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Confidence Intervals for Successive Comparisons of Ordered Treatment Effects

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Abstract

Lee and Spurrier[8] present a new procedure for making successive comparisons between ordered treatments. Their procedure has important applications for problems where the treatments can be assumed to satisfy a simple ordering, such as for a sequence of increasing dose-levels of a drug. The advantage of their procedure is that it provides more chance to detect when changes in the treatment means occur than other test procedures (for example: test in Hayter[4]). The disadvantage of their procedure is that it is not as powerful as other test procedures. In this paper we propose a test procedure which try to keep the advantage of Lee and Spurrier's procedure and promote the power performance of their test procedure.

Keywords: Simple ordering, pairwise comparisons, simultaneous confidence intervals.
AMS Subject Classification: 62H15.

§1. Introduction

Consider the one-way analysis of variance model

$$X_{ij} = \mu_i + \varepsilon_{ij}, \qquad 1 \le i \le k,\ 1 \le j \le n_i,$$

where the ε_{ij} are independent $N(0, \sigma^2)$ random variables. Let $\overline{X}_i$, $1 \le i \le k$, be the ith sample mean based upon n_i observations, and let S^2 be an unbiased estimate of σ^2 distributed independently of the $\overline{X}_i$ as $S^2 \sim \sigma^2\chi_v^2/v$ for some degrees of freedom v. Usually, the mean squared error in the analysis of variance will be used as the estimate S^2 with $v = \sum\limits_{i=1}^{k} n_i - k$.

Suppose that the data represent information on k treatments which can be assumed to satisfy the simple ordering $\mu_1 \le \mu_2 \le \cdots \le \mu_k$. A problem that has received considerable attention is that testing the null hypothesis

$$H_0 : \mu_1 = \cdots = \mu_k$$

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against the simple ordered alternative hypothesis

$$H_A: \mu_1 \le \cdots \le \mu_k$$

with at least one strict inequality. Bartholomew[1] derived the likelihood ratio test for this problem. Williams [15, 16] proposed a different test procedure for this problem based on the statistic $\widehat{\mu}_k - \overline{X}_1$, and Marcus[10] discussed a modification of Williams's test, which uses the statistic $\widehat{\mu}_k - \widehat{\mu}_1$, where $\widehat{\mu}_1, \widehat{\mu}_2, \cdots, \widehat{\mu}_k$ are the maximum likelihood estimators of $\mu_1, \mu_2, \cdots, \mu_k$ under the order restriction $\mu_1 \le \mu_2 \le \cdots \le \mu_k$.

It is also useful to be able to make direct comparisons of the treatments through the generation of a set of simultaneous confidence intervals. Hayter[4] proposed a one-sided studentized range test (OSRT) which provides simultaneous one-sided lower confidence bounds for the ordered pairwise comparisons $\mu_i - \mu_j$, $1 \le j < i \le k$. Lee and Spurrier[8] proposed successive comparisons test (LST) for ordered treatments which provides simultaneous one-sided lower confidence bounds for the successive ordered pairwise comparisons $\mu_{i+1} - \mu_i$, $1 \le i \le k-1$. LST procedure provides more chance to detect when changes in the treatment means occur than ORST. However, this procedure is not as powerful as ORST in rejection the null hypothesis that the treatment effects are all equal in favour of the alternative hypothesis of simple order. In this paper we propose a test procedure which try to keep the advantage of Lee and Spurrier's procedure and promote the power performance of their test procedure.

Consider the subset of pairwise differences

$$\mu_2 - \mu_1, \mu_3 - \mu_2, \cdots, \mu_k - \mu_{k-1}, \mu_k - \mu_1.$$

Let the one-sided critical points $d = d_{k,\alpha,\nu,R}$ be defined by the equation

$$\mathsf{P}_{H_0}\Big(\max_{1\le i\le k-1} \frac{\overline{X}_{i+1} - \overline{X}_i}{S\sqrt{1/n_{i+1} + 1/n_i}} \le d, \frac{\overline{X}_k - \overline{X}_1}{S\sqrt{1/n_k + 1/n_1}} \le d\Big) = 1 - \alpha, \tag{1.1}$$

the two-sided critical points $d' = d'_{k,\alpha,\nu,R}$ be defined by the equation

$$\mathsf{P}_{H_0}\Big(\max_{1\le i\le k-1} \frac{|\overline{X}_{i+1} - \overline{X}_i|}{S\sqrt{1/n_{i+1} + 1/n_i}} \le d', \frac{|\overline{X}_k - \overline{X}_1|}{S\sqrt{1/n_k + 1/n_1}} \le d'\Big) = 1 - \alpha. \tag{1.2}$$

