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Testing Unit Roots of Financial Time Series: An Application to Major Stock Markets in Asia-Pacific Area *

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Abstract

In this article, we examine the daily structure of stock price indices in the major stock markets in Asia-Pacific area using fractional integrated techniques. According to the long memory characteristics of the data, a particular version of Robinson's (1994) test is proposed for testing unit roots and non-stationarity in the financial data. The results show that the long memory behavior of the stock price indices in this region is different but quite similar.

Keywords: Unit root test, stock price index, long memory, stochastic process.

AMS Subject Classification: 62N03, 62-07.

§1. Introduction

The modeling of stock prices has attracted a great deal of attention because it provides useful information for the decision-makers of the governments, financial institutions, investment agents, etc. In the last two decades, many statistical models have been proposed, while there is still little consensus about appropriate modeling. The existing modeling of stock prices includes random walk (Fama, 1970), ARCH(p) model (Engle, 1982), GARCH(p,q) model (Bollerslev, 1986), volatility model (Taylor, 1986), long memory models (Grene and Fielitz, 1977; Lo, 1991; Cheung and Lai, 1995; Barkoulas and Baum, 1996; Crato, 1994), etc. The overall evidence suggests that stochastic long memory is absent in stock market returns but it may be a feature of some stock price indices. However, most of the historical work focuses on the stock markets of the developed countries and regions, especially in America and Europe. Therefore we are much more interested in the modeling of stock prices in relatively small markets in Asia-Pacific area. In this article, we consider eleven stock price indices in Asia-Pacific area, including that of Hong Kong, South Korea,

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Japan, Singapore, India, Malaysia, Taiwan, Pakistan and China, etc. And we prefer to employ the stochastic long memory processes to account for the correlation structure of financial series at long lags. In fact, the presence of long memory dynamics in asset prices would provide evidence against the weak form of market efficiency as it implies non-linear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics.

The technique we adopt in this article is Robinson's (1994) test which is the most efficient test when directed against the appropriate fractional alternatives. It possesses many distinguishing advantages compared with other procedures. It has a standard null limit distribution and it can be carried out without the estimation of the parameter a priori.

The outline of the article is as follows: Section 2 briefly presents the procedure for testing unit roots in raw time series. Section 3 is the empirical application to the daily stock market price indices in Asia-Pacific Area. Section 4 is the concluding comments.

§2. Robinson's (1994) Test

Now we briefly describe Robinson's test (1994) which is a Lagrange Multiplier test for testing unit roots and other fractionally hypotheses when the roots are located at any frequency on the interval $[0,\pi]$. The test is derived via the score principle and its asymptotic critical values obey the Chi-squared distribution. Let $(Y_t)_t$ be a stochastic process such that:

$$Y_t = \beta' Z_t + X_t, \tag{2.1}$$

where $(Z_t)_t$ is a $k\times 1$ observable vector, β an unknown $k\times 1$ vector and $(X_t)_t$ a process which follows equation

$$F(B)X_t = (I-B)^{d_0+\theta_0}\prod_{i=1}^{k-1}(I-2\nu_i B+B^2)^{d_i+\theta_i}(I+B)^{d_k+\theta_k}X_t = \varepsilon_t, \tag{2.2}$$

where B is the backshift operator. For $i=1,\ldots,k-1$, $\nu_i=\cos\lambda_i$, λ_i being any frequency between 0 and π. For $i=0,1,\ldots,k$, θ_i belongs to $[-1,1]$ and d_i is such that: $|d_i|<1/2$, implying thus that the spectral density is unbounded at λ_i. Moreover, $(\varepsilon_t)_t$ is an innovation process to be specified. The process described by equation (2.2) nests all the specific fractional integration processes generally used in applications. It is referred to as Seasonal/Cyclical Long Memory (SCLM henceforth) process. It has been first discussed by Robinson (1994) in order to test whether the data stemmed from a stationary or a non-stationary process, under uncorrelated and weak-correlated innovations $(\varepsilon_t)_t$.

