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A New Look at the Adjustment Coefficient in the Compound Poisson Model Perturbed by Diffusion *

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Abstract

In this paper, we study the adjustment coefficient as a function of the retention levels for combinations of quota-share with excess of loss reinsurance in the compound Poisson model perturbed by diffusion. We calculate the quota-share on original terms and the excess of loss reinsurance premium according to the expected value principle. Then the result that the adjustment coefficient is a unimodal function of the retention limit for excess of loss reinsurance in this risk model is obtained. In the last part of this paper, an upper bound for the probability of ruin in finite horizon is given.

Keywords: Adjustment coefficient, combinations of excess of loss with quota-share reinsurance, diffusion, the probability of ruin, upper bound.

AMS Subject Classification: 60J25, 91B30.

§1. Introduction

The adjustment coefficient plays an important role in risk theory. Several have concentrated their attention on the effects of reinsurance by the adjustment coefficient. Waters[5] proved that the adjustment coefficient is a unimodal function of the retention level in case of proportional reinsurance. He proved that the adjustment coefficient was a unimodal function of the retention limit for excess of loss reinsurance, assuming that the reinsurance premium calculation principle was the expected value principle and the annual claims had a compound Poisson distribution. In Centeno[1], combinations of quota-share with excess of loss reinsurance were considered. Centeno[2] studied the insurer's adjustment coefficient as function of the retention levels for combinations of quota-share with excess of loss reinsurance in the Sparre Anderson model, generalizing some results of Centeno[1].

In this paper, we study the adjustment coefficient as function of the retention levels for combinations of quota-share with excess of loss reinsurance in the compound Poisson

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model perturbed by diffusion. We calculate the quota-share on original terms and the excess of loss reinsurance premium according to the expected value principle.

The paper is structured as follows: in Section 2, we give some assumptions about the compound Poisson risk model after reinsurance. Two lemmas are given for the proof of the theorem in Section 3. In Section 3, an important result about the adjustment coefficient is given. We give an upper bound for the probability of ruin in finite horizon in Section 4, following by an example.

§2. Some Assumptions

We assume the number of claims $\{N(t)\}_{t\geq 0}$ follows a Poisson process, i.e. the number of claims, $N(t)$, that occur in the interval $(0,t]$ can be written as

$$N(t)=\sup\{n: S_n\leq t\} \tag{2.1}$$

with $S_0=0$, $S_n=T_1+T_2+\cdots+T_n$ for $n\geq 1$, where $\{T_i\}_{i=1}^{\infty}$ are independent and identically distributed non-negative random variables. S_n denotes the epoch of the nth claim and T_i is the time between the $i-1$th and the ith claim. Let the expected value of T_i be $1/\lambda$. $\{X_i\}_{i=1}^{\infty}$ are independent random variables with common distribution $F(x): F(0)=0$ and $0<F(x)<1$ for $0<x<+\infty$. For simplicity we assume that $F(x)$ is differentiable, with $F'(x)=f(x)$ being the individual claim amount probability density function. Let μ be the expected value of X_i. We also assume that $M_X(r)$, the moment generating function of $F(x)$, exists for some $0<\tau\leq+\infty$ and

$$\lim_{r\to\tau}M_X(r)=\lim_{r\to\tau}\mathsf{E}[e^{rX}]=+\infty. \tag{2.2}$$

Now we consider the surplus process of the insurance company as follows

$$U(t)=u+Pt-\sum_{i=1}^{N(t)}X_i+W_t, \tag{2.3}$$

where $u>0$ is the insurer's initial surplus and the premiums are received continuously at a constant rate P per unit of time. $\{W_t\}$ is a Wiener process with infinitesimal drift 0 and infinitesimal variance $2D>0$. Thus W_t is normally distributed with mean 0 and variance $2Dt$.

Let us consider the case that insurer is willing to reduce his or her risk by means of reinsurance under the form of combination of quota-share with excess of loss reinsurance. We assume that $e>c$ and $(1-e)P-(1+\alpha)\lambda\mu<0$, where eP is the amount used to cover the insurer's expenses per unit of time, α is the loading coefficient. We note that for the quota-share retention level a, the insurer pays the reinsurance premium $(1-a)P$ less a commission $c(1-a)P$.