We construct the following sets of one-sided simultaneous confidence intervals for successive comparisons and $\mu_k - \mu_1$:

$$\begin{aligned}
&\mu_{i+1} - \mu_i \in \Big(\overline{X}_{i+1} - \overline{X}_i - Sd\sqrt{\frac{1}{n_{i+1}} + \frac{1}{n_i}}, +\infty\Big), \qquad 1 \le i \le k-1,\\
&\mu_k - \mu_1 \in \Big(\overline{X}_k - \overline{X}_1 - Sd\sqrt{\frac{1}{n_k} + \frac{1}{n_1}}, +\infty\Big),
\end{aligned} \tag{1.3}$$

and two-sided simultaneous confidence intervals

$$\mu_{i+1}-\mu_i \in \Big(\overline{X}_{i+1}-\overline{X}_i-Sd'\sqrt{\frac{1}{n_{i+1}}+\frac{1}{n_i}},\overline{X}_{i+1}-\overline{X}_i+Sd'\sqrt{\frac{1}{n_{i+1}}+\frac{1}{n_i}}\Big),\quad 1\le i\le k-1,$$

$$\mu_k-\mu_1 \in \Big(\overline{X}_k-\overline{X}_1-Sd'\sqrt{\frac{1}{n_k}+\frac{1}{n_1}},\overline{X}_k-\overline{X}_1+Sd'\sqrt{\frac{1}{n_k}+\frac{1}{n_1}}\Big). \tag{1.4}$$

The critical points $d_{k,\alpha,\nu,R}$ and $d'_{k,\alpha,\nu,R}$ depend upon the covariance matrix R of the normal random variables

$$t_i=\frac{\overline{X}_{i+1}-\overline{X}_i}{\sigma\sqrt{1/n_{i+1}+1/n_i}},\quad 1\le i\le k-1,\qquad t_k=\frac{\overline{X}_k-\overline{X}_1}{\sigma\sqrt{1/n_k+1/n_1}},$$

which has 1's on the diagonal and 0's everywhere else except for

$$\begin{aligned} r_{i,i+1}=r_{i+1,i}&=-\frac{\sqrt{n_in_{i+2}}}{\sqrt{(n_i+n_{i+1})(n_{i+1}+n_{i+2})}},\qquad 1\le i\le k-2,\\ r_{1,k}=r_{k,1}&=\frac{\sqrt{n_2n_k}}{\sqrt{(n_1+n_2)(n_1+n_k)}},\\ r_{k-1,k}=r_{k,k-1}&=\frac{\sqrt{n_1n_{k-1}}}{\sqrt{(n_1+n_{k-1})(n_1+n_k)}}.\end{aligned}$$

Obviously, the critical values d, d' are larger than those of LST, but smaller than those of OSRT. So this procedure provides simultaneous confidence intervals for successive comparisons which are shorter than those provided by OSRT, but longer than those provided by LST. By the following discussion, we will find that this procedure has a substantially more power than LST, but less power than OSRT. Along with the increasing of k, the inferior positions of the proposed procedure decrease sharply.

We organized this paper in the following way. In Section 2, we calculate critical points $d_{k,\alpha,\nu,R}$ and $d'_{k,\alpha,\nu,R}$ by simulation. In Section 3, we show how to short the length of the confidence intervals given in (1.3), (1.4) using the relationship between step-down decision procedures and confidence sets. In Section 4, we do a simulation for a comparison of the power performance of the proposed procedure with those of LST and OSRT, and compare their abilities of detecting when changes in the treatment means occur when the equality is rejected. Section 5 illustrates the procedure with an example.