Robinson (1994) works with the general model (2.2) for a fixed d and tests the assumption

$$H_0 : \theta = (\theta_0, \dots, \theta_k)' = 0,$$

against the alternative:

$$H_a : \theta \neq 0.$$

The test statistic is defined by:

$$\widetilde{R} = \frac{T}{\widetilde{\sigma}^4} \frac{\widetilde{a}^2}{\widetilde{A}}, \tag{2.3}$$

where T is the length of the raw time series and

$$\widetilde{\sigma}^2 = \frac{2\pi}{T} \sum_j^* I_{\widetilde{\varepsilon}}(\lambda_j).$$

$I_{\widetilde{\varepsilon}}(\lambda_j)$ is the periodogram of $\widetilde{\varepsilon}_t$ with $\widetilde{\varepsilon}_t = F(B)Y_t$, $F(B)$ being given in equation (2.2) under the null. Moreover, we get:

$$\widetilde{A} = \frac{2}{T} \sum_j^* \psi(\lambda_j) \cdot \psi(\lambda_j)',$$

and

$$\widetilde{a}^2 = \frac{-2\pi}{T} \sum_j^* \psi(\lambda_j) I_{\widetilde{\varepsilon}}(\lambda_j),$$

where $\sum_j^*$ is the sum over $\lambda_j = 2\pi j/T \in M = \{\lambda : -\pi < \lambda < \pi, \lambda \notin (\rho_l - \eta, \rho_l + \eta)\}$ such that ρ_l are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$, η is a given positive constant. Finally, we get:

$$\psi(\lambda_j) = (\psi_l(\lambda_j)),$$

with

$$\psi_l(\lambda_j) = \delta_{0l} \log \left| 2 \sin \frac{1}{2} \lambda_j \right| + \delta_{kl} \log \left(2 \cos \frac{1}{2} \lambda_j \right) + \sum_{i=1}^{k} \delta_{il} \log(|2(\cos \lambda_j - \cos \omega_i)|,$$

for $l = 0, 1, \dots, k$, where $\delta_{il} = 1$ if $i = l$ and 0 otherwise.

Under certain regularity conditions, Robinson (1994) established that:

$$\widetilde{R} \to_d \chi^2_{k+1},$$

where $k+1 = \dim(\theta)$. If χ^2_{k+1} represents the χ^2 distribution with $k+1$ degrees of freedom then $\chi^2_{k+1,\alpha}$ represents a quantile for a given level α. As soon as $\widetilde{R} > \chi^2_{k+1,\alpha}$, we reject H_0, with a risk α.

In particular, in model (2.2), if $d_i = 0$, and $\theta_i = 0$, $i = 1, \ldots, k$, we get the FI(d) (Fractionally Integrated) process if $(\varepsilon_t)_t$ is a white noise:

$$(I - B)^d X_t = \varepsilon_t, \tag{2.4}$$

proposed by Granger and Joyeux (1980) and Hosking (1981). If we assume that $(\varepsilon_t)_t$ follows a GARCH noise, we get the FIGARCH model (fractionally integrated and GARCH) (Baillie, Bollerslev and Mikkelsen, 1996). This class of models permits to take into account the existence of an infinite cycle, as well as the spectral density's typical shape of macroeconomics data, namely an explosion for the very low frequencies. This version of Robinson's (1994) test was used in empirical applications in Gil-Alana and Robinson (1997) and Gil-Alana (2000, 2002) and other versions of his test, based on seasonal and cyclical data, can be found in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001).

§3. Empirical Application

3.1 Data

The time series data analyzed in this section correspond to the log-transformation of the daily closing price of the stock market prices: Hang Seng index of Hongkong (1986.12.31–2010.3.22), KOSPI index of South Korea (2005.4.22–2008.11.7), Nikkei 225 index of Japan (1984.1.5–2010.3.19), STI index of Singapore (1987.12.28–2010.3.22), BSE30 index of India (1997.7.1–2010.3.22), KLSE index of Malaysia (1993.12.3–2010.3.22), TWII index of Taiwan (1997.7.2–2010.3.22), KSE-100 index of Pakistan (1997.7.2–2010.3.22), SSE composite index of China (1994.12.6–2010.3.22), SZSE component index of China (1991.4.3–2010.3.22), CSI 300 index of China (2002.1.4–2010.3.22). The data in this study is obtained on the Financial Research Database of RESSET. The programming language is R.

3.2 Estimation and Interpretation of Results

3.2.1 Estimation Method

In fact, under the null hypotheses, the Robinson's (1994) test can work as a parameter estimation method since under the null, the test chooses the best long memory parameter which corresponds to the greatest p-value of the Chi-squared test. We accept the null hypotheses if the p-value is greater than the significant level and we reject it if the p-value is smaller than or equal to the significance level. Thus, using the grid-search method, we can obtain the best estimation of the fractional parameter. In this sense, the test appears as a good method to test and to estimate the long memory parameters in the SCLM models.