After the reinsurance arrangement the insurer's surplus process becomes

$$U_{a,M}(t) = u + ((1-e)P - P_{a,M})t - \sum_{i=1}^{N(t)} \min(aX_i, M) + W_t, \tag{2.4}$$

where $P_{a,M} = (1-c)(1-a)P + (1+\alpha)\lambda \int_{M/a}^{\infty} (ax - M)\mathrm{d}F(x)$ is the reinsurance premium. The parameter a denotes the quota-share retention level and M denotes the excess of loss retention limit. The process is a martingale if and only if

$$\mathsf{E}[e^{-rU_{a,M}(t)} | U_{a,M}(0) = u] = e^{-ru}. \tag{2.5}$$

That is

$$\lambda\Big[\int_0^{M/a} e^{rax}\mathrm{d}F(x) + \int_{M/a}^{\infty} e^{rM}\mathrm{d}F(x) - 1\Big] + Dr^2 - ((1-e)P - P_{a,M})r = 0. \tag{2.6}$$

If we let

$$H_{a,M}(r) = \lambda\Big[\int_0^{M/a} e^{rax}\mathrm{d}F(x) + \int_{M/a}^{\infty} e^{rM}\mathrm{d}F(x) - 1\Big] + Dr^2 - ((1-e)P - P_{a,M})r, \tag{2.7}$$

then the positive root $R_{a,M}$ of $H_{a,M}(r) = 0$ is the adjustment coefficient in risk theory. The adjustment coefficient is very important in estimating the ruin probability. The interested reader can see [3] or [4] for detailed introduction. Let $\mathsf{E}[W(a,M)]$ denote the insurer's expected net profit per period of time after reinsurance and expenses, i.e. $\mathsf{E}[W(a,M)] = (1-e)P - P_{a,M} - \lambda\mathsf{E}[X_{a,M}]$, where $X_{a,M} = \min(aX_i, M)$.

Let L be the set of points for which the insurer's net expected profit is positive, i.e.

$$L = \{(a,M) : 0 \le a \le 1, M \ge 0 \ \text{ and } \ \mathsf{E}[W(a,M) > 0\}. \tag{2.8}$$

The next two lemmas are useful in the proof of the theorem in Section 3.

Lemma 2.1 1. The adjustment coefficient is positive if and only if $(a,M) \in L$.
2. For any $(a,M) \in L$, $H'_{a,M}(r)$ is positive at $r = R_{a,M}$.

Proof 1. Fix (a,M), we have

$$H'_{a,M}(r) = \lambda\mathsf{E}[X_{a,M}e^{rX_{a,M}}] + 2Dr - ((1-e)P - P_{a,M}). \tag{2.9}$$

Let

$$\xi = \begin{cases} +\infty, & \text{if } \ M < +\infty; \\ \tau, & \text{if } \ M = +\infty, \end{cases}$$

where $M=+\infty$ means no excess of loss reinsurance. Noting that

$$H_{a,M}(0)=0, \tag{2.10}$$

$$\lim_{r\to\xi}H_{a,M}(r)=+\infty, \tag{2.11}$$

$$H''_{a,M}(r)=\lambda\mathsf{E}[X_{a,M}^2e^{rX_{a,M}}]+2D>0, \tag{2.12}$$

we conclude that $H_{a,M}(r)$ is a convex function. So we can say that the adjustment coefficient is positive if and only if

$$H'_{a,M}(0)<0. \tag{2.13}$$

It is equivalent to

$$\lambda\mathsf{E}[X_{a,M}]-((1-e)P-P_{a,M})<0, \tag{2.14}$$

that is, $\mathsf{E}[W(a,M)]>0$. The first part of the Lemma 2.1 is proved.

2. Suppose $(a,M)\in L$. From the proof of part 1, we can see $H'_{a,M}(0)<0$ and $H_{a,M}(R_{a,M})=0$. Given the convexity of $H_{a,M}(r)$, we conclude that $H'_{a,M}(R_{a,M})>0$, which completes the proof. $\square$

Note that the above proof does not depend on the reinsurance premium calculation principles used for both the arrangements.

