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On the Expected Penalty Functions in a Discrete Semi-Markov Risk Model *

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Abstract: This paper considers the expected penalty functions for a discrete semi-Markov risk model, which includes several existing risk models such as the compound binomial model (with time-correlated claims) and the compound Markov binomial model (with time-correlated claims) as special cases. Recursive formulae and the initial values for the discounted free penalty functions are derived in the two-state model by an easy method. We also give some applications of our results.

Keywords: expected penalty function; generating function; recursive formula; semi-Markov risk model

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§1. Introduction

Survival probability in a semi-Markov risk model was first investigated by Janssen and Reinhard [1], where they assumed that the surplus process not only depend on the current state but also depend on the next state of an environmental Markov chain. Albrecher and Boxma [2] and Cheung and Landriault [3] further studied the discounted penalty function in such a risk model by some generalized methods.

For the discrete-time semi-Markov risk model, Reinhard and Snoussi [4,5] derived recursive formulae for the distribution of the surplus just prior to ruin and that of the deficit at ruin in a special case, where a strict restriction was imposed on the total claim sizes. Chen et al. [6,7] removed the restriction of [4,5] and derived recursive formulae for computing the expected discounted dividends and survival probabilities for the model. As was mentioned in [7], the discrete-time semi-Markov risk model without restriction embraces some existing discrete-time risk models including the compound binomial model (with

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time-correlated claims) and the compound Markov binomial model (with time-correlated claims). So it is interesting to consider some further problems in this model.

The compound (Markov) binomial model (with time-correlated claims) has been extensively studied by various authors. For example, ruin problems in the compound binomial model were considered by Gerber [8], Shiu [9], Willmot [10], Dickson [11], Cheng et al. [12] and so on. Ruin problems in the compound binomial model with time-correlated claims were investigated by Yuen and Guo [13] and Xiao and Guo [14], the discounted free Gerber-Shiu penalty function for the compound binomial model with randomized dividends were treated in [15] and [16]. Ruin probability and the Gerber-Shiu penalty function for the compound Markov binomial model were studied by Cossette et al. [17,18] and Yuen and Guo [19]. Ruin problems in some other modified compound binomial risk model can be found in [20].

In this paper, we consider the discounted free Gerber-Shiu penalty function for the discrete-time semi-Markov risk model. Recursive formulae and the initial values are derived by an easy method in the two-state model.

The rest of the paper is organized as follows. In Section 2, we present the mathematical formulation of the discrete semi-Markov model. In Section 3, we derive recursive formulae for computing the discounted free Gerber-Shiu penalty function for the model. Section 4 is devoted to finding the initial values of the penalty function for applying the recursive formula. Some applications of our model are considered in Section 5.

§2. The Model

The model considered in this paper is the same as that in [6, 7]. For the reader's convenience, we also give some details here. Let $(J_n, n \in \mathbb{N})$ be a homogeneous, irreducible and aperiodic Markov chain with finite state space $M = \{1, 2, \ldots, m\}$ $(1 \leqslant m < \infty)$. Its one-step transition probability matrix is given by

$$\boldsymbol{P} = (p_{ij})_{i,j\in M}, \qquad p_{ij} = \mathsf{P}(J_n = j \mid J_{n-1} = i, J_k, k \leqslant n-1),$$

with a unique stationary distribution $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_m)$. The insurer's surplus at the end of the t-th period $(t \in \mathbb{N}_+)$ is given by

$$U_t = u + t - \sum_{i=1}^{t} Y_i, \qquad t \in \mathbb{N}_+, \tag{1}$$

where $u \in \mathbb{N}$ is the initial surplus and Y_i denotes the total amount of claims in the i-th period. We further assume that a premium of 1 is received at the beginning of each time period and Y_t's are nonnegative integer-valued random variables. The distribution of Y_t's is influenced by the environmental Markov chain $(J_n, n \in \mathbb{N})$ in the way that (J_t, Y_t) depends on $\{J_k, Y_k; k \leqslant t-1\}$ only through J_{t-1}. Define

$$g_{ij}(l) = \mathsf{P}(Y_t = l, J_t = j \mid J_{t-1} = i, J_k, Y_k, k \leqslant t-1), \qquad l \in \mathbb{N},$$

which describes the conditional joint distribution of Y_t and J_t given the previous state J_{t-1} and plays a key role in the following derivations.