§2. Computation of Critical Points

There are many statist focusing on the calculation of the critical points for multiple comparisons problems. Among them, Genz and Bretz[3] proposed a numerical computation method. Liu et al.[9] used a recursive integration technique.

Table 1 Values of d(upper) and d'(lower) for $\alpha = 0.05$

$\nu\backslash k$	3	4	5	6	7	8	9
5	2.737	3.006	3.204	3.353	3.483	3.590	3.691
	3.252	3.489	3.675	3.809	3.923	4.035	4.130
10	2.369	2.561	2.694	2.808	2.891	2.966	3.033
	2.740	2.906	3.029	3.130	3.216	3.287	3.348
16	2.256	2.416	2.537	2.628	2.703	2.767	2.826
	2.578	2.720	2.827	2.916	2.992	3.050	3.100
20	2.217	2.374	2.485	2.576	2.659	2.708	2.762
	2.531	2.663	2.769	2.847	2.916	2.976	3.025
25	2.190	2.341	2.445	2.530	2.602	2.661	2.711
	2.492	2.621	2.716	2.795	2.862	2.920	2.964
30	2.171	2.318	2.420	2.504	2.574	2.631	2.680
	2.464	2.592	2.687	2.760	2.826	2.881	2.928
40	2.146	2.288	2.388	2.475	2.538	2.592	2.641
	2.434	2.553	2.648	2.717	2.783	2.832	2.877
60	2.126	2.264	2.362	2.437	2.501	2.556	2.599
	2.402	2.524	2.608	2.680	2.739	2.788	2.830
120	2.101	2.235	2.333	2.406	2.467	2.519	2.564
	2.371	2.486	2.570	2.640	2.695	2.744	2.782
∞	2.077	2.211	2.303	2.375	2.434	2.483	2.528
	2.344	2.452	2.537	2.600	2.656	2.699	2.738

Miwa et al.[13] developed a method based on the computation of the orthant probability for a general multivariate normal vector with a positive definite correlation matrix. However, It's a pity their methods can't be employed here for our special correlation matrix R. We advise to get the critical points $d_{k,\alpha,\nu,R}$ and $d'_{k,\alpha,\nu,R}$ by simulation. We present some critical points in the balanced cases when all of the sample sizes n_i are equal. Table 1 shows the simulation results for $\alpha = 0.05$, different k and ν. The case $\nu = \infty$ denotes that σ^2 is known. To get one critical value we perform 100,000 simulated samples generated by MATLAB. We further repeated this process 10 times and computed the average of the 10 estimated upper α points. For other cases, the critical values can be simulated similarly.

To compare the critical points among the proposed procedure, LST and ORST, some values of them in the balanced cases are reported together in Table 2, where $c = c_{k,\alpha,\nu,R^*}$,

$c' = c'_{k,\alpha,\nu,R^*}$, which can be found in Liu et al.[9], satisfy

$$\mathsf{P}_{H_0}\Big(\max_{1\le i\le k-1}\frac{\overline{X}_{i+1}-\overline{X}_i}{S\sqrt{1/n_{i+1}+1/n_i}}\le c\Big)=1-\alpha,\quad \mathsf{P}_{H_0}\Big(\max_{1\le i\le k-1}\frac{|\overline{X}_{i+1}-\overline{X}_i|}{S\sqrt{1/n_{i+1}+1/n_i}}\le c'\Big)=1-\alpha,$$

and $h = h_{k,\alpha,\nu}$, which can be found in Hayter[4], satisfy

$$\mathsf{P}_{H_0}\Big(\max_{1\le i<j\le k}\frac{\overline{X}_j-\overline{X}_i}{S\sqrt{1/n_j+1/n_i}}\le h\Big)=1-\alpha.$$

Table 2 Compare critical points h(first), d(second) with c(third), d'(fourth) with c'(fifth) when $\alpha = 0.05$