Let

$$a_0=\frac{(e-c)P}{(1-c)P-\lambda\mathsf{E}[X]} \tag{2.15}$$

and

$$A=\{a:0<a\le 1 \text{ and there exists an } M \text{ such that } \mathsf{E}[W(a,M)]=0\}. \tag{2.16}$$

Lemma 2.2 Under the assumptions on the reinsurance premium $P_{a,M}$, we have

1. $A=(a_0,1]$.

2. For each $a\in A$ there is a unique M such that $\mathsf{E}[W(a,M)]=0$, i.e. there is a function Φ mapping A into $(0,+\infty)$ such that $M=\Phi(a)$ is equivalent to $\mathsf{E}[W(a,M)]=0$.

3. $\Phi(a)$ is convex.

4. $\lim\limits_{a\to a_0}\Phi(a)=+\infty$.

Proof 1. If let $\mathsf{E}[W(a,+\infty)]=\lim\limits_{M\to+\infty}\mathsf{E}[W(a,M)]$, then

$$\begin{aligned}\mathsf{E}[W(a,+\infty)] &= \lim_{M\to+\infty}\mathsf{E}[W(a,M)]\\ &= \lim_{M\to+\infty}\Big\{(1-e)P-(1-c)(1-a)P-\lambda(1+\alpha)\int_{M/a}^{\infty}(ax-M)\mathrm{d}F(x)\\ &\quad -\lambda\Big(\int_0^{M/a}ax\mathrm{d}F(x)+M\Big(1-F\Big(\frac{M}{a}\Big)\Big)\Big)\Big\}\\ &= (1-e)P-(1-c)(1-a)P-\lambda a\mathsf{E}[X]. \end{aligned} \tag{2.17}$$

Differentiating $\mathsf{E}[W(a,+\infty)]$, we get

$$\frac{\mathrm{d}\mathsf{E}[W(a,+\infty)]}{\mathrm{d}a}=(1-c)P-\lambda\mathsf{E}[X]>0$$

and

$$\mathsf{E}[W(a_0,+\infty)]=(1-e)-(1-c)(1-a_0)-\lambda a_0\mathsf{E}[X]=0, \tag{2.18}$$

hence when $0\le a<a_0$, $\mathsf{E}[W(a,+\infty)]<0$.

Calculating the derivative of $\mathsf{E}[W(a,M)]$ with respect to M, we get

$$\begin{aligned}\frac{\partial\mathsf{E}[W(a,M)]}{\partial M} &= (1+\alpha)\lambda\Big(1-F\Big(\frac{M}{a}\Big)-\lambda\Big(1-F\Big(\frac{M}{a}\Big)\Big)\Big)\\ &= \Big[1-F\Big(\frac{M}{a}\Big)\Big]\alpha\lambda,\end{aligned} \tag{2.19}$$

which is positive, so when $0\le a\le a_0$, $\mathsf{E}[W(a,M)]<\mathsf{E}[W(a,+\infty)<0$.

Now we prove there is a finite solution to $\mathsf{E}[W(a,M)]=0$, when $a_0<a\le 1$.

Since $\mathsf{E}[W(a_0,0)]<\mathsf{E}[W(a_0,+\infty)]=0$ and

$$\frac{\mathrm{d}\mathsf{E}[W[(a,0)]}{\mathrm{d}a}=(1-c)P-(1+\alpha)\lambda\mathsf{E}[X]<0,$$

we get that when $a_0<a\le 1$, $\mathsf{E}[W(a,0)]<0$. Fix $a\in(a_0,1]$, we have $\mathsf{E}[W(a,+\infty)]>\mathsf{E}[W(a_0,+\infty)]=0$, so we conclude that there is a unique M such that $\mathsf{E}[W(a,M)]=0$.

2. In the proof of part 1 we have already proved that

$$\frac{\partial\mathsf{E}[W(a,M)]}{\partial M}>0,$$

so $\mathsf{E}[W(a,M)]$ is increasing in M and there exists a unique M such that $\mathsf{E}[W(a,M)]=0$.

