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# Usual Multivariate Stochastic Order on the Proportional Reversed Hazard Rates Model *

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**Abstract**

Let $X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i = 1, 2, \ldots, n$, be all independent PRHR variables. Firstly, we show that $(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$ implies $(Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}) \geq_{\rm st} (X_{1:n}, X_{2:n}, \ldots, X_{n:n})$. Secondly, we consider the comparison of convolutions of independent heterogeneous PRHR variables with respect to the usual stochastic ordering. Suppose $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$ and $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$, we prove that $(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$ implies $\sum\limits_{r=k}^{n} Y_r \geq_{\rm st} \sum\limits_{r=k}^{n} X_r$, for all $1 \leq k \leq n$. The results established here strengthen some of the results known in the literature.

**Keywords:** Order statistics, Beta distribution, usual multivariate stochastic order, proportional hazard rates model, proportional reversed hazard rates model.

**AMS Subject Classification:** 60E15, 62N05, 62G30, 62D05.

## §1. Introduction

Order statistics play an important role in statistical inference, life testings, reliability theory and many other areas. Let $X_1, X_2, \ldots, X_n$ be $n$ random variables and let $X_{i:n}$ denotes their $i$th order statistic, $i = 1, 2, \ldots, n$. In reliability theory, the $k$th order statistic $X_{k:n}$ corresponds to the lifetime of a $(n - k + 1)$-out-of-$n$ system. Parallel and series systems are the building blocks of more complex coherent systems, wherein the lifetime of a series system corresponds to the smallest order statistic $X_{1:n}$ and the lifetime of a parallel system corresponds to the largest order statistic $X_{n:n}$. Many authors have studied stochastic comparisons of lifetimes of series and parallel systems, for example, see Khaledi and Kochar (2000, 2006), Dykstra et al. (1997), Fang and Zhang (2011, 2012), and references cited therein.

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Proportional reversed hazard rates (PRHR) model and proportional hazard rates (PHR) model are two important models in reliability theory. Let $X_1, X_2, \ldots, X_n$ denote the lifetimes of $n$ components of a system with distribution functions $F_1, F_2, \ldots, F_n$, respectively. If there exist positive constants $\alpha_1, \alpha_2, \ldots, \alpha_n$ and a distribution function $F(x)$ with corresponding density function $f(x)$ and survival function $\overline{F}(x)$ such that

$$F_i(x) = F^{\alpha_i}(x), \qquad i = 1, 2, \ldots, n,$$

we say that random variables $X_1, X_2, \ldots, X_n$ follow the PRHR model. Meanwhile, $F(x)$ and $\widetilde{r}(x) = f(x)/F(x)$ are called the baseline distribution and baseline reversed hazard functions, respectively, and $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the proportional reversed hazard rate parameters. We all know that generalized exponential distribution and exponentiated Weibull distribution are special cases of this model. This model have discussed in Chapter 7 of Marshall and Olkin (2007). If there exist positive constants $\lambda_1, \lambda_2, \ldots, \lambda_n$ such that

$$\overline{F}_i(x) = [\overline{F}(x)]^{\lambda_i}, \qquad i = 1, 2, \ldots, n, \tag{1.1}$$

we say that random variables $X_1, X_2, \ldots, X_n$ follow the PHR model. In this case, $r(x) = f(x)/\overline{F}(x)$ is the hazard rate corresponding to the baseline distribution $F(x)$, then the hazard rate of $X_i$ is $\lambda_i r(x)$, $i = 1, 2, \ldots, n$. So, (1.1) can be expressed as

$$\overline{F}_i(x) = \mathrm{e}^{-\lambda_i R(x)}, \qquad i = 1, 2, \ldots, n,$$

where $R(x) = \int_0^x r(t)\mathrm{d}t$, is the cumulative hazard rate of $X$. For example, exponential random variables with hazard rates $\lambda_1, \lambda_2, \ldots, \lambda_n$ is follows the PHR model with $R(x) = x$. Many interesting results have been obtained about the PHR model in the literature. Pledger and Proschan (1971) have proved that if $(X_1, X_2, \ldots, X_n)$ and $(Y_1, Y_2, \ldots, Y_n)$ have proportional hazard rate vectors $(\mu_1, \mu_2, \ldots, \mu_n)$ and $(\nu_1, \nu_2, \ldots, \nu_n)$, then

