) + (1-\alpha)(b + \lambda_2\beta_2)) \\ &\leqslant \max\{U(b + \lambda_1\beta_2), U(b + \lambda_2\beta_2)\} \\ &= \max\{g(\lambda_1), g(\lambda_2)\}. \end{aligned}$$

Thus $g(\lambda)$ is a quasi-convex polynomial. By the quasi-convexity of U,

$$\begin{aligned} g(\lambda) = U(b + \lambda\beta_2) &= U\Big(\frac{1}{2}(2b - \beta_1) + \frac{1}{2}(\beta_1 + 2\lambda\beta_2)\Big) \\ &\leqslant \max\{U(2b - \beta_1), U(\beta_1 + 2\lambda\beta_2)\}. \end{aligned} \tag{11}$$

By (11) and the assumption that $U(\beta_1 + \lambda\beta_2)$ is a constant function of λ, we get that the quasi-convex polynomial $g(\lambda)$ has an upper bound. Hence by Corollary 3, we know that $U(b + \lambda\beta_2)$ is a constant function of λ. $\Box$

Corollary 5 Let $U : \mathbb{R}^n \to \mathbb{R}$ be a quasi-convex polynomial with $U(0) = 0$. Define

$$S_U := \{\alpha : U(\lambda\alpha) = 0, \forall\, \lambda \in \mathbb{R}\}. \tag{12}$$

Then S_U is a vector subspace of $\mathbb{R}^n$.

Proof Let $\alpha_1, \alpha_2 \in S_U$. For any $c_1, c_2, \lambda \in \mathbb{R}$, by Corollary 4, we get that

$$U(\lambda(c_1\alpha_1 + c_2\alpha_2)) = U(\lambda c_1\alpha_1 + \lambda c_2\alpha_2) = U(\lambda c_1\alpha_1) = 0.$$

Hence $c_1\alpha_1 + c_2\alpha_2 \in S_U$, and thus S_U is a vector subspace of $\mathbb{R}^n$. $\Box$

Now suppose that U and V are two quasi-convex polynomials from $\mathbb{R}^n$ into $\mathbb{R}$ satisfying that $U(0) = V(0) = 0$. Define S_U by (12). Similarly, define S_V.

Definition 6 U and V are said to be concordant of order r, if

$$\dim(S_U^{\perp}) - \dim(S_U^{\perp} \cap S_V) = r. \tag{13}$$

Note that this definition is symmetric in U and V, i.e. if (13) holds, then (see [2])

$$\dim(S_V^{\perp}) - \dim(S_V^{\perp} \cap S_U) = r.$$

Theorem 7 Let X be an $n \times 1$ random vector distributed as $\mathrm{N}(0, I_n)$. Let U and V be two symmetric (i.e. $U(x) = U(-x)$, $V(x) = V(-x)$) quasi-convex polynomials on $\mathbb{R}^n$ satisfying $\mathsf{Cov}\,(U(X), V(X)) = 0$. Furthermore, assume that $U(0) = V(0) = 0$, and U and V are concordant of order r. Then there exists an orthogonal transformation $Y = LX$ such that U and V can be expressed as functions of two different sets of components of Y, i.e. U and V are unlinked.

Proof Based on the lemmas and corollaries established above, the proof of this theorem is similar to the one of [2]. For the reader's convenience, we spell out the details in the following.

Let $\{\alpha_1, \alpha_2, \cdots, \alpha_{r+t}\}$, $\{\alpha_{r+1}, \alpha_{r+2}, \cdots, \alpha_{r+t}\}$, $\{\alpha_1, \alpha_2, \cdots, \alpha_{r+t+m}\}$ and $\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ be orthonormal bases of $S_U^{\perp}$, $S_U^{\perp} \cap S_V$, $S_U^{\perp} + S_V^{\perp}$, and $\mathbb{R}^n$, respectively. We will show that if $r > 0$ then $\mathsf{Cov}\,(U(X), V(X)) > 0$, which contradicts the condition given in the theorem, and so we get $r = 0$, and thus U and V are unlinked.