3. Calculating the derivative of both sides of $\mathsf{E}[W(a,M)]=0$ at $M=\Phi(a)$, we get

$$\frac{\mathrm{d}M}{\mathrm{d}a} = -\Big\{\frac{\partial\mathsf{E}[W(a,M)]}{\partial a}\Big\}\Big/\Big\{\frac{\partial\mathsf{E}[W(a,M)]}{\partial M}\Big\}\Big|_{M=\Phi(a)}, \tag{2.20}$$

$$\begin{aligned}\frac{\mathrm{d}^2M}{\mathrm{d}a^2} = -\Big\{\Big[&\frac{\partial^2\mathsf{E}[W(a,M)]}{\partial a^2}\Big(\frac{\partial\mathsf{E}[W(a,M)]}{\partial M}\Big)^2\\ &-2\frac{\partial^2\mathsf{E}[W(a,M)]}{\partial a\partial M}\frac{\partial\mathsf{E}[W(a,M)]}{\partial a}\frac{\partial\mathsf{E}[W(a,M)]}{\partial M}\\ &+\frac{\partial^2\mathsf{E}[W(a,M)]}{\partial M^2}\Big(\frac{\partial\mathsf{E}[W(a,M)]}{\partial a}\Big)^2\Big]\Big/\Big(\frac{\partial\mathsf{E}[W(a,M)]}{\partial M}\Big)^3\Big\}\Big|_{M=\Phi(a)}.\end{aligned} \tag{2.21}$$

Due to the fact that $\mathsf{E}[W(a,M)]=0$, we can calculate that

$$\begin{aligned}\frac{\partial\mathsf{E}[W(a,M)]}{\partial a} &= (1-c)P-\lambda(1+\alpha)\int_{M/a}^{\infty}x\mathrm{d}F(x)-\lambda\int_0^{M/a}x\mathrm{d}F(x)\\ &= \frac{(e-c)P-\lambda\alpha M(1-F(M/a))}{a}.\end{aligned} \tag{2.22}$$

The other partial derivatives of $\mathsf{E}[W(a,M)]$ can be calculated as follows,

$$\frac{\partial^2\mathsf{E}[W(a,M)]}{\partial a^2} = -\frac{M^2}{a^3}\alpha\lambda f\Big(\frac{M}{a}\Big), \tag{2.23}$$

$$\frac{\partial\mathsf{E}[W(a,M)]}{\partial M} = \lambda\alpha\Big[1-F\Big(\frac{M}{a}\Big)\Big], \tag{2.24}$$

$$\frac{\partial^2\mathsf{E}[W(a,M)]}{\partial M^2} = -\frac{\lambda\alpha}{a}f\Big(\frac{M}{a}\Big), \tag{2.25}$$

$$\frac{\partial\mathsf{E}[W(a,M)]}{\partial a\partial M} = \frac{\lambda\alpha M}{a^2}f\Big(\frac{M}{a}\Big). \tag{2.26}$$

Substitute (2.22)-(2.26) into (2.21), we get

$$\frac{\mathrm{d}^2M}{\mathrm{d}a^2} = \frac{(e-c)^2P^2f(M/a)}{(\lambda\alpha)^2a^3[1-F(M/a)]^3} > 0. \tag{2.27}$$

Hence $\Phi(a)$ is convex.

4. We can see that $\lim\limits_{a\to a_0}\Phi(a)=+\infty$ is equivalent to $\mathsf{E}[W(a,+\infty)]=0$. From the proof of part 1, the conclusion is right. $\square$

§3. An Important Result About the Adjustment Coefficient

Theorem 3.1 For a fixed value of $a\in(a_0,1]$, $R_{a,M}$ is a unimodal function of M, attaining its maximum value at the only point satisfying

$$M = \frac{1}{R_{a,M}}\ln(1+\alpha). \tag{3.1}$$

Proof The adjustment coefficient $R_{a,M}$ is, for fixed $a\in(a_0,1]$ and $M>\Phi(a)$, the only root of $H_{a,M}(r)=0$, where $H_{a,M}(r)=\lambda\mathsf{E}[e^{rX_{a,M}}]-\lambda+Dr^2-((1-e)-P_{a,M})r$.