$$(\mu_1, \mu_2, \ldots, \mu_n) \succeq_{\mathrm{m}} (\nu_1, \nu_2, \ldots, \nu_n),$$

implies that, $X_{i:n} \geq_{\mathrm{st}} Y_{i:n}$, $i = 1, 2, \ldots, n$. Subsequently, Proschan and Sethuraman (1976) generalized the above result from componentwise stochastic ordering to multivariate stochastic ordering, that is, $(X_{1:n}, X_{2:n}, \ldots, X_{n:n}) \geq_{\mathrm{st}} (Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n})$.

Recently, Balakrishnan et al. (2014) studied stochastic comparison of vectors of order statistics in the PRHR model with respect to usual multivariate stochastic order. Let $X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i = 1, 2$, be all independent PRHR variables, then

$$(\alpha_1, \alpha_2) \succeq_{\mathrm{m}} (\gamma_1, \gamma_2)$$

implies

$$(Y_{1:2}, Y_{2:2}) \ge_{\rm st} (X_{1:2}, X_{2:2}).$$

The above result only consider stochastic comparison of vectors of order statistics between two PRHR variables. In this paper, we obtain some new results about stochastic comparison of vector of order statistics in the PRHR model. Specifically, let $X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i = 1, 2, \ldots, n$, be all independent variables. We will prove that

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$$

implies

$$(Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}) \ge_{\rm st} (X_{1:n}, X_{2:n}, \ldots, X_{n:n}).$$

Furthermore, suppose

$$\alpha_1 \ge \alpha_2 \ge \cdots \ge \alpha_n \qquad \text{and} \qquad \gamma_1 \ge \gamma_2 \ge \cdots \ge \gamma_n,$$

we show that

$$(\alpha_1, \alpha_2, \cdots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$$

implies

$$\sum_{r=k}^{n} Y_r \ge_{\rm st} \sum_{r=k}^{n} X_r,$$

for all $1 \le k \le n$.

## §2. Preliminaries and Main Results

First, we give the definition of the usual multivariate stochastic ordering.

**Definition 2.1** Let $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ and $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_n)$ be two random vectors, $\boldsymbol{X}$ is said to be larger than $\boldsymbol{Y}$ in the usual stochastic order if $f(\boldsymbol{X}) \ge f(\boldsymbol{Y})$ for all increasing function $f(x_1, x_2, \ldots, x_n)$ (i.e. increasing in each coordinate); in symbols, $\boldsymbol{X} \ge_{\rm st} \boldsymbol{Y}$.

Majorization is a very interesting topic in statistics, which is a pre-ordering on vectors by sorting all components in decreasing order.

**Definition 2.2** Let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\boldsymbol{\lambda}^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_n^*)$ denote two $n$-dimensional real vectors. Let $\lambda_{[1]} \ge \lambda_{[2]} \ge \cdots \ge \lambda_{[n]}$, $\lambda_{[1]}^* \ge \lambda_{[2]}^* \ge \cdots \ge \lambda_{[n]}^*$ be their ordered components. $\boldsymbol{\lambda}^*$ is said to be majorized by $\boldsymbol{\lambda}$, in symbols $\boldsymbol{\lambda} \succeq_{\rm m} \boldsymbol{\lambda}^*$, if $\sum\limits_{i=1}^{k} \lambda_{[i]} \ge \sum\limits_{i=1}^{k} \lambda_{[i]}^*$; for $k = 1, 2, \ldots, n-1$, and $\sum\limits_{i=1}^{n} \lambda_i = \sum\limits_{i=1}^{n} \lambda_i^*$.

The following four results will be needed to prove our main results.

**Lemma 2.1** (Shaked and Shanthikumar, 2007; p.273) Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be two $n$-dimensional random vectors. If $\boldsymbol{X} \geq_{\rm st} \boldsymbol{Y}$ and $\boldsymbol{h}: \Re^n \to \Re^k$ is any $k$-dimensional increasing (decreasing) function, then for any positive integer $k$, the $k$-dimensional vectors $\boldsymbol{h}(\boldsymbol{X})$ and $\boldsymbol{h}(\boldsymbol{Y})$ satisfy the multivariate ordering $\boldsymbol{h}(\boldsymbol{X}) \geq_{\rm st} (\leq_{\rm st})\, \boldsymbol{h}(\boldsymbol{X})$.

