应用概率统计 第 40 卷
第 4 期 2024 年 8 月

Chinese Journal of Applied Probability and Statistics
Aug., 2024, Vol. 40, No. 4, pp. 572–587
doi: 10.12460/j.issn.1001-4268.aps.2024.2022067

Pricing Catastrophe Options with Credit Risk in a Regime-Switching Model*

XU Yajuan[1] WANG Guojing[2]

[1]Department of Mathematics and Physics, Suzhou Vocational University, Suzhou, 215104, China

[2]The Center for Financial Engineering and Department of Mathematics, Soochow University, Suzhou, 215006, China

Abstract: In this paper, we consider the price of catastrophe options with credit risk in a regime-switching model. We assume that the macroeconomic states are described by a continuous-time Markov chain with a finite state space. By using the measure change technique, we derive the price expressions of catastrophe put options. Moreover, we conduct some numerical analysis to demonstrate how the parameters of the model affect the price of the catastrophe put option.

Keywords: pricing; catastrophe option; credit risk; regime-switching; measure change

2020 Mathematics Subject Classification: 91B24

Citation: XU Y J, WANG G J. Pricing catastrophe options with credit risk in a regime-switching model [J]. Chinese J Appl Probab Statist, 2024, 40(4): 572–587.

1 Introduction

There have been many severe catastrophes in the past few decades, which have attracted increasing attention to catastrophe derivatives pricing. Cox and Pedersen[1] examined the price of catastrophe bonds by briefly discussing the equilibrium price and its relationship to the standard arbitrage-free pricing framework. Cox et al.[2] assumed that the price process of the asset is driven by a geometric Brownian motion with additional downward jumps of a prespecified size in the event of a catastrophe. They applied this model to price catastrophe options. Jaimungal and Wang[3] generalized the results of Cox et al.[2]. They assumed that the losses follow a compound Poisson process and that the drop in asset price depends on the total loss level. They obtained the closed-form formulae for the price of catastrophe put options. Jiang et al.[4] presented a catastrophe put option pricing model that accounts for interest rate uncertainty. Xu and Wang[5]

*The project was supported by the Jiangsu University Philosophy and Social Science Research Project (Grant No. 2019SJA1326).

Received April 25, 2022. Revised September 9, 2022.

assumed that the interest rate process and the default intensity process are modeled through the Vasicek model and provided the price expressions of catastrophe put options with credit risk.

In recent years, the pricing frameworks with a regime-switching model have been used by many researchers in modern financial economics, see, for example, Elliott et al.[6], Wang and Wang[7], and Wang et al.[8]. In the pricing frameworks with a regime-switching model, the market is assumed to be in different states depending on the state of the economy. A regime shift from one economic state to another may occur due to various financial factors, such as changes in business conditions, management decisions and other macroeconomic conditions. In this paper, we incorporate the model of Jiang et al.[4] into the pricing framework with a regime-switching model, where the issuing company's share price process, the loss process and the price process of the asset are all related to the macroeconomic states. We study the price of the catastrophe put option with credit risk in the proposed model.

The rest of this paper is organized as follows. In Section 2, we present the basic assumptions and the dynamics of the issuing company's share price process, the loss process and the price process of the asset. In Section 3, we adopt a measure change to determine an equivalent martingale probability measure for pricing catastrophe put options. In Section 4, we obtain some closed-form results for pricing catastrophe put options with credit risk in a regime-switching model. We present some numerical analysis to examine how the parameters of the model affect the price of the catastrophe put option in Section 5.

2 Modeling assumptions

Given a filtered complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=0}^{T}, P)$, where $\mathcal{F} = \mathcal{F}_T$ and P is a real-word probability measure. In this paper, the macroeconomic states are described by a continuous-time irreducible Markov chain $\{X(t)\}_{t\geqslant 0}$ with a finite state space $D = \{\mathbf{e}_1, \mathbf{e}_2, \cdots, \mathbf{e}_\mathrm{N}\}$, $\mathbf{e}_\mathrm{i} = (0, \cdots, 0, 1, 0, \cdots, 0)' \in R^N$, where $'$ denotes the transpose of a vector or a matrix. As in Elliott et al.[9], the process $\{X(t)\}_{t\geqslant 0}$ has the following decomposition:

$$X(t) = X(0) + \int_0^t AX(u)\mathrm{d}u + M(t), \tag{1}$$

where $A = (a_{ij})_{i,j=1,2,\cdots,N}$ is the generator of the process $X(t)$ and $M(t)$ is a martingale with