Let us consider $R_{a,M}$ as a function of $(a,M)\in L$. From the implicit function theorem it follows that

$$\frac{\partial R_{a,M}}{\partial M} = -\Big[\frac{\partial H_{a,M}(r)}{\partial M}\Big]\Big/\Big[\frac{\partial H_{a,M}(r)}{\partial r}\Big]\Big|_{r=R_{a,M}}.$$

We have already known

$$\frac{\partial H_{a,M}(r)}{\partial r}\Big|_{r=R_{a,M}} > 0.$$

Since

$$\begin{aligned}\frac{\partial H_{a,M}(r)}{\partial M} &= \lambda re^{rM}\Big(1-F\Big(\frac{M}{a}\Big)\Big)-r(1+\alpha)\lambda\Big(1-F\Big(\frac{M}{a}\Big)\Big)\\ &= r\Big(1-F\Big(\frac{M}{a}\Big)\Big)[\lambda e^{rM}-\lambda(1+\alpha)],\end{aligned} \tag{3.2}$$

we know

$$\frac{\partial H_{a,M}(r)}{\partial M}\Big|_{r=R_{a,M}}=0$$

is equivalent, for finite M, to $\lambda e^{R_{a,M}}-\lambda(1+\alpha)=0$. That is

$$M=\frac{1}{R_{a,M}}\ln(1+\alpha).$$

Calculate the second derivative with respect to M of $R_{a,M}$, we get

$$\frac{\partial^2 R_{a,M}}{\partial M^2}|_{\partial R_{a,M}/\partial M=0}=-\Big[\frac{\partial^2 H_{a,M}(r)}{\partial M^2}\Big]\Big/\Big[\frac{\partial H_{a,M}(r)}{\partial r}\Big]\Big|_{r=R_{a,M},\partial R_{a,M}/\partial M=0}. \tag{3.3}$$

We know that

$$\frac{\partial H_{a,M}(r)}{\partial r}\Big|_{r=R_{a,M},\partial R_{a,M}/\partial M=0}>0,$$

and

$$\begin{aligned}&\frac{\partial^2 H_{a,M}(r)}{\partial M^2}\Big|_{r=R_{a,M},\partial R_{a,M}/\partial M=0}\\ =\ &\frac{R_{a,M}}{a}f\Big(\frac{M}{a}\Big)[\lambda e^{R_{a,M}M}-\lambda(1+\alpha)]+R_{a,M}\Big(1-F\Big(\frac{M}{a}\Big)\Big)\lambda R_{a,M}e^{R_{a,M}M}\\ =\ &R_{a,M}^2\Big(1-F\Big(\frac{M}{a}\Big)\Big)\lambda e^{R_{a,M}M}>0,\end{aligned} \tag{3.4}$$

so we conclude that

$$\frac{\partial^2 R_{a,M}(r)}{\partial M^2}|_{\partial R_{a,M}/\partial M=0}<0. \tag{3.5}$$

From the above proof, we can see, for fixed $a\in(a_0,1]$, $R_{a,M}$ has at most one turning point, and when such a point exists it is a maximum. The maximum will exist if we can guarantee that there is a finite solution to $\lambda e^{R_{a,M}}-\lambda(1+\alpha)=0$.

Fix $a\in(a_0,1]$, let

$$D_{a,M}(R_{a,M})=\lambda e^{R_{a,M}}-\lambda(1+\alpha). \tag{3.6}$$

Since $\lim\limits_{M\to\Phi(a)}R_{a,M}=0$, we get

$$\lim_{M\to\Phi(a),R_{a,M}\to 0}D_{a,M}(R_{a,M})=-\alpha\lambda<0,$$

and

$$\lim_{M\to+\infty,R_{a,M}\to 0}D_{a,M}(R_{a,M})=+\infty,$$

so there exists one solution and it is unique. $\Box$

§4. Upper Bound for the Probability of Ruin in Finite Horizon

In this section, we give the upper bound for the probability of ruin in finite horizon under the risk model (2.4). Let a time horizon $t > 0$ be given and let T_u denote the time of ruin. The finite time ruin probability $\Psi(u,t)$ is defined by

$$\Psi(u,t) = \mathsf{P}\{T_u \le t\}. \tag{4.1}$$

In the classical risk model

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i, \tag{4.2}$$

put $t = yu$, we have

$$\Psi(u,t) = \Psi(u,yu) \le \exp\{-uR_y\}, \tag{4.3}$$

where $R_y = \sup\limits_{r\ge R}(r - y\lambda h(r) + yrc)$. The result (4.3) can be found in [page 136] in [4].