Define $Y_1, Y_2, \cdots, Y_n$ by $X = \sum\limits_{i=1}^{n} Y_i \alpha_i$, i.e. Y_i is the i-th component of X. Then $Y_1, Y_2, \cdots, Y_n$ are i.i.d. as $\mathrm{N}(0, 1)$. By Corollary 4,

$$U(X) = U\Big(\sum_{i=1}^{n} Y_i \alpha_i\Big) = U\Big(\sum_{i=1}^{r} Y_i \alpha_i + \sum_{i=r+1}^{r+t} Y_i \alpha_i\Big),$$
$$V(X) = V\Big(\sum_{i=1}^{n} Y_i \alpha_i\Big) = V\Big(\sum_{i=1}^{r} Y_i \alpha_i + \sum_{i=r+t+1}^{r+t+m} Y_i \alpha_i\Big).$$

Assume that $r > 0$. Let $y^* = (y_1, y_2, \cdots, y_r)'$ be a nonzero vector in $\mathbb{R}^r$. Define

$$U^*(y^*) := \mathsf{E}\Big[U\Big(\sum_{i=1}^{r} y_i \alpha_i + \sum_{i=r+1}^{r+t} Y_i \alpha_i\Big)\Big],$$
$$V^*(y^*) := \mathsf{E}\Big[V\Big(\sum_{i=1}^{r} y_i \alpha_i + \sum_{i=r+t+1}^{r+t+m} Y_i \alpha_i\Big)\Big].$$

Then by the fact that U and V are two symmetric quasi-convex polynomials and the condition that $Y_1, Y_2, \cdots, Y_n$ are i.i.d. as $\mathrm{N}(0, 1)$, which implies that $-Y_1, -Y_2, \cdots, -Y_n$ are i.i.d. as $\mathrm{N}(0, 1)$, we get that U^* and V^* are two symmetric quasi-convex polynomials of y^*.

By the choice of the bases, $U\big(\lambda \sum\limits_{i=1}^{r} y_i\alpha_i\big)$ is not a zero function of λ. By Corollary 4 and the condition $U(0)=0$, we know that $U\big(\lambda \sum\limits_{1}^{r} y_i x_i + \sum\limits_{r+1}^{r+t} y_i x_i\big)$ is not a constant of λ. In addition, by the symmetry and quasi-convexity of U, $U(x) \geqslant U(0)$, $\forall\, x \in \mathbb{R}^n$. Hence by Lemma 2, we get that when $\lambda \to \infty$,

$$U\Big(\lambda \sum_{1}^{r} y_i\alpha_i + \sum_{r+1}^{r+t} Y_i x_i\Big) + U\Big(-\lambda \sum_{1}^{r} y_i\alpha_i + \sum_{r+1}^{r+t} Y_i x_i\Big) \to \infty. \tag{14}$$

Taking the expectation of (14) with respect to $Y_{i+1}, Y_{i+2}, \cdots, Y_{r+t}$ and using Egoroff's theorem (see e.g. [10; Theorem 21.3] or [11; Remark 2.3.6(1)]), we obtain

$$U^*(\lambda y^*) \to \infty \qquad \text{as } \lambda \to \infty. \tag{15}$$

Similarly,

$$V^*(\lambda y^*) \to \infty \qquad \text{as } \lambda \to \infty. \tag{16}$$

Define $Y^* = (Y_1, Y_2, \cdots, Y_r)'$. By the independence of components of $X = (Y_1, Y_2, \cdots, Y_r, Y_{r+1}, \cdots, Y_n)'$ and simple calculations, we have

$$\begin{aligned}
&\mathsf{Cov}\,(U(X), V(X)) \\
&= \mathsf{E}[U(X)V(X)] - \mathsf{E}[U(X)]\mathsf{E}[V(X)] \\
&= \mathsf{E}[U^*(Y^*)V^*(Y^*)] - \mathsf{E}[U^*(Y^*)]\mathsf{E}[V^*(Y^*)] \\
&= \int_0^\infty \int_0^\infty [\mathsf{P}(Y^* \in A_{k_1}^c \cap B_{k_2}^c) - \mathsf{P}(Y^* \in A_{k_1}^c)\mathsf{P}(Y^* \in B_{k_2}^c)]\mathrm{d}k_1\mathrm{d}k_2 \\
&= \int_0^\infty \int_0^\infty [\mathsf{P}(Y^* \in A_{k_1} \cap B_{k_2}) - \mathsf{P}(Y^* \in A_{k_1})\mathsf{P}(Y^* \in B_{k_2})]\mathrm{d}k_1\mathrm{d}k_2,
\end{aligned} \tag{17}$$

where

$$A_{k_1} = \{y^* : U^*(y^*) \leqslant k_1\}, \qquad B_{k_2} = \{y^* : V^*(y^*) \leqslant k_2\}.$$

