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Some Results on Bivariate Compound Poisson Risk Model *

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Abstract

In this paper we consider a risk model with two correlated classes of insurance business. Asymptotic results for the deficit at ruin caused by different classes of insurance business are obtained. Explicit expression for the deficit at ruin caused by different classes of insurance business are given when the original claim size random variables are exponentially distributed. In addition we also give a brief discussion on the classical risk model perturbed by the Gamma process.

Keywords: Deficit at ruin, correlated aggregate claims, ruin probability, compound Poisson, Gamma process.

AMS Subject Classification: 91B30, 60G51.

§1. Introduction

In this paper we consider a risk model with two correlated classes of insurance business. Let $X_i^{(j)}$ $(j=1,2)$ be claim size random variables for the j-th class with common distribution function $F_j(x)$ and μ_j, $j=1,2$ be their means. Then the risk model generated from the two correlated classes of business is given by

$$\widetilde{S}(t)=u+ct-\sum_{i=1}^{K_1(t)}X_i^{(1)}-\sum_{i=1}^{K_2(t)}X_i^{(2)}, \tag{1.1}$$

where $K_j(t)$ is the claim number process for class j $(j=1,2)$. Assume that $\{X_i^{(1)}, i=1,2,\ldots,\}$ and $\{X_i^{(2)}, i=1,2,\ldots,\}$ are independent claim size random variables, and they are independent of claim number processes $K_j(t)$. The two claim number processes are correlated as follows

$$K_1(t)=N_1(t)+N_3(t) \qquad \text{and} \qquad K_2(t)=N_2(t)+N_3(t). \tag{1.2}$$

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Obviously the correlation in (1.1) is caused by the common component $N_3(t)$. In recent years, various kinds of correlations among claim amounts and claim numbers have been studied by many authors. Goovaerts and Dhaene (1996) gave a compound Poisson approximation for a portfolio of dependent risks; Ambagaspitiya (1999) considered two classes of correlated aggregate claims distributions; A more general correlated aggregate claims risk model has been studied by Wang and Yuen (2005), in their model, not only the continuous-time Poisson model with common shock was studied, but also the so-called thinning-dependence structure was investigated. For some recent references, one can see Yuen (2006), Cai (2006) and Zhang (2007) and Li (2007).

Yuen et al. (2002) also considered the same model (1.1) and derived ruin probabilities for the risk process of the sum of correlated aggregate claims with Erlang common shock in continuous time. However in our paper we will study the ruin probability and the deficit at ruin caused by the jump of $N_j(t)$ respectively. These are useful variables if the insurer wants to known the impact of different classes of claims caused by different classes of $N_j(t)$.

We assume that $N_1(t)$, $N_2(t)$, $N_3(t)$ are three independent Poisson processes with intensity λ_1, λ_2, λ_3. It can be shown that $S(t)$ becomes a compound Poisson process with parameter $\lambda=\lambda_1+\lambda_2+\lambda_3$ (see Yuen and Wang (2002)). In order to depict clearly, we can rewrite the model (1.1) as the following one

$$S(t)=u+ct-\Big(\sum_{i=1}^{N_1(t)}X_i^{(1)}+\sum_{i=1}^{N_2(t)}X_i^{(2)}+\sum_{i=1}^{N_3(t)}X_i^{(3)}\Big), \tag{1.3}$$

where $X_i^{(3)}=X_i^{(1)}+X_i^{(2)}$ and we denote $F_3(x)=F_1*F_2(x)$. It is obvious that $\widetilde{S}(t)$ and $S(t)$ have the same distribution. As usual, we assume $c>\lambda_1\mu_1+\lambda_2\mu_2+\lambda_3(\mu_1+\mu_2)$, that is the insurance company has the positive safety loading.

The paper is organized as follows. In Section 2, we consider the distributions of deficit at ruin caused by the jump of $N_j(t)$ $(j=1,2,3)$. Asymptotic results for these distributions are given. Section 3 is devoted to studying the deficit at ruin caused by the jump of $N_j(t)$ when the original claim size random variables are exponential. Explicit expressions for these distributions are obtained. The classic risk model perturbed by the Gamma process is studied in Section 4. Asymptotic results for the ruin probability due to the Gamma process and compound Poisson process are also given respectively in this section.