**Lemma 2.2** (Proschan and Sethuraman, 1976) Let $X_i \sim [\overline{F}(x)]^{\mu_i}$, $Y_i \sim [\overline{F}(x)]^{\nu_i}$, $i = 1, 2, \ldots, n$, be all independent. Then

$$(\mu_1, \mu_2, \ldots, \mu_n) \succeq_{\rm m} (\nu_1, \nu_2, \ldots, \nu_n) \Longrightarrow (X_{1:n}, X_{2:n}, \ldots, X_{n:n}) \geq_{\rm st} (Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}).$$

**Lemma 2.3** Let $X_1, X_2, \ldots, X_n$ be independent random variables with $X_i \sim \text{beta}\,(1, \alpha_i)$, $i = 1, 2, \ldots, n$. Then $X_1, X_2, \ldots, X_n$ follow the PHR model.

**Proof** From $X_i \sim \text{beta}\,(1, \alpha_i)$, $i = 1, 2, \ldots, n$, we can obtain, for $\forall\, x \in (0, 1)$,

$$\overline{F}_i(x) = (1 - x)^{\alpha_i}.$$

So, the result is hold according to the definition of PHR model. $\square$

From the definition of beta distribution, we have the following result.

**Lemma 2.4** Let $X_1, X_2, \ldots, X_n$ be independent random variables with $X_i \sim \text{beta}\,(\alpha_i, 1)$, $i = 1, 2, \ldots, n$. Then $1 - X_i \sim \text{beta}\,(1, \alpha_i)$, $i = 1, 2, \ldots, n$.

Now, we give a sufficient condition for the usual multivariate stochastic ordering between vectors of order statistics from two sets of heterogeneous beta random variables.

**Theorem 2.1** Let $X_i \sim \text{beta}\,(\alpha_i, 1)$, $Y_i \sim \text{beta}\,(\gamma_i, 1)$, $i = 1, 2, \ldots, n$, be all independent random variables. If

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n),$$

then

$$(Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}) \geq_{\rm st} (X_{1:n}, X_{2:n}, \ldots, X_{n:n}).$$

**Proof** Firstly, from Lemma 2.4, we get

$$1 - X_i \sim \text{beta}\,(1, \alpha_i), \qquad 1 - Y_i \sim \text{beta}\,(1, \gamma_i), \qquad i = 1, 2, \ldots, n.$$

So, from Lemmas 2.2 and 2.3,

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$$

implies

$$(1 - X_{1:n}, 1 - X_{2:n}, \ldots, 1 - X_{n:n}) \geq_{\rm st} (1 - Y_{1:n}, 1 - Y_{2:n}, \ldots, 1 - Y_{n:n}). \tag{2.1}$$

Now, let us consider the function

$$\boldsymbol{h}(x_1, x_2, \ldots, x_n) = (1 - x_1, 1 - x_2, \ldots, 1 - x_n),$$

where $0 < x_i < 1$, $i = 1, 2, \ldots, n$. Obviously, the function $\boldsymbol{h}(x_1, x_2, \ldots, x_n)$ is a $n$-dimensional decreasing function. Therefore, from Lemma 2.1 and the ordering in (2.1), we have

$$\begin{aligned}(Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}) &= \boldsymbol{h}(1 - Y_{1:n}, 1 - Y_{2:n}, \ldots, 1 - Y_{n:n}) \\ &\geq_{\rm st} \boldsymbol{h}(1 - X_{1:n}, 1 - X_{2:n}, \ldots, 1 - X_{n:n}) = (X_{1:n}, X_{2:n}, \ldots, X_{n:n}).\end{aligned}$$

Secondly, we extend the above result to the PRHR model. □

**Theorem 2.2** Let $X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i = 1, 2, \ldots, n$, be all independent random variables. Then

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$$

implies

$$(Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}) \geq_{\rm st} (X_{1:n}, X_{2:n}, \ldots, X_{n:n}).$$