Let $\tau = \inf\{t \in \mathbb{N}_+ : U_t < 0\}$ be the time of ruin. The Gerber-Shiu expected discount -ed penalty function given the initial surplus u and the initial environment state i is defined as

$$m_i(u) = \mathsf{E}(v^\tau \omega(U_{\tau-}, |U_\tau|)\mathbf{1}_{(\tau<\infty)} \mid U_0 = u, J_0 = i), \qquad i \in M,\ u \in \mathbb{N}, \tag{2}$$

where $\omega(x, y)$ is a nonnegative bounded function and $0 < v \leqslant 1$ is the discounted factor.

Assume that for all i and j,

$$\mu_{ij} = \sum_{k=0}^{\infty} k g_{ij}(k) < \infty,$$

and define

$$\mu_i = \sum_{j=1}^{m} \mu_{ij}, \qquad i \in M.$$

We further assume that the positive safety loading condition holds, that is,

$$\sum_{i=1}^{m} \pi_i \mu_i < 1, \tag{3}$$

which ensures that ruin is not certain.

Here, we only consider the case with $v = 1$ and $m = 2$. The rest of this paper aims to derive a recursive formula for computing $m_i(u)$. Obviously, the result obtained in this paper is an extension of [7]. Besides, as was shown in Section 5 of [7], our result is also an extension of [14], [19] and [21] in some aspect.

§3. Recursive Formula for $m_i(u)$

This section devotes to the derivation of the recursive formulae for $m_i(u)$, $i = 1, 2$. Let

$$g_i(k) = \sum_{j=1}^{2} g_{ij}(k), \quad \xi_i(u) = \sum_{k=u+2}^{\infty} g_i(k)\omega(u+1, k-u-1), \qquad i = 1, 2.$$

Considering the first time period, it is easy to see that

$$m_i(u)=\sum_{j=1}^{2}\sum_{k=0}^{u+1} g_{ij}(k)m_j(u+1-k)+\xi_i(u), \qquad i=1,2,\ u\in\mathbb{N}. \tag{4}$$

Now we employ the technique of generating functions to derive the recursive formulae for $m_i(u)$. Let $\widetilde{m}_i(s)$, $\widetilde{g}_{ij}(s)$ and $\widetilde{\xi}_i(s)$ denote the generating functions of $m_i(u)$, $g_{ij}(u)$ and $\xi_i(u)$ respectively. By multiplying both sides of (4) by s^{u+1} and summing over u from 0 to ∞, we obtain

$$s\widetilde{m}_i(s)=\sum_{j=1}^{2}\widetilde{g}_{ij}(s)\widetilde{m}_j(s)+s\widetilde{\xi}_i(s)-\sum_{j=1}^{2}g_{ij}(0)m_j(0), \qquad i=1,2. \tag{5}$$

Let $e_i=\sum\limits_{j=1}^{2}g_{ij}(0)m_j(0)$, $i=1,2$. Then we have

$$\begin{cases}[\widetilde{g}_{11}(s)-s]\widetilde{m}_1(s)+\widetilde{g}_{12}(s)\widetilde{m}_2(s)=e_1-s\widetilde{\xi}_1(s),\\ \widetilde{g}_{21}(s)\widetilde{m}_1(s)+[\widetilde{g}_{22}(s)-s]\widetilde{m}_2(s)=e_2-s\widetilde{\xi}_2(s).\end{cases} \tag{6}$$

It follows from (6) that

$$\begin{aligned}&[(\widetilde{g}_{11}(s)-s)(\widetilde{g}_{22}(s)-s)-\widetilde{g}_{21}(s)\widetilde{g}_{12}(s)]\widetilde{m}_1(s)\\ =\,&[e_1-s\widetilde{\xi}_1(s)](\widetilde{g}_{22}(s)-s)-[e_2-s\widetilde{\xi}_2(s)]\widetilde{g}_{12}(s).\end{aligned} \tag{7}$$