§2. The Distribution of the Deficit at Ruin

Let T_j be the ruin time caused by the jump of $N_j(t)$. What we are concerned with in this section is not the distribution of the deficit at ruin $G(u,y)=\mathsf{P}(T<\infty,|S(T)|>y)$

but the following three ones.

$$\begin{aligned} G_1(u,y) &= \mathsf{P}(T=T_1<\infty, |S(T)|>y), \\ G_2(u,y) &= \mathsf{P}(T=T_2<\infty, |S(T)|>y), \\ G_3(u,y) &= \mathsf{P}(T=T_3<\infty, |S(T)|>y). \end{aligned}$$

That is $G_i(u,y)$ $(j=1,2,3)$ are the deficit distribution that due to the arrival of claim $X^{(i)}$. Note that with probability λ_1/λ, λ_2/λ and λ_3/λ the first jump of $S(t)$ comes from $N_1(t)$, $N_2(t)$ and $N_3(t)$ respectively. Thus by conditioning on whether there is a claim in the small time interval $(0,t)$ and the type of the claim we obtain, for $i=1,2,3$,

$$\begin{aligned} G_i(u,y) &= (1-\lambda t)G_i(u+ct,y) + \sum_{j=1}^{3}\lambda_j \int_0^{u+ct} G_i(u+ct-x,y)\mathrm{d}F_j(x) \\ &\quad + \lambda_i \overline{F}_i(u+ct+y). \end{aligned}$$

Let $t\to 0$ gives

$$cG_i'(u,y) = \lambda G_i(u,y) - \sum_{j=1}^{3}\lambda_j \int_0^{u} G_i(u-x,y)\mathrm{d}F_j(x) - \lambda_i\overline{F}_i(u+y). \tag{2.1}$$

Note that

$$\psi_i(u) := \mathsf{P}(T=T_i<\infty) = G_i(u,0). \tag{2.2}$$

Therefore, let $y=0$ in (2.1), we have

$$c\psi_i'(u) = \lambda\psi_i(u) - \sum_{j=1}^{3}\lambda_j \int_0^{u} \psi_i(u-x)\mathrm{d}F_j(x) - \lambda_i\overline{F}_i(u). \tag{2.3}$$

By integrating with respect to u in (2.1), we obtain,

$$\begin{aligned} c(G_i(u,y)-G_i(0,y)) &= \lambda\int_0^u G_i(s,y)\mathrm{d}s - \lambda_i\int_0^u \overline{F}_i(s+y)\mathrm{d}s \\ &\quad - \sum_{j=1}^{3}\lambda_j \int_0^u\int_0^s G_i(s-x,y)\mathrm{d}F_j(x)\mathrm{d}s. \end{aligned}$$

From this we can verify that

$$cG_i(u,y) = cG_i(0,y) - \lambda_i\int_0^u \overline{F}_i(s+y)\mathrm{d}s + \sum_{j=1}^{3}\lambda_j \int_0^u G_i(u-x,y)\overline{F}_j(x)\mathrm{d}x. \tag{2.4}$$

Letting $u\to\infty$ yields

$$cG_i(0,y) = \lambda_i\int_0^\infty \overline{F}_i(s+y)\mathrm{d}s, \tag{2.5}$$

which in turn implies

$$cG_i(u,y) = \lambda_i\int_u^\infty \overline{F}_i(s+y)\mathrm{d}s + \sum_{j=1}^{3}\lambda_j \int_0^u G_i(u-x,y)\overline{F}_j(x)\mathrm{d}x, \tag{2.6}$$

which is a defective renewal equation. By renewal theorem, we can get the following theorem.

Theorem 2.1 Let R be a constant such that

$$\int_0^\infty e^{Rx}\sum_{j=1}^3 \lambda_j \overline{F}_j(x)\mathrm{d}x = c. \tag{2.7}$$

Then, as $u\to\infty$,

$$G_i(u,y)\sim \frac{\lambda_i\displaystyle\int_0^\infty (e^{Rx}-1)\overline{F}_i(x+y)\mathrm{d}x}{\displaystyle\sum_{j=1}^3 \lambda_j \mathsf{E}X^{(j)}e^{RX^{(j)}}-c}e^{-Ru}, \tag{2.8}$$

where $X^{(j)}$ is a generic random variable which is the same distributed as $X_i^{(j)}$.