**Proof** Let $X_i^* \sim \text{beta}\,(\alpha_i, 1)$ and $Y_i^* \sim \text{beta}\,(\gamma_i, 1)$, $i = 1, 2, \ldots, n$, be all independent. Then, according to Theorem 2.1, we get

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$$

implies

$$(Y^*_{1:n}, Y^*_{2:n}, \ldots, Y^*_{n:n}) \geq_{\rm st} (X^*_{1:n}, X^*_{2:n}, \ldots, X^*_{n:n}).$$

Since the baseline distribution function $F(x)$ is increasing in $x$, $F^{-1}(x)$ is also an increasing function, and so from Lemma 2.1, we have

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\rm m} (\gamma_1, \gamma_2, \ldots, \gamma_n)$$

implies

$$(F^{-1}(Y^*_{1:n}), F^{-1}(Y^*_{2:n}), \ldots, F^{-1}(Y^*_{n:n})) \geq_{\rm st} (F^{-1}(X^*_{1:n}), F^{-1}(X^*_{2:n}), \ldots, F^{-1}(X^*_{n:n})).$$

Note that,

$$(X_{1:n}, X_{2:n}, \ldots, X_{n:n}) \overset{\rm st}{=} (F^{-1}(X^*_{1:n}), F^{-1}(X^*_{2:n}), \ldots, F^{-1}(X^*_{n:n}))$$

and

$$(Y_{1:n}, Y_{2:n}, \ldots, Y_{n:n}) \stackrel{\text{st}}{=} (F^{-1}(Y_{1:n}^*), F^{-1}(Y_{2:n}^*), \ldots, F^{-1}(Y_{n:n}^*)).$$

Therefore, the required result follows immediately. □

An interesting and useful special case of Theorem 2.2 is given in the following corollary.

**Corollary 2.1** Under the conditions of Theorem 2.2, $\sum\limits_{r\in I} Y_{r:n} \geq_{\text{st}} \sum\limits_{r\in I} X_{r:n}$, for each subset $I$ of $\{1, 2, \ldots, n\}$. Thus, for $k = 1, 2, \ldots, n$, we have

$$\sum_{r=1}^{k} Y_{r:n} \geq_{\text{st}} \sum_{r=1}^{k} X_{r:n} \tag{2.2}$$

holds; in particular,

$$\sum_{r=1}^{n} Y_{r:n} \geq_{\text{st}} \sum_{r=1}^{n} X_{r:n}. \tag{2.3}$$

**Proof** Since $\sum\limits_{r\in I} x_{r:n}$ is an increasing function of $(X_{1:n}, X_{2:n}, \ldots, X_{n:n})$, we immediately obtain the results according to Theorem 2.2 and Definition 2.1. □

Let $X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i = 1, 2, \ldots, n$, be all independent random variables. Suppose $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$ and $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$. Thus, we can obtain $X_1 \geq_{\text{st}} X_2 \geq_{\text{st}} \cdots \geq_{\text{st}} X_n$ and $Y_1 \geq_{\text{st}} Y_2 \geq_{\text{st}} \cdots \geq_{\text{st}} Y_n$ holding from the definition of distribution function. Next, we consider the comparison of convolutions of independent heterogeneous PRHR variables with respect to the usual stochastic order.

**Theorem 2.3** Let $X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i = 1, 2, \ldots, n$, be all independent random variables, where $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$ and $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$. Then if

$$(\alpha_1, \alpha_2, \ldots, \alpha_n) \succeq_{\text{m}} (\gamma_1, \gamma_2, \ldots, \gamma_n),$$

we have

$$\sum_{r=k}^{n} Y_r \geq_{\text{st}} \sum_{r=k}^{n} X_r, \tag{2.4}$$

for all $1 \leq k \leq n$.