$\nu\backslash k$	3	4	5	6	7	8	9
5	3.872	4.520	5.000	5.380	5.696	5.961	6.197
	2.737	3.006	3.204	3.353	3.483	3.590	3.691
	2.565	2.881	3.103	3.275	3.415	3.532	3.634
	3.252	3.489	3.675	3.809	3.923	4.035	4.130
	3.031	3.319	3.531	3.697	3.835	3.951	4.053
10	3.353	3.833	4.180	4.452	4.676	4.864	5.029
	2.369	2.561	2.694	2.808	2.891	2.966	3.033
	2.227	2.456	2.615	2.737	2.835	2.918	2.990
	2.740	2.906	3.029	3.130	3.216	3.287	3.348
	2.569	2.778	2.929	3.047	3.143	3.225	3.296
30	3.070	3.460	3.736	3.948	4.121	4.265	4.391
	2.171	2.318	2.420	2.504	2.574	2.631	2.680
	2.042	2.227	2.353	2.449	2.525	2.589	2.644
	2.464	2.592	2.687	2.760	2.826	2.881	2.928
	2.321	2.488	2.606	2.697	2.770	2.832	2.885
∞	2.943	3.295	3.539	3.725	3.875	4.000	4.107
	2.077	2.211	2.303	2.375	2.434	2.483	2.528
	1.960	2.126	2.238	2.322	2.389	2.445	2.493
	2.344	2.452	2.537	2.600	2.656	2.699	2.738
	2.212	2.361	2.464	2.543	2.607	2.659	2.704

Table 2 shows that $c < d < h$ and $c' < d'$ are true. Along with the increasing of k, the degrees of the differences between c and d and c' and d' become narrower, but the differences between c and h become larger.

§3. Step-Down

In this section we short the confidence intervals produced by the proposed procedure based on a step-down test. First, we introduce the step-down test we need here. It should be noted that the step-down procedure here is not the classical step-down. In fact it only contains some sub-steps of the classical step-down. The reason for that we do not use the classical one is that the classical one does not afford manageable inversions to confidence sets to provide more information about the bounds for $\mu_{i+1}-\mu_i$ (see Hayter and Hsu[5]). Second, we derive the confidence intervals based on the duality between confidence intervals and hypothesis testing. The technique by which we derive the confidence intervals has been used in Hayter and Hsu[5] and Hayter et al.[6].

3.1 Step-Down Test

Here we give a step-down test for testing the null hypothesis $H_0 : \mu_1 = \cdots = \mu_k$ against the simple ordered alternative hypothesis $H_A : \mu_1 \le \cdots \le \mu_k$ with at least one strict inequality as follows:

(1) If $(\overline{X}_k - \overline{X}_1)/(S\sqrt{1/n_k + 1/n_1}) \le d$ and $(\overline{X}_{i+1} - \overline{X}_i)/(S\sqrt{1/n_{i+1} + 1/n_i}) \le d$, $1 \le i \le k-1 \to$ not reject H_0, no declaration is made about $\mu_1, \cdots, \mu_k$.

If $(\overline{X}_k - \overline{X}_1)/(S\sqrt{1/n_k + 1/n_1}) \le d$ and $\max\limits_{1\le i\le k-1} (\overline{X}_{i+1} - \overline{X}_i)/(S\sqrt{1/n_{i+1} + 1/n_i}) > d$ $\to$ reject H_0, declare $\mu_{i+1} > \mu_i$ if $(\overline{X}_{i+1} - \overline{X}_i)/(S\sqrt{1/n_{i+1} + 1/n_i}) > d$.

If $(\overline{X}_k - \overline{X}_1)/(S\sqrt{1/n_k + 1/n_1}) > d \to$ reject H_0, declare $\mu_k > \mu_1$, go to the next step.

(2) If $\max\limits_{1\le i\le k-1} (\overline{X}_{i+1} - \overline{X}_i)/(S\sqrt{1/n_{i+1} + 1/n_i}) \le c \to$ no declaration is made about the relationship between μ_{i+1} and μ_i for $1 \le i \le k-1$.

If $\max\limits_{1\le i\le k-1} (\overline{X}_{i+1} - \overline{X}_i)/(S\sqrt{1/n_{i+1} + 1/n_i}) > c \to$ declare $\mu_{i+1} > \mu_i$ if $(\overline{X}_{i+1} - \overline{X}_i)$ $/(S\sqrt{1/n_{i+1} + 1/n_i}) > c$.