Since $U^*(y^*)$ and $V^*(y^*)$ are symmetric, quasi-convex polynomials of y^*, A_{k_1} and B_{k_2} are both symmetric convex sets (see [9; Table II]). By the Gaussian correlation inequality (see [3] or [4]),

$$\mathsf{P}(Y^* \in A_{k_1} \cap B_{k_2}) - P(Y^* \in A_{k_1})\mathsf{P}(Y^* \in B_{k_2}) \geqslant 0. \tag{18}$$

Define a set

$$M = \{(k_1, k_2) \in (0,\infty) \times (0,\infty) \,|\, A_{k_1} \subset B_{k_2}, \mathsf{P}(Y^* \in B_{k_2}^c) > 0, \mathsf{P}(Y^* \in A_{k_1}) > 0\}.$$

When $A_{k_1} \subset B_{k_2}$, we have

$$\begin{aligned} & \mathsf{P}(Y^* \in A_{k_1} \cap B_{k_2}) - \mathsf{P}(Y^* \in A_{k_1})\mathsf{P}(Y^* \in B_{k_2}) \\ = & \mathsf{P}(Y^* \in A_{k_1})[1 - \mathsf{P}(Y^* \in B_{k_2})] \\ = & \mathsf{P}(Y^* \in A_{k_1})\mathsf{P}(Y^* \in B_{k_2}^c). \end{aligned}$$

Hence we obtain

$$M \subset \{(k_1, k_2) \in (0,\infty)\times(0,\infty) \,|\, \mathsf{P}(Y^* \in A_{k_1} \cap B_{k_2}) - \mathsf{P}(Y^* \in A_{k_1})\mathsf{P}(Y^* \in B_{k_2}) > 0\}. \quad (19)$$

By (15), (16), and Lemma 2, the Lebesgure measure of M is positive. Hence by (17), (18) and (19), we obtain

$$\mathsf{Cov}\,(U(X), V(X)) > 0,$$

which contradicts the assumption, and so $r = 0$. □

Proof of Theorem 1 Let X be an $n \times 1$ random vector distributed as $\mathrm{N}(0, I_n)$, and f, g be two symmetric, quasi-convex polynomials satisfying that $f(X)$ and $g(X)$ are independent. By the symmetry and quasi-convexity of f and g, we have that $f(x) \geqslant f(0)$, $g(x) \geqslant g(0)$ for all $x \in \mathbb{R}^n$. Define

$$U(x) := f(x) - f(0), \qquad V(x) := g(x) - g(0).$$

Then U and V are two symmetric quasi-convex polynomials on $\mathbb{R}^n$ satisfying the conditions in Theorem 7, and thus U and V are unlinked. It follows that f and g are unlinked. □

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关于对称拟凸多项式的不相连定理

洪和静

(南京大学数学系, 南京, 210093; 南京方腾医药技术有限公司, 南京, 211100)

胡泽春

(四川大学数学学院, 成都, 610065)

摘　要: 假定 μ_n 为 $\mathbb{R}^n$ 上的标准高斯测度, X 为 $\mathbb{R}^n$ 上的随机向量, 分布为 μ_n. 不相连猜测说的是: 如果 f 与 g 为 $\mathbb{R}^n$ 上的两个多项式, 而且 $f(X)$ 与 $g(X)$ 相互独立, 则存在 $\mathbb{R}^n$ 上的正交变换 $Y=LX$ 及整数 k 使得 $f\circ L^{-1}$ 为 $(y_1,y_2,\cdots,y_k)$ 的函数, $g\circ L^{-1}$ 为 $(y_{k+1},y_{k+2},\cdots,y_n)$ 的函数. 此时, 称 f 与 g 不相连. 在这篇注记中, 我们证明: 对于两个对称拟凸多项式 f 与 g, 如果 $f(X)$ 与 $g(X)$ 相互独立, 则 f 与 g 不相连.

关键词: 不相连猜测; 拟凸多项式; 高斯相关猜测

中图分类号: O211.3; O211.5