For notational convenience, we define

$$\begin{aligned}&\overline{g}_{ii}(1)=g_{ii}(1)-1, \quad \overline{g}_{ii}(k)=g_{ii}(k), \qquad i=1,2,\ k\in\mathbb{N}\backslash\{1\},\\ &h_i(0)=e_i, \quad h_i(k)=-\xi_i(k-1), \qquad i=1,2,\ k\in\mathbb{N}\backslash\{0\},\\ &f_k=\sum_{n=0}^{k}[\overline{g}_{11}(n)\overline{g}_{22}(k-n)-g_{21}(n)g_{12}(k-n)], \qquad g_k^{(1)}=\sum_{n=0}^{k}m_1(n)f_{k-n},\\ &A_k^{(1)}=\sum_{n=0}^{k}[h_1(n)\overline{g}_{22}(k-n)-h_2(n)g_{12}(k-n)], \qquad k\in\mathbb{N}.\end{aligned}$$

Let $\widetilde{g^{(1)}}(s)$, $\widetilde{f}(s)$ and $\widetilde{A^{(1)}}(s)$ denote the generating functions of $g_k^{(1)}$, f_k and $A_k^{(1)}$ respectively. Note that for any two sequences $\{a(n), n=0,1,\ldots\}$ and $\{b(n), n=0,1,\ldots\}$ with generating functions $\widetilde{a}(s)$ and $\widetilde{b}(s)$, we have the following property

$$\widetilde{a}(s)\widetilde{b}(s)=\sum_{n=0}^{\infty}a*b(n)s^n=\sum_{n=0}^{\infty}\sum_{k=0}^{n}a(k)b(n-k)s^n.$$

Applying this property to (7) yields

$$\widetilde{g^{(1)}}(s)=\widetilde{f}(s)\widetilde{m}_1(s)=\widetilde{A^{(1)}}(s), \tag{8}$$

where $\widetilde{A^{(1)}}(s)$ is the expression on the right hand side of equation (7). Then comparing the coefficients of s^k in both sides of the above equation gives $g_k^{(1)} = A_k^{(1)}$, $k \in \mathbb{N}$, that is,

$$\sum_{n=0}^{k} m_1(n) f_{k-n} = A_k^{(1)}, \qquad k \in \mathbb{N}. \tag{9}$$

Similarly, one can obtain

$$\sum_{n=0}^{k} m_2(n) f_{k-n} = A_k^{(2)}, \qquad k \in \mathbb{N}, \tag{10}$$

where $A_k^{(2)} = \sum\limits_{n=0}^{k} [-h_1(n) g_{21}(k-n) + h_2(n) \overline{g}_{11}(k-n)]$, $k \in \mathbb{N}$.

According to Proposition 1 of [7], we know that $f_1 \neq 0$ if $f_0 = 0$. Hence, from (9) and (10), we obtain the following recursive formula

$$m_i(k) = \begin{cases} \dfrac{1}{f_0}\Big[A_k^{(i)} - \sum\limits_{n=0}^{k-1} m_i(n) f_{k-n}\Big] & \text{if } f_0 \neq 0; \\ \dfrac{1}{f_1}\Big[A_{k+1}^{(i)} - \sum\limits_{n=0}^{k-1} m_i(n) f_{k+1-n}\Big] & \text{if } f_0 = 0 \text{ and } f_1 \neq 0, \end{cases} \tag{11}$$

for $i = 1, 2$ and $k \in \mathbb{N}_+$.

§4. The Initial Value for $m_i(u)$

After obtaining recursive formula (11) for $m_i(u)$, we need to determine the initial values $m_i(0)$, $i = 1, 2$. So we should make an effort to find two equations associated with them in this section.

4.1 The First Equation

Assume that

$$\lim_{s \to 1} \widetilde{m}_i(s) = \sum_{u=0}^{\infty} m_i(u) < \infty, \qquad i = 1, 2,$$

then it follows from (6) that

$$\begin{cases} p_{12}\Big(\sum\limits_{u=0}^{\infty} m_1(u) - \sum\limits_{u=0}^{\infty} m_2(u)\Big) = -\sum\limits_{j=1}^{2} g_{1j}(0) m_j(0) + \sum\limits_{u=0}^{\infty} \xi_1(u), \\ p_{21}\Big(\sum\limits_{u=0}^{\infty} m_1(u) - \sum\limits_{u=0}^{\infty} m_2(u)\Big) = \sum\limits_{j=1}^{2} g_{2j}(0) m_j(0) - \sum\limits_{u=0}^{\infty} \xi_2(u), \end{cases}$$

which yields

$$m_1(0)(g_{11}(0) p_{21} + g_{21}(0) p_{12}) + m_2(0)(g_{12}(0) p_{21} + g_{22}(0) p_{12})$$

$$= p_{21} \sum_{u=0}^{\infty} \xi_1(u) + p_{12} \sum_{u=0}^{\infty} \xi_2(u). \tag{12}$$

Remark 1 Assume that $\omega(x, y) \leqslant K$ for some constant K, then we can see that

$$\sum_{u=0}^{\infty} \xi_i(u) \leqslant K \sum_{u=0}^{\infty} \sum_{k=u+2}^{\infty} g_i(k) = K \sum_{k=2}^{\infty} \sum_{u=0}^{k-2} g_i(k) \leqslant K\mu_i < \infty, \qquad i = 1, 2.$$

4.2 The Second Equation

In this subsection, we use an alternative method to find another relation between $m_1(0)$ and $m_2(0)$. To do it, we consider several cases of $f_0 = g_{11}(0)g_{22}(0) - g_{12}(0)g_{21}(0)$.

Case 1: If $f_0 = 0$, it follows from (9) that $f_1 m_1(0) = A_1^{(1)}$, which yields

$$K_1 m_1(0) + K_2 m_2(0) = g_{12}(0)\xi_2(0) - g_{22}(0)\xi_1(0), \tag{13}$$

where

$$\begin{aligned} K_1 &= g_{22}(0)g_{11}(1) - g_{12}(0)g_{21}(1) - g_{22}(0) \leqslant 0, \\ K_2 &= g_{22}(0)g_{12}(1) - g_{12}(0)g_{22}(1) + g_{12}(0) \geqslant 0. \end{aligned}$$

Furthermore,

$$K_1 = K_2 = 0 \iff g_{12}(0) = g_{22}(0) = 0.$$

In this case, we have $e_1 = g_{11}(0)m_1(0)$, $e_2 = g_{21}(0)m_1(0)$, and by (10),

$$\begin{aligned} f_1 m_2(0) = {} & \{g_{21}(0)g_{11}(1) - g_{11}(0)g_{21}(1) - g_{21}(0)\}m_1(0) \\ & + g_{21}(0)\xi_1(0) - g_{11}(0)\xi_2(0). \end{aligned} \tag{14}$$

Case 2: If $f_0 > 0$, then $\widetilde{f}(0) = f_0 > 0$. Note that

$$\begin{aligned} \widetilde{f}'(s) = {} & (\widetilde{g}'_{11}(s) - 1)(\widetilde{g}_{22}(s) - s) + (\widetilde{g}_{11}(s) - s)(\widetilde{g}'_{22}(s) - 1) \\ & - [\widetilde{g}'_{12}(s)\widetilde{g}_{21}(s) + \widetilde{g}_{12}(s)\widetilde{g}'_{21}(s)], \end{aligned}$$

we have

$$\begin{aligned} \widetilde{f}'(1) &= -p_{21}(\mu_{11} - 1) - p_{12}(\mu_{22} - 1) - p_{21}\mu_{12} - p_{12}\mu_{21} \\ &= p_{21}(1 - \mu_1) + p_{12}(1 - \mu_2). \end{aligned}$$

It is easy to see that the unique stationary distribution is

$$\boldsymbol{\pi} = (\pi_1, \pi_2) = \Big(\frac{p_{21}}{p_{21} + p_{12}}, \frac{p_{12}}{p_{21} + p_{12}}\Big).$$

Then according to the positive safety loading condition (3), we obtain that $\widetilde{f}'(1) > 0$. On the other hand, since $\widetilde{f}(1) = 0$, then there exists a $\delta > 0$ such that $\widetilde{f}(s) < 0$ for any $s \in (1-\delta, 1)$. As a consequence, there exists a $\rho \in (0,1)$ such that $\widetilde{f}(\rho) = 0$, which in turn implies that $\widetilde{A^{(1)}}(\rho) = 0$, that is,

$$\begin{aligned}
&\{g_{11}(0)(\widetilde{g}_{22}(\rho) - \rho) - g_{21}(0)\widetilde{g}_{12}(\rho)\}m_1(0)\\
&+ \{g_{12}(0)(\widetilde{g}_{22}(\rho) - \rho) - g_{22}(0)\widetilde{g}_{12}(\rho)\}m_2(0)\\
=\,& \rho(\widetilde{\xi}_1(\rho)[\widetilde{g}_{22}(\rho) - \rho] - \widetilde{\xi}_2(\rho)\widetilde{g}_{12}(\rho)).
\end{aligned} \tag{15}$$

Case 3: If $f_0 < 0$, then $\widetilde{f}(0) = f_0 < 0$. Besides,

$$\begin{aligned}
\widetilde{f}(-1) &= [\widetilde{g}_{11}(-1) + 1][\widetilde{g}_{22}(-1) + 1] - \widetilde{g}_{21}(-1)\widetilde{g}_{12}(-1)\\
&> (1 - \widetilde{g}_{11}(1))(1 - \widetilde{g}_{22}(1)) - \widetilde{g}_{21}(1)\widetilde{g}_{12}(1)\\
&= (1 - p_{11})(1 - p_{22}) - p_{21}p_{12} = 0.
\end{aligned}$$

So there exists a $\rho \in (-1, 0)$ such that $\widetilde{f}(\rho) = 0$, which yields that $\widetilde{A^{(1)}}(\rho) = 0$. That is, (15) also holds in this case.

Remark 2 For Case 2, the method used in [7] is invalid in this paper.

§5. Applications

In this section, we discuss a few special cases of the model (1) which have been considered in the literature.

5.1 The Compound Markov Binomial Model

As stated in [7], for $i = 1, 2$, if we let

$$g_{i1}(k) = \begin{cases} p_{i1}, & k = 0; \\ 0, & k > 0, \end{cases} \qquad g_{i2}(k) = \begin{cases} 0, & k = 0; \\ p_{i2}f(k), & k > 0, \end{cases}$$

then one can see that the compound Markov binomial model is a special case of model (1). In this case, we have

$$\begin{aligned}
&f_0 = 0,\\
&f_1 = p_{11}(p_{22}f(1) - 1) - p_{21}p_{12}f(1) = -p_{11} + (p_{11} - p_{21})f(1),
\end{aligned}$$

$$\begin{aligned}
f_2 &= p_{11}p_{22}f(2) + 1 - p_{22}f(1) - p_{21}p_{12}f(2) \\
&= 1 - p_{22}f(1) + (p_{11} - p_{21})f(2), \\
f_k &= p_{11}p_{22}f(k) - p_{22}f(k-1) - p_{21}p_{12}f(k) \\
&= -p_{22}f(k-1) + (p_{11} - p_{21})f(k), \qquad k \geqslant 3, \\
A_{k+1}^{(2)} &= p_{12}\theta(k), \ \text{ where } \ \theta(k) = \sum_{n=k+2}^{\infty} f(n)\omega(k+1, n-k-1), \qquad k \geqslant 0.
\end{aligned}$$

For $u = 1, 2, \ldots$, it follows from (11) that

$$\begin{aligned}
&[p_{11} - (p_{11} - p_{21})f(1)]m_2(u) \\
&= m_2(u-1) + \sum_{n=0}^{u-1} m_2(n)[(p_{11} - p_{21})f(u+1-n) - p_{22}f(u-n)] - p_{12}\theta(u). \qquad (16)
\end{aligned}$$

By (12) and (14), it is easy to see that

$$m_2(0) = \frac{(p_{11} - p_{21})\theta(0) + p_{12}\sum\limits_{u=0}^{\infty}\theta(u)}{p_{11} - (p_{11} - p_{21})f(1)},$$

which is the same as (2.17) of [21]. Besides, by mathematical induction, we can prove that (16) is equivalent to (2.16) of [21].

5.2 Some Important Actuarial Quantities

Survival (or ruin) probability for the model has been studied in [7]. Here we consider the joint probability function of the surplus immediately before ruin and the deficit at ruin, and the probability function of the claim causing ruin.

5.2.1 The Surplus Immediately before Ruin and the Deficit at Ruin

For $x = 0, 1, \ldots$ and $y = 1, 2, \ldots$, let $\omega(z_1, z_2) = \mathbf{1}_{(z_1 = x, z_2 = y)}$ in (2). Then

$$m_i(u) = f_i(u, x, y) = \mathsf{P}(U_{\tau-} = x, |U_\tau| = y, \tau < \infty \,|\, U_0 = u, J_0 = i),$$

which is the joint probability function of the surplus immediately before ruin and the deficit at ruin. In this case, for $i = 1, 2$, we have

$$\begin{aligned}
\xi_i(u) &= \sum_{k=u+2}^{\infty} g_i(k)\mathbf{1}_{(u+1=x, k-u-1=y)} = g_i(x+y)\mathbf{1}_{(u=x-1)}, \qquad u \in \mathbb{N}, \\
h_i(0) &= \sum_{j=1}^{2} g_{ij}(0)f_j(0, x, y), \qquad h_i(u) = -\mathbf{1}_{(u=x)}g_i(x+y), \quad u \in \mathbb{N}\backslash\{0\}.
\end{aligned}$$

Let $\theta_k = h_1(0)\overline{g}_{22}(k) - h_2(0)g_{12}(k)$. Then

$$A_k^{(1)} = \begin{cases} f_0 f_1(0,x,y), & k=0; \\ \theta_k, & 1 \leqslant k < x; \\ \theta_k - [g_1(x+y)\overline{g}_{22}(0) - g_2(x+y)g_{12}(0)], & k = x; \\ \theta_k - [g_1(x+y)\overline{g}_{22}(k-x) - g_2(x+y)g_{12}(k-x)], & k > x. \end{cases}$$

$A_k^{(2)}$ can be obtained similarly and the recursive formula for $f_i(u,x,y)$ follows from (11).

Besides, it is easy to see that

$$\sum_{u=0}^{\infty} \xi_i(u) = g_i(x+y)\mathbf{1}_{(x\geqslant 1)}, \qquad i=1,2.$$

Then the right hand side of (12) becomes

$$[p_{21}g_1(x+y) + p_{12}g_2(x+y)]\mathbf{1}_{(x\geqslant 1)}.$$

Finally, the initial values $f_i(0,x,y)$ can be derived from (12) and (13) (or (14), (15)).

5.2.2 The Probability Function of the Claim Causing Ruin

Now, consider $\omega(z_1,z_2) = \mathbf{1}_{(z_1+z_2=x)}$ in (2) for $x=1,2,\ldots$. Then

$$m_i(u) = q_i(x\,|\,u) = \mathsf{P}(U_{\tau-} + |U_\tau| = x, \tau<\infty\,|\,U_0=u, J_0=i),$$

which is the probability function of the claim causing ruin. In this case, for $i=1,2$, we have

$$\xi_i(u) = \sum_{k=u+2}^{\infty} g_i(k)\mathbf{1}_{(k=x)} = g_i(x)\mathbf{1}_{(u\leqslant x-2)}, \qquad u\in\mathbb{N},$$
$$h_i(0) = \sum_{j=1}^{2} g_{ij}(0)q_i(x|0), \qquad h_i(u) = -\mathbf{1}_{(u\leqslant x-1)}g_i(x), \quad u\in\mathbb{N}\backslash\{0\}.$$

Let $\theta_k = h_1(0)\overline{g}_{22}(k) - h_2(0)g_{12}(k)$. Then

$$A_k^{(1)} = \begin{cases} f_0 q_1(x\,|\,0), & k=0; \\ \theta_k - \sum\limits_{n=1}^{k}[g_1(x)\overline{g}_{22}(k-n) - g_2(x)g_{12}(k-n)], & 1\leqslant k<x; \\ \theta_k - \sum\limits_{n=1}^{x-1}[g_1(x)\overline{g}_{22}(k-n) - g_2(x)g_{12}(k-n)], & k\geqslant x. \end{cases}$$

$A_k^{(2)}$ can be obtained similarly and the recursive formula for $q_i(x\,|\,u)$ follows from (11).

Besides, it is easy to see that

$$\sum_{u=0}^{\infty} \xi_i(u) = (x-1)g_i(x)\mathbf{1}_{(x\geqslant 2)}, \qquad i=1,2.$$

Then the right hand side of (12) becomes

$$[p_{21}g_1(x)+p_{12}g_2(x)](x-1)\mathbf{1}_{(x\geqslant 2)}.$$

Finally, the initial values $q_i(x\,|\,0)$ can be derived from (12) and (13) (or (14), (15)).

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离散半马氏风险模型中的期望罚金函数

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摘　要： 本文研究离散半马氏风险模型中的期望罚金函数, 所考虑的模型包含了多个已有的风险模型, 如(具有延迟索赔)复合二项模型和(具有延迟索赔)复合马氏二项模型. 通过一个简单的方法得到了两状态模型中期望罚金函数的递推公式和初始值. 我们也对所得结果给出了一些应用.

关键词： 期望罚金函数; 母函数; 递推公式; 半马氏风险模型

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