This step-down only contains some sub-steps of classical step-down, so it controls the familywise error rate at α. Another thing should be noted is that this step-down procedure has the same power performance as the procedure (1.3) for our test problem.

3.2 Confidence Intervals

Consider the acceptance sets given by

$$\begin{aligned}\mu_k - \mu_1 \le 0 \quad \Rightarrow \quad & A(\mu_i, 1 \le i \le k) = \big\{(\overline{X}_i, 1 \le i \le k) : \\ & \overline{X}_{i+1} - \overline{X}_i - (\mu_{i+1} - \mu_i) \le dS\sqrt{1/n_{i+1} + 1/n_i}\,, 1 \le i \le k-1, \\ & \overline{X}_k - \overline{X}_1 - (\mu_k - \mu_1) \le dS\sqrt{1/n_k + 1/n_1}\,\big\}\end{aligned}$$

and

$$\mu_k - \mu_1 > 0 \quad \Rightarrow \quad A(\mu_i, 1 \le i \le k) = \big\{(\overline{X}_i, 1 \le i \le k) : \\ \overline{X}_{i+1} - \overline{X}_i - (\mu_{i+1} - \mu_i) \le cS\sqrt{1/n_{i+1} + 1/n_i}\,, 1 \le i \le k-1\big\}.$$

Each of these acceptance sets has a coverage probability of exactly $1-\alpha$. These acceptance sets $A(\mu_i, 1 \le i \le k)$ can be inverted to form the following $1-\alpha$ level confidence sets for $\mu_k - \mu_1$ and $\mu_{i+1} - \mu_i$, $1 \le i \le k-1$:

If

$$\overline{X}_k - \overline{X}_1 > dS\sqrt{1/n_k + 1/n_1} \\ \Rightarrow \quad \mu_{i+1} - \mu_i \in \big[\overline{X}_{i+1} - \overline{X}_i - cS\sqrt{1/n_{i+1} + 1/n_i}, +\infty\big);\ \mu_k - \mu_1 \in (0, +\infty).$$

If

$$\overline{X}_k - \overline{X}_1 \le dS\sqrt{1/n_k + 1/n_1} \\ \Rightarrow \quad \mu_{i+1} - \mu_i \in \big[\overline{X}_{i+1} - \overline{X}_i - dS\sqrt{1/n_{i+1} + 1/n_i}, +\infty\big); \\ \mu_k - \mu_1 \in \big[\overline{X}_k - \overline{X}_1 - dS\sqrt{1/n_k + 1/n_1}, +\infty\big). \tag{3.1}$$

Notice that the generated confidence intervals always correspond to the step-down procedure. These confidence intervals provide the same confidence intervals for successive pairwise comparisons as (1.3) when $\overline{X}_k - \overline{X}_1 \le dS\sqrt{1/n_k + 1/n_1}$ and shorter ones when $\overline{X}_k - \overline{X}_1 > dS\sqrt{1/n_k + 1/n_1}$.

In the same way we can short the two-sided confidence intervals for successive comparisons in (1.4). We don't discuss it in more detail.

§4. Power Study

To test k ordered treatments being all equal and detect when changes in the treatment means occur when the equality is rejected, we can use our proposed procedure, LST or OSRT. In this section our primary goal is to compare the power performance and the ability of detecting between these procedures.

Let $\mu = (\mu_1, \cdots, \mu_k)'$ and $A = \{\mu : \mu_1 \le \cdots \le \mu_k\}$, then A is a convex set. Table 3 gives simulation results to compare the power performance of the proposed procedure (3.1), denoted by 'PRO', with those of OSRT and LST for $3 \le k \le 9$, $\alpha = 0.05$ and σ^2 being known to be 1. Here for simplicity we fix $n = 1$ and consider the center direction $(1, 2, \cdots, k)'/\sqrt{2}$ and the edge direction $(1, \cdots, 1, 2)'/\sqrt{2}$ in the set A, because they are among the extreme cases.

Table 3 demonstrates that the power performance of ORST procedure is the best, the power performance of LST is the worst. Both ORST and the proposed procedure have substantially more power than LST. The power dominations of ORST and PRO over LST are getting greater with k becoming larger when the true direction is the center direction.