Proof Multiply both side of (2.6) by e^{Ru} to obtain a renewal equation

$$ce^{Ru}G_i(u,y)=\lambda_i e^{Ru}\int_u^\infty \overline{F}_i(s+y)\mathrm{d}s+\int_0^u G_i(u-x,y)\sum_{j=1}^3\lambda_j e^{Ru}\overline{F}_j(x)\mathrm{d}x. \tag{2.9}$$

By renewal theorem we have

$$\begin{aligned}\lim_{u\to\infty} e^{Ru}G_(u,y) &= \frac{\lambda_i\displaystyle\int_0^\infty\int_u^\infty \overline{F}_i(x+y)\mathrm{d}xe^{Ru}\mathrm{d}u}{\displaystyle\int_0^\infty ye^{Ry}\sum_{j=1}^3\lambda_j\overline{F}_j(y)\mathrm{d}y}\\ &= \frac{\lambda_i\displaystyle\int_0^\infty (e^{Rx}-1)\overline{F}_i(x+y)\mathrm{d}x}{\displaystyle\sum_{j=1}^3 \lambda_j \mathsf{E}X^{(j)}e^{RX^{(j)}}-c}. \qquad \square\end{aligned} \tag{2.10}$$

Let $y=0$ in Theorem 2.1, we get the following corollary.

Corollary 2.1 Let R be the constant defined in the Theorem 2.1, then, as $u\to\infty$,

$$\psi_i(u)\sim \frac{\lambda_i\displaystyle\int_0^\infty e^{Rx}\overline{F}_i(x)\mathrm{d}x-\lambda_i\mu_i}{\displaystyle\sum_{j=1}^3 \lambda_j \mathsf{E}X^{(j)}e^{RX^{(j)}}-c}e^{-Ru}. \tag{2.11}$$

§3. Exponential Claims

In this section, we will consider the case of exponential claims. Suppose $X_i^{(1)}$ and $X_i^{(2)}$ are exponentially distributed with means μ_1 and μ_2. Then it is easy to get the distribution of $X_i^{(3)}=X_i^{(1)}+X_i^{(2)}$,

$$F_3(x)=1-\frac{\mu_2}{\mu_2-\mu_1}e^{-x/\mu_2}-\frac{\mu_1}{\mu_1-\mu_2}e^{-x/\mu_1}. \tag{3.1}$$

By equation (2.1), it is seen that

$$\begin{aligned} cG_i'(u,y) &= \lambda G_i(u,y) - K_1\int_0^u G_i(x,y)e^{-(u-x)/\mu_1}\mathrm{d}x \\ &\quad - K_2\int_0^u G_i(x,y)e^{-(u-x)/\mu_2}\mathrm{d}x - \lambda_i\overline{F}_i(u+y), \end{aligned} \tag{3.2}$$

where

$$K_1 = \frac{\lambda_1}{\mu_1} - \frac{\lambda_3}{\mu_2-\mu_1}, \qquad K_2 = \frac{\lambda_2}{\mu_2} - \frac{\lambda_3}{\mu_1-\mu_2}.$$

In order to get rid of the integral term, differentiate the above equation twice to get

$$cG_i'''(u,y) + \Big(\frac{c}{\mu_1}+\frac{c}{\mu_2}-\lambda\Big)G_i''(u,y) + \Big(\frac{c}{\mu_1\mu_2} - \frac{\lambda_2+\lambda_3}{\mu_1} - \frac{\lambda_1+\lambda_3}{\mu_2}\Big)G_i'(u,y) = 0. \tag{3.3}$$