From a practical point of view, before implementing a fractional process on real data, it is warmly recommended to carry out a statistical test to show evidence of persistence in the data. We can refer to Ferrara et al. (2010) for their Monte Carlo simulation results. For the processes with infinite cycle, when sample size is greater than 1000, we have the confidence of more than 95% to attain in mean the correct estimated value.

3.2.2 Modeling

Carrying out some statistical analysis, we find that the financial series that we study all exhibit long memory behavior with infinite cycle, for the evidence of slow decay in their autocorrelation function (ACF) and the existence of the explosion at the zero frequency in their spectrum, which indicate the fact that the impact of the shocks to the series is permanent. See Figure 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 for more details.

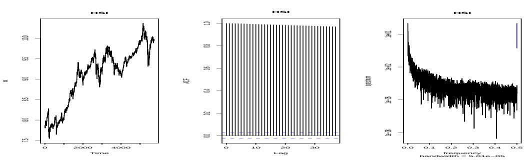

Figure 1 Trajectory, ACF and spectrum of log-transformation of HSI index

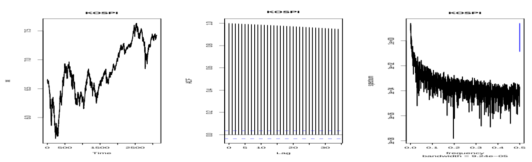

Figure 2 Trajectory, ACF and spectrum of log-transformation of KOSPI index

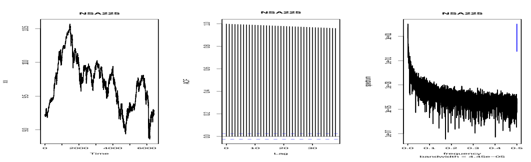

Figure 3 Trajectory, ACF and spectrum of log-transformation of NSA 225 index

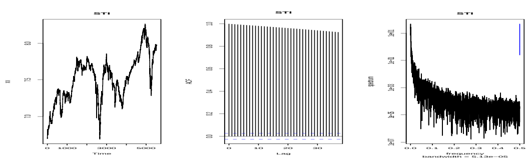

Figure 4 Trajectory, ACF and spectrum of log-transformation of STI index

According to the information provided by the ACF and spectrum, we employ the model in (2.1) and (2.4), denoting the time series by y_t, with $z_t = (1, t)'$, $t \geq 1$, i.e.

$$\begin{cases} y_t = a + bt + x_t; \\ (I - B)^d x_t = \varepsilon_t. \end{cases} \tag{3.1}$$