Table 3　Power comparison between the three procedures

Direction	Procedure	k						
		3	4	5	6	7	8	9
Center	OSRT	0.198	0.316	0.480	0.666	0.830	0.937	0.984
	PRO	0.198	0.301	0.436	0.594	0.747	0.863	0.938
	LST	0.145	0.152	0.161	0.166	0.168	0.172	0.173
Edge	OSRT	0.113	0.101	0.095	0.090	0.087	0.083	0.080
	PRO	0.113	0.099	0.091	0.088	0.081	0.080	0.077
	LST	0.097	0.086	0.078	0.074	0.070	0.069	0.066

Table 4　Detecting ability comparison between three procedures

Direction	λ	procedure	k						
			3	4	5	6	7	8	9
Center	2	PRO	0.160	0.130	0.107	0.093	0.081	0.076	0.067
		LST	0.168	0.131	0.107	0.093	0.081	0.076	0.067
		OSRT	0.141	0.092	0.066	0.051	0.040	0.034	0.029
	6	PRO	0.852	0.810	0.777	0.752	0.729	0.710	0.694
		LST	0.852	0.810	0.777	0.752	0.729	0.710	0.694
		OSRT	0.822	0.749	0.691	0.644	0.601	0.568	0.538
Edge	2	PRO	0.148	0.117	0.100	0.087	0.076	0.071	0.064
		LST	0.170	0.130	0.109	0.093	0.081	0.075	0.068
		OSRT	0.141	0.092	0.068	0.051	0.040	0.034	0.029
	6	PRO	0.841	0.800	0.770	0.745	0.723	0.706	0.688
		LST	0.850	0.808	0.777	0.751	0.728	0.710	0.690
		OSRT	0.820	0.746	0.690	0.643	0.601	0.567	0.538

Table 4 compares the abilities of detecting between this three procedures. For simplicity we let $n=1$, $\alpha=0.05$ and suppose σ^2 is known to be 1. We consider the center direction $\lambda\times(1,2,\cdots,k)'$ and the edge direction $\lambda\times(1,\cdots,1,2)'$, $\lambda=2,6$. We repeat 100,000 samples to get the probability of detecting $\mu_k>\mu_{k-1}$ for each procedure. Table 4 tells that, along the edge direction, the detecting ability of the PRO procedure is close to the detecting ability of LST, and, lager k is, higher the close degree is. Along the center direction, the PRO procedure has the same detecting ability as LST when $k \geq 5$. The detecting ability of the OSRT always becomes weaker with k being larger.

So, when our interest is to test k ordered treatments being all equal and provide useful information regarding the changes occurring in adjacent populations, the PRO procedure is an attractive alternative, especially when k is large.

§5. Example

The construction of simultaneous confidence intervals for successive differences of treatment effects is illustrated by the following example.

Example 1 Bhalla and Sokal[2] studied the effect of density on the dry weight of hybrid houseflies. In this experiment $k = 7$ different densities were considered corresponding to 2560, 1280, 640, 320, 160, 80 and 40 individuals per 36g of medium. In each case $n = 5$ replicates were obtained and the sample averages in milligrams were reported as $\overline{X}_1 = 0.74$; $\overline{X}_2 = 0.73$; $\overline{X}_3 = 1.40$; $\overline{X}_4 = 1.57$; $\overline{X}_5 = 2.24$; $\overline{X}_6 = 2.63$; $\overline{X}_7 = 3.23$ and a pooled estimate $S = 0.283$ was obtained with $\nu = 28$ degrees of freedom.

Liu et al.[9] derived the 99% confidence level confidence intervals for the successive differences of the treatment effects using LST as following

$$\begin{aligned} &\mu_2 - \mu_1 \in (0.584, +\infty), \quad \mu_3 - \mu_2 \in (0.096, +\infty), \quad \mu_4 - \mu_3 \in (-0.404, +\infty), \\ &\mu_5 - \mu_4 \in (0.096, +\infty), \quad \mu_6 - \mu_5 \in (-0.184, +\infty), \quad \mu_7 - \mu_6 \in (0.026, +\infty). \end{aligned} \tag{5.1}$$

By these confidence intervals they established that $\mu_3 > \mu_2$, $\mu_5 > \mu_4$ and $\mu_7 > \mu_6$. If the one-sided studentized range test was used, they obtained the interval for $\mu_7 - \mu_6$ is $(-0.047, +\infty)$ and found the inference $\mu_7 > \mu_6$ cannot be drawn.