Its characteristic equation

$$cx^2 + \Big(\frac{c}{\mu_1}+\frac{c}{\mu_2}-\lambda\Big)x + \Big(\frac{c}{\mu_1\mu_2} - \frac{\lambda_2+\lambda_3}{\mu_1} - \frac{\lambda_1+\lambda_3}{\mu_2}\Big) = 0 \tag{3.4}$$

has two negative roots say R_1, R_2. Since $\lim\limits_{u\to\infty} G_i(u,y) = 0$, the general solutions of equation (3.3) takes the form

$$G_i(u,y) = g_{i1}(y)e^{R_1u} + g_{i2}e^{R_2u}. \tag{3.5}$$

To determine $g_{i1}(y), g_{i2}(y)$, it is suffice to substitute (3.5) in (3.2). We thus get

$$\begin{aligned} &g_{i1}(y)h(R_1)e^{R_1u} + g_{i1}(y)h(R_2)e^{R_2u} \\ = &\Big[\frac{K_1}{R_1+1/\mu_1}g_{i1}(y) + \frac{K_1}{R_2+1/\mu_1}g_{i2}(y)\Big]e^{-u/\mu_1} \\ &+\Big[\frac{K_2}{R_1+1/\mu_2}g_{i1}(y) + \frac{K_2}{R_2+1/\mu_2}g_{i2}(y)\Big]e^{-u/\mu_2} - \lambda_i\overline{F}_i(u+y), \end{aligned}$$

where

$$h(x) = cx + \frac{K_1}{x+1/\mu_1} + \frac{K_2}{x+1/\mu_2} - \lambda.$$

Since R_1, R_2 are the roots of equation (3.4), it is easy to verify that $h(R_1) = h(R_2) = 0$. Therefore we get following equations for $g_{i1}(y)$ and $g_{i2}(y)$.

$$\begin{aligned} &\Big[\frac{K_1}{R_1+1/\mu_1}g_{i1}(y) + \frac{K_1}{R_2+1/\mu_1}g_{i2}(y)\Big]e^{-u/\mu_1} \\ &+\Big[\frac{K_2}{R_1+1/\mu_2}g_{i1}(y) + \frac{K_2}{R_2+1/\mu_2}g_{i2}(y)\Big]e^{-u/\mu_2} = \lambda_i\overline{F}_i(u+y). \end{aligned} \tag{3.6}$$

Solve above equations yield the following theorem.

Theorem 3.1 Let K_1, K_2 be defined as above and

$$h_i(R_1,R_2)=\frac{(R_1+1/\mu_1)(R_1+1/\mu_2)(R_2+1/\mu_1)(R_2+1/\mu_2)}{K_i(R_1-R_2)(1/\mu_1-1/\mu_2)}.$$

Then for $i=1,2,3$,

$$G_i(u,y)=g_{i1}(y)e^{R_1u}+g_{i2}(y)e^{R_2u}, \tag{3.7}$$

where $g_{ij}(y)$ are given by

$$g_{ij}(y)=(-1)^{j+1}\lambda_i e^{-y/\mu_i}\frac{h_i(R_1,R_2)}{\prod\limits_{k\neq j,l\neq i}(R_k+1/\mu_l)}, \qquad i,j=1,2, \tag{3.8}$$

and

$$g_{3j}(y)=\frac{\lambda_3}{\mu_1-\mu_2}\Big[\frac{\mu_1}{\lambda_1}g_{1j}(y)+\frac{\mu_2}{\lambda_2}g_{2j}(y)\Big], \qquad j=1,2. \tag{3.9}$$

Proof We only prove the case of $i=1$, the other cases can be proved similarly. In the case of $i=1$, $\overline{F}_1(u+y)=e^{-(u+y)/\mu_1}$. Substituting this into equation (3.6) yields that

$$\begin{aligned}&\Big[\frac{K_1}{R_1+1/\mu_1}g_{11}(y)+\frac{K_1}{R_2+1/\mu_1}g_{12}(y)-\lambda_1e^{y/\mu_1}\Big]e^{-u/\mu_1}\\&+\Big[\frac{K_2}{R_1+1/\mu_2}g_{11}(y)+\frac{K_2}{R_2+1/\mu_2}g_{12}(y)\Big]e^{-u/\mu_2}=0.\end{aligned} \tag{3.10}$$

Since above equation is satisfied for all $u>0$, thus we must have

$$\begin{cases}\dfrac{K_1}{R_1+1/\mu_1}g_{11}(y)+\dfrac{K_1}{R_2+1/\mu_1}g_{12}(y)=\lambda_1e^{y/\mu_1}\\ \dfrac{K_2}{R_1+1/\mu_2}g_{11}(y)+\dfrac{K_2}{R_2+1/\mu_2}g_{12}(y)=0.\end{cases} \tag{3.11}$$

Solve the above equations we get

$$\begin{aligned}g_{11}(y)&=\lambda_1e^{-y/\mu_1}\frac{(R_1+1/\mu_1)(R_1+1/\mu_2)(R_2+1/\mu_1)}{K_1(R_1-R_2)(1/\mu_1-1/\mu_2)},\\ g_{12}(y)&=-\lambda_1e^{-y/\mu_1}\frac{(R_1+1/\mu_1)(R_2+1/\mu_1)(R_2+1/\mu_2)}{K_1(R_1-R_2)(1/\mu_1-1/\mu_2)}.\end{aligned} \qquad \square$$

From equation (2.2), we have the following corollary.

Corollary 3.1 Let K_1, K_2 be defined as above and

$$h_i(R_1,R_2)=\frac{(R_1+1/\mu_1)(R_1+1/\mu_2)(R_2+1/\mu_1)(R_2+1/\mu_2)}{K_i(R_1-R_2)(1/\mu_1-1/\mu_2)}.$$

Then for $i=1,2,3$,

$$\psi_i(u)=g_{i1}e^{R_1u}+g_{i2}e^{R_2u}, \tag{3.12}$$

where g_{ij} are given by

$$g_{ij} = (-1)^{j+1} \frac{\lambda_i h_i(R_1, R_2)}{\prod\limits_{k\neq j, l\neq i} (R_k + 1/\mu_l)}, \qquad i, j = 1, 2, \tag{3.13}$$

and

$$g_{3j} = \frac{\lambda_3}{\mu_1 - \mu_2} \Big[\frac{\mu_1}{\lambda_1} g_{1j} + \frac{\mu_2}{\lambda_2} g_{2j}\Big], \qquad j = 1, 2. \tag{3.14}$$

§4. Other Extensions

In this section, we will give a brief discussion on the classical risk model perturbed by the Gamma process. First we give a brief introduction on the Gamma process. The Gamma process is a subordinator with Laplace exponent given by

$$l(s) = -a \ln\Big(1 + \frac{s}{b}\Big) = \int_0^\infty (e^{-sx} - 1) a x^{-1} e^{-bx} \mathrm{d}x, \qquad s > 0, \tag{4.1}$$

where $a, b > 0$. Easy to see that the Lévy measure of gamma process is given by $\nu(\mathrm{d}x) = ax^{-1}e^{-bx}\mathrm{d}x$, $x > 0$ and the mean of this process at time one is $\mu_G = a/b$. By Dufresne et al. (1991), we know that Gamma process has nonnegative increment and the number of claims in any time interval is infinite with probability one. However, $G(t)$ is finite, because the majority of the claims are very small in some sense.

Now we give the risk model perturbed by the Gamma process,

$$U(t) = u + ct - \sum_{i=1}^{N(t)} X_i - G(t), \tag{4.2}$$

where $u \geq 0$, $c = (1+\theta)(\lambda\mu + \mathsf{E}G(1))$, $N(t)$ is a Poisson claim number process with intensity λ and is independent of i.i.d. claim size random variables X_i with distribution $P(x)$ and mean μ, $G(t)$ is a Gamma process and is independent of $N(t)$ and X_i.

Let $Y(t) \,\widehat{=}\, U(t) - u$, note that $Y(t)$ has no positive jumps, it is a spectrally negative Lévy process with an initial value of zero. Easy to see that $\mathsf{E}e^{sY_t} = e^{t\phi(s)}$, $s \geq 0$, where $\phi(s) = cs - \lambda(\widehat{F}(s) - 1) + l(s)$. We now give a useful result in risk theory which is due to Zolotarev (1964).

Lemma 4.1 Let $\{Y(t) : t \geq 0\}$ is a spectrally negative Lévy process with initial value of zero, and $\gamma = \mathsf{E}Y(1) \geq 0$. Define

$$\psi(x) = \mathsf{P}\Big(\inf_{t\geq 0} Y(t) < -x\Big) \qquad \text{for } x \geq 0.$$

Then the function $\psi(x)$ can be defined from the Laplace exponent $\phi(\lambda)$ by the equation

$$s\int_0^\infty e^{-sx}\psi(x)\mathrm{d}x = 1 - \frac{\gamma s}{\phi(s)}, \qquad s \geq 0.$$

Following the method used in Furrer (1998), we obtain

Theorem 4.1 Consider the risk process (4.2), the ruin probability $\psi(u)$ satisfies

$$1-\psi(u)=(1-\rho)\sum_{n=0}^{\infty}\rho^n H^{*n}(u), \tag{4.3}$$

where $H(x)=pP_I(x)+qF(x)$,

$$p=\frac{\lambda\mu}{\lambda\mu+\mathsf{E}G(1)},\qquad \rho=\frac{1}{1+\theta},$$
$$P_I(x)=\frac{1}{\mu}\int_0^x(1-P(y))\mathrm{d}y,\qquad F(x)=\int_0^x\int_y^\infty\frac{b}{z}e^{-bz}\mathrm{d}z\mathrm{d}y$$

is the exponential integral distribution function with density function

$$Q(x)=\int_x^\infty\frac{b}{z}e^{-bz}\mathrm{d}z.$$

Proof $\{R(t)-u:t\geq 0\}$ is a spectrally negative Lévy process with an initial value of zero. Applying Lemma 4.1, we know that the ruin probability $\psi(u)$ satisfies

$$s\int_0^\infty e^{-sx}\psi(x)\mathrm{d}x=1-\frac{\gamma s}{\phi(s)},\qquad s\geq 0, \tag{4.4}$$

where $\gamma=\mathsf{E}Y(1)=c-\lambda\mu-\mathsf{E}G(1)$, $\phi(s)=cs-\lambda(\widehat{F}(s)-1)+l(s)$. Using a similar method to that in Furrer (1998), we have

$$\begin{aligned}
\int_0^\infty e^{-su}\mathrm{d}(1-\psi(u)) &= 1-s\int_0^\infty e^{-su}\psi(u)\mathrm{d}u\\
&= \frac{\gamma s}{cs-\lambda(1-\widehat{P}(s))+l(s)}\\
&= \frac{\gamma}{c}\frac{c}{cs+\dfrac{\lambda(1-\widehat{P}(s))}{s}+\dfrac{l(s)}{s}}\\
&= (1-\rho)\frac{1}{1-\rho\Big[\dfrac{\lambda\mu}{\lambda\mu+\mathsf{E}G(1)}\cdot\widehat{P}_I(s)-\dfrac{\mathsf{E}G(1)}{\lambda\mu+\mathsf{E}G(1)}\cdot\dfrac{1}{\mathsf{E}G(1)}\cdot\dfrac{l(s)}{s}\Big]}\\
&= (1-\rho)\frac{1}{1-\rho[p\widehat{P}_I(s)+q\widehat{F}(s)]}\\
&= (1-\rho)\sum_{n=0}^{\infty}(\rho\widehat{H}(s))^n.
\end{aligned}\tag{4.5}$$

Inverting the expression (4.5) yields (4.3). Thus we complete the proof of the theorem. $\square$

What we will discuss next is the ruin probabilities caused by compound Poisson process and Gamma process respectively. To solve this problem we can refer to the topic discussed in Dufresne et al. (1991). Noting that Gamma process is the limit of compound Poisson processes. In order to explain clearly we replace $N(t)$ by $N_1(t)$ and the correspond parameter is λ_1. Thus we can get the corresponding results for the model (4.2) from the following risk process,

$$U(t) = u + ct - \sum_{i=1}^{N_1(t)} X_i - \sum_{i=1}^{N_2(t)} Y_i, \qquad t \geq 0 \tag{4.6}$$

in which we substitute $G(t)$ by $\sum\limits_{i=1}^{N_2(t)} Y_i$, where $N_1(t)$ and $N_2(t)$ are two independent Poisson processes with intensity λ_1, λ_2, claims size X_i, Y_i, are independent with distributions $P_1(x)$, $P_2(x)$ and means μ_1, μ_2 respectively. Let

$$\begin{aligned} \psi_1(u) &= \mathsf{P}(T < \infty,\ T \ \text{ is some jump time of } \ N_1(t)), \\ \psi_2(u) &= \mathsf{P}(T < \infty,\ T \ \text{ is some jump time of } \ N_2(t)). \end{aligned}$$

By a similar argument as in Section 2 we can obtain the following theorem for the model (4.6).

Theorem 4.2 Let R be a constant such that

$$\int_0^{\infty} e^{Rx} \sum_{j=1}^{2} \lambda_j \overline{P}_j(x) \mathrm{d}x = c. \tag{4.7}$$

Then, as $u \to \infty$,

$$\lim_{u \to \infty} e^{Ru} \psi_i(u) = \frac{\dfrac{\lambda_i}{c} \displaystyle\int_0^{\infty} e^{Rx} \int_x^{\infty} (1 - P_i(y)) \mathrm{d}y \mathrm{d}x}{\dfrac{1}{R}\Big(h'(R)\dfrac{\lambda_1 + \lambda_2}{c} - 1\Big)},$$

where

$$h(r) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \int_0^{\infty} e^{rz} \mathrm{d}P_1(z) + \frac{\lambda_2}{\lambda_1 + \lambda_2} \int_0^{\infty} e^{rz} \mathrm{d}P_2(z) - 1.$$

The proof of this theorem is similar as that of Corollary 2.1.

Remark 1 Note that

$$\int_0^{\infty} e^{Rx} \lambda_i \overline{P}_i(x) \mathrm{d}x = \frac{1}{R} \int_0^{\infty} (e^{Rx} - 1) \lambda_i P_i(\mathrm{d}x) = \frac{\lambda_i(\widehat{P}_i(-R) - 1)}{R}. \tag{4.8}$$

Then equation (4.7) is equivalent to the following equation

$$\lambda_1(\widehat{P}_1(-R) - 1)) + \lambda_2(\widehat{P}_2(-R) - 1)) = cR, \tag{4.9}$$

which is often called the Lunderberg equation.

Theorem 4.3 Let R be the root of equation

$$\lambda(\widehat{P}(-r)-1)+a\ln\frac{b}{b-r}=cr \tag{4.10}$$

and denote by $\psi_1(u)$ and $\psi_2(u)$ the ruin probabilities of risk model (4.2) caused by compound Poisson process and Gamma process respectively. Then,

$$\lim_{u\to\infty}e^{Ru}\psi_1(u)=\frac{\lambda\displaystyle\int_0^\infty e^{Rx}\int_x^\infty(1-P_1(y))\mathrm{d}y\mathrm{d}x}{\lambda\displaystyle\int_0^\infty xe^{Rx}(1-P_1(x))\mathrm{d}x+\frac{1}{R}\Big(\frac{a}{b-R}-\frac{a}{R}\ln\frac{b}{b-R}\Big)}, \tag{4.11}$$

$$\lim_{u\to\infty}e^{Ru}\psi_2(u)=\frac{\dfrac{1}{R}\Big(\dfrac{a}{R}\ln\dfrac{b}{b-R}-\dfrac{a}{b}\Big)}{\lambda\displaystyle\int_0^\infty xe^{Rx}(1-P(x))\mathrm{d}x+\frac{1}{R}\Big(\frac{a}{b-R}-\frac{a}{R}\ln\frac{b}{b-R}\Big)}. \tag{4.12}$$

Proof Since

$$\theta(r)=\lambda(\widehat{P}_1(-r)-1)+a\ln\frac{b}{b-r}-cr$$

is convex, it is easy to see that equation (4.10) indeed has a unique positive root R in $(0,b)$.

From the method used in Dufresne et al. (1991), we only need to substitute $\nu(\mathrm{d}y)$ for $\lambda_2P_2(\mathrm{d}y)$ to obtain the corresponding results for the model (4.2). Note that equation (4.7) and (4.9) are equivalent, then by Theorem 4.2 we complete the proof of the theorem. □

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二元复合Poisson风险模型的几个结果

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本文研究具有相依关系的一类风险模型. 得到了由不同类别的索赔产生的破产时赤字分布的渐近结果以及指数索赔下的精确结果. 同时研究了带伽玛过程干扰的古典风险过程.

关 键 词: 破产时赤字, 相依索赔, 破产概率, 复合Poisson过程, Gamma过程.

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