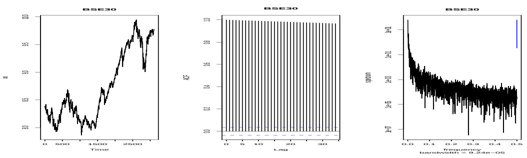

Figure 5 Trajectory, ACF and spectrum of log-transformation of BSE 30 index

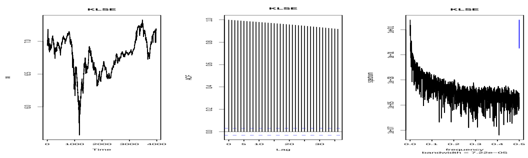

Figure 6 Trajectory, ACF and spectrum of log-transformation of KLSE index

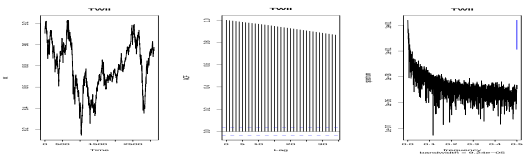

Figure 7 Trajectory, ACF and spectrum of log-transformation of TWII index

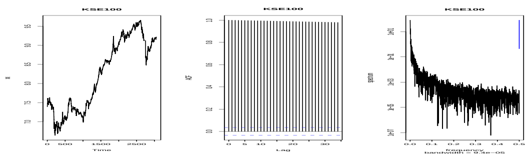

Figure 8 Trajectory, ACF and spectrum of log-transformation of KSE 100 index

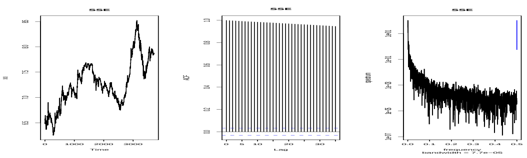

Figure 9 Trajectory, ACF and spectrum of log-transformation of SSE index

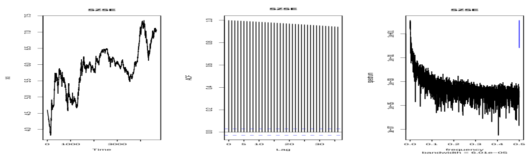

Figure 10 Trajectory, ACF and spectrum of log-transformation of SZSE index

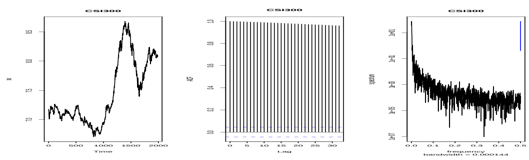

Figure 11 Trajectory, ACF and spectrum of log-transformation of CSI 300 index

Table 1 presents the estimation results of the parameters a, b, d and the corresponding p-value using Robinson's (1994) test. The numbers in the brackets are the standard deviations of the estimation. For the deterministic structure, significant positive intercepts can be observed together with the small slopes near 0. With respect to the long-run dynamics, the infinite cycles are common in these eleven markets with similar fractional differencing orders. More precisely, among these eleven stock indices, ten possess the fractional orders slightly greater than 1, which indicates the permanent shocks. One exception is the Nikkei 225 index, whose fractional order is slightly less than 1, which indicates that the series is long memory and mean reverting with the effect of shock disappearing in the long run, i.e. the shocks are transitory. The disturbance of the models are GARCH-type (generalized autoregressive conditional heteroskedasticity) noise. Here we do not describe too much the estimation of the parameters of noise, since our interest is to have an overall idea of the fractional orders in different markets. Therefore, it is indicated that the major stock markets in Asia-Pacific area possess the similar fractional differencing orders and the similar long memory behavior.

Table 1 Empirical estimation of the parameters in model in (2.1) and (2.4)

Index	# Observations	Estimation of a	Estimation of b	Estimation of d	p-value
	T	$\widehat{a}$	$\widehat{b}$	$\widehat{d}$	
Hang Seng	5733	8.055	3.667e-04	1.104	0.9748427
		(7.854e-03)	(2.373e-06)		
KOSPI	3107	6.130	4.300e-04	1.021	0.9965444
		(8.531e-03)	(4.760e-06)		
Nikkei 225	6423	1.001e+01	-9.766e-05	0.978	0.9898395
		(7.685e-03)	(2.072e-06)		
STI	5527	7.175	1.231e-04	1.053	0.9761241
		(6.341e-03)	(1.987e-06)		
BSE 30	3107	7.816	5.898e-04	1.024	0.9985739
		(1.099e-02)	(6.126e-06)		
KLSE	3986	6.674	5.351e-05	1.001	0.9866103
		(8.581e-03)	(30728e-06)		
TWII	3101	8.803	-8.417e-06	1.036	0.9809926
		(7.969e-03)	(4.450e-06)		
KSE 100	3054	6.776	9.533e-04	1.067	0.9956605
		(1.359e-02)	(7.706e-06)		
SSE	3710	6.721	3.005e-04	1.01	0.9905831
		(1.090e-02)	(5.088e-06)		
SZSE	4677	4.924	4.061e-04	1.052	0.9746710
		(1.264e-02)	(4.679e-06)		
CSI300	1987	6.772	6.919e-04	1.022	0.9892952
		(1.705e-02)	(1.486e-05)		

§4. Conclusion

In this paper, we have examined the degree of persistence over time for major price indices in Asia-Pacific area using a version of Robinson's (1994) test. The results show that there exists a common pattern of long range dependence for the stock market prices in these markets in Asia-Pacific Area. All series seem to present the non-stationarity and same type of long range of dependence. The market which is a little different from others is the Japanese market (Nikkei 225 index) with a smaller fractional differencing order. And two small markets in this region that we do not include in this paper are the Indonesia stock market (JKSE index) and Israel stock market (TA 100 index) since the existence of seasonality in their spectrum, which is one of our interest in the future study. Another possible extension is that we can make the comparison of the persistence behavior in the American and European stock markets.

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金融时间序列中单位根的检验: 在亚太地区主要股票市场中的应用

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本文运用分数差分的方法研究了亚太地区主要股票市场的日股票价格. 根据数据特征, 我们运用了Robinson (1994)年提出的检验统计量的一种特殊形式对金融数据的单位根和不稳定性进行检验. 结果证明, 该地区股票价格长记忆行为各不相同但十分相似.

关 键 词: 单位根检验, 股票价格指数, 长记忆, 随机过程.

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