Next we show what the conclusion is when the proposed procedure (3.1) is used for this example. We can get the critical point d satisfying (1.1) as $d = 3.250$ by simulation for this example. Notice that $\overline{X}_7 - \overline{X}_1 = 2.49 > dS\sqrt{2/7} = 0.492$. By the procedure (3.1), we can arrive the same confidence intervals and the same conclusions for successive comparisons as (5.1).

References

[1] Bartholomew, D.J., A test of homogeneity for ordered alternatives, *Biometrika*, **41**(1959), 36–48.

[2] Bhalla, S.C. and Sokal, R.R., Competition among genotypes in the housefly at varying densities and proportions (the green strain), *Evolution*, **18**(1964), 312–330.

[3] Genz, A. and Bretz, F., Methods for the computation of multivariate t-probabilities, *J. Comp. Graph. Stat.*, **11**(2002), 950–970.

[4] Hayter, A.J., A one-sided studentized range test for testing against a simple ordered alternative, *J. Amer. Statist. Assoc.*, **85(411)**(1990), 778–785.

[5] Hayter, A.J. and Hsu, J.C., On the relationship between stepwise decision procedures and confidence sets, *J. Amer. Statist. Assoc.*, **89(425)**(1994), 128–136,

[6] Hayter, A.J., Miwa T. and Liu, W., Combining the advantages of on-sided and two-sided procedures for comparing several treatments with a control, *J. Statist. Plann. Inference*, **86**(2000), 323–330.

[7] Hochberg, Y. and Tamhane, A.C., *Multiple Comparison Procedures*, Wiley, New York, 1987.

[8] Lee, R.E. and Spurrier, J.D., Successive comparisons between ordered treatments, *J. Statist. Plann. Inference*, **43**(1995), 323–330.

[9] Liu, W., Hayter, A.J. and Miwa T., Simultaneous confidence interval estimation for successive comparisons of ordered treatment effects, *J. Statist. Plann. Inference*, **88**(2000), 75–86.

[10] Marcus, R., The powers of some tests of the equality of normal means against an ordered alternative, *Biometrika*, **63**(1976), 177–183.

[11] Miller, R.G., *Simultaneous Statistical Inference*, 2nd Edition, Springer, Berlin, 1981.

[12] Miwa, T., Hayter, A.J. and Liu, W., Calculations of level probabilities for normal random variables with unequal variances with applications to Bartholomew's test in unbalanced one-way models, *Comput. Statist. Data Anal.*, **34**(2000), 17–32.

[13] Miwa, T., Hayter, A.J. and Kuriki, S., The evaluation of general non-centered orthant probabilities, *Journal of the Royal Statistical Society Series B*, **65**(2003), 223–234.

[14] Tukey, J.W., The problem of multiple comparisons, Unpublished manuscript, Princeton University, 1953.

[15] Williams, D.A., A test for differences between treatment means when several dose levels are compared with a zero dose control, *Biometrics*, **27**(1971), 103–117.

[16] Williams, D.A., The comparison of several dose levels with a zero dose control, *Biometrics*, **28**(1972), 519–531.

有序处理连续比较的置信区间

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Lee and Spurrier[8]为有序均值的连续比较提出了一个新的检验过程. 他们的过程对满足简单序的均值有重要的应用价值, 例如在研究增长剂量对药物效用的影响. 与其它检验过程相比(例: Hayter[4]中的检验), 其优点在于产生了更短的连续比较的置信限, 从而能够提供更多机会发现在何药剂量处有不同的效用. 但作为有序均值的齐性检验, 它的势表现远劣于其它检验. 本文的目的是提出一检验过程在尽量保持Lee and Spurrier[8]检验的优点的同时大大地提高其势表现.

关 键 词: 简单序, 成对比较, 同时置信区间.

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