**Proof** For $k = 1$, (2.4) coincides with (2.3). Let $2 \leq k \leq n$. Since $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ $\succeq_{\text{m}} (\gamma_1, \gamma_2, \ldots, \gamma_n)$, we denote $\theta = \sum\limits_{r=k}^{n} \gamma_r - \sum\limits_{r=k}^{n} \alpha_r \geq 0$. Define

$$\delta_k = \lambda_k + \theta, \qquad \delta_i = \lambda_i, \qquad i = k+1, k+2, \ldots, n,$$

we have $(\delta_k, \delta_{k+1}, \ldots, \delta_n) \succeq_{\text{m}} (\gamma_k, \gamma_{k+1}, \ldots, \gamma_n)$. Let $Z_i \sim F^{\delta_i}$. By (2.3), we get

$$\sum_{r=k}^{n} Y_r \geq_{\text{st}} \sum_{r=k}^{n} Z_r. \tag{2.5}$$

Since $\delta_k = \lambda_k + \theta \geq \lambda_k$, then $Z_k \geq_{\rm st} X_k$. Also $X_k$ and $\sum\limits_{r=k+1}^{n} X_r$, $Z_k$ and $\sum\limits_{r=k+1}^{n} Z_r$ are independent, respectively. Therefore, we have

$$\sum_{r=k}^{n} Z_r \geq_{\rm st} \sum_{r=k}^{n} X_r. \tag{2.6}$$

Combining (2.5) and (2.6) yields the desired result (2.4). $\square$

Last, we give a typical direct application of Theorem 2.2.

A random variable $X$ is said to have the exponentiated Weibull (EW) distribution if its cumulative distribution function and probability density function are given by

$$F(x;\alpha,\beta,\lambda) = \big(1 - {\rm e}^{-(\lambda x)^{\beta}}\big)^{\alpha}, \qquad x > 0,\ \alpha > 0,\ \beta > 0,\ \lambda > 0,$$

and

$$f(x;\alpha,\beta,\lambda) = \alpha\beta\lambda^{\beta}x^{\beta-1}{\rm e}^{-(\lambda x)^{\beta}}\big(1 - {\rm e}^{-(\lambda x)^{\beta}}\big)^{\alpha-1}, \qquad x > 0,\ \alpha > 0,\ \beta > 0,\ \lambda > 0.$$

Here, $\alpha$ and $\beta$ are shape parameters and $\lambda$ is a scale parameter, respectively. ${\rm EW}(\beta,\alpha,\lambda)$ would be used to denote a EW distribution. The EW distribution including many distributions as special cases, for example, the standard two-parameter Weibull distribution, the generalized exponential (GE) distribution, the Burr type $X$ distribution.

**Corollary 2.2** Let $X_i \sim {\rm EW}(\beta,\alpha_i,\lambda)$, $Y_i \sim {\rm EW}(\beta,\gamma_i,\lambda)$, $i = 1,2,\ldots,n$, be all independent random variables. For all $\beta > 0$, $\lambda > 0$, if

$$(\alpha_1,\alpha_2,\ldots,\alpha_n) \succeq_{\rm m} (\gamma_1,\gamma_2,\ldots,\gamma_n),$$

we have $(Y_{1:n},Y_{2:n},\ldots,Y_{n:n}) \geq_{\rm st} (X_{1:n},X_{2:n},\ldots,X_{n:n})$.

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# PRHR模型次序统计量的普通多元随机序

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设$X_i \sim F^{\alpha_i}$, $Y_i \sim F^{\gamma_i}$, $i=1,2,\ldots,n$, 都是独立的PRHR随机变量. 首先, 我们由$(\alpha_1,\alpha_2,\ldots,\alpha_n) \succeq_{\mathrm{m}} (\gamma_1,\gamma_2,\ldots,\gamma_n)$, 可得$(Y_{1:n},Y_{2:n},\ldots,Y_{n:n}) \geq_{\mathrm{st}} (X_{1:n},X_{2:n},\ldots,X_{n:n})$成立. 其次, 我们考虑了独立的PRHR随机变量卷积的一般序比较. 假定$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$和$\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$, 我们证明了由$(\alpha_1,\alpha_2,\ldots,\alpha_n) \succeq_{\mathrm{m}} (\gamma_1,\gamma_2,\ldots,\gamma_n)$, 可得对任意的$1 \leq k \leq n$, 有$\sum\limits_{r=k}^{n} Y_r \geq_{\mathrm{st}} \sum\limits_{r=k}^{n} X_r$成立. 本文中建立的结果推广了已有文献的相关结论.

**关键词:** 次序统计量, Beta分布, 普通多元随机序, 比例故障率模型, 比例反故障率模型.

**学科分类号:** O212.4, O213.2.