In our model defined by (2.4), we have

$$R_y = \sup_{r\ge R_{a,M}} (r - yH_{a,M}(r)), \tag{4.4}$$

where $H_{a,M}(r)$ is defined by (2.7). Put

$$g(r) = r - yH_{a,M}(r), \tag{4.5}$$

we have

$$g'(r) = 1 - yH'_{a,M}(r) \tag{4.6}$$

and

$$g''(r) = -yH''_{a,M}(r), \tag{4.7}$$

which is negative by the proof of Lemma 2.1, so $g(r)$ is concave.

Result Considering the risk model defined by (2.4), for $(a,M)\in L$, we have

$$\Psi(u,yu) \le \begin{cases} e^{-uR_{a,M}}, & \text{if } y \ge \dfrac{1}{H'_{a,M}(R_{a,M})}; \\[2ex] e^{-u\widehat{R}_{a,M}}, & \text{if } y < \dfrac{1}{H'_{a,M}(R_{a,M})}, \end{cases}$$

where $\widehat{R}_{a,M}$ is the solution to $g'(r) = 0$.

Proof For $(a, M) \in L$, we have $R_{a,M}$ is positive by Lemma 2.1. Note that $g(r) = r - yH_{a,M}(r)$, which yields $g(R_{a,M}) = R_{a,M}$. We have known $g(r)$ is concave, hence when $r \geq R_{a,M}$, it can be seen that

$$R_y = R_{a,M} \tag{4.8}$$

is equivalent to

$$g'(R_{a,M}) \leq 0, \tag{4.9}$$

that is

$$y \geq \frac{1}{H'_{a,M}(R_{a,M})}. \tag{4.10}$$

When

$$g'(R_{a,M}) > 0, \tag{4.11}$$

we can see that R_y is the solution to

$$g'(r) = 1 - yH'_{a,M}(r) = 0, \tag{4.12}$$

we denote this solution $\widehat{R}_{a,M}$. □

Example Assume that the individual claim amount distribution is Exp(1), i.e. $F(x) = 1 - \exp(-x)$, $x > 0$. We assume that the inter arrival times have mean 1, i.e. $\lambda = 1$.

We have

$$H_{a,M}(r) = \int_0^{M/a} \exp(rax - x)\mathrm{d}x + \exp\left(rM - \frac{M}{a}\right) + Dr^2 - ((1-e)P - P_{a,M})r$$

and

$$H'_{a,M}(r) = a\int_0^{M/a} x\exp(rax - x)\mathrm{d}x + M\exp\left(rM - \frac{M}{a}\right) + 2Dr - ((1-e)P - P_{a,M}).$$

Let $D = 0.5$, $a = 0.6$, $M = 30$, $P = 1.6$, $e = 0.3$, $c = 0.2$ and $\alpha = 0.8$.

We have $R_{a,M} = 0.0093$ and $H'_{a,M}(R_{a,M}) = 0.0081$.

Let $1/y = 0.0090$, so

$$y < \frac{1}{H'_{a,M}(R_{a,M})}.$$

$\widehat{R}_{a,M}$ is the solution to $H'_{a,M}(r) = 0.0090$, and we have $\widehat{R}_{a,M} = 0.0098$.

Then $R_{a,M} < \widehat{R}_{a,M}$, that is what we have expected.

Let $1/y = 0.0060$, so

$$y > \frac{1}{H'_{a,M}(R_{a,M})}.$$

$\widehat{R}_{a,M}$ is the solution to $H'_{a,M}(r) = 0.0060$, and we have $\widehat{R}_{a,M} = 0.0081$.

Then $R_{a,M} > \widehat{R}_{a,M}$, that is what we have expected.

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带干扰复合泊松模型下对调整系数的新研究

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本文研究了带干扰复合泊松模型中采用成数再保与超额损失再保险混合策略时作为自留额水平函数的调整系数. 我们按照原始条款计算成数再保费, 按照期望值保费原则计算超额损失再保费, 这样得到了调整系数是超额损失自留额极限的单峰函数的结论. 本文最后部分给出了有限时间破产概率的上界.

关键词: 调整系数, 超额损失再保险与成数分保的组合, 干扰, 破产概率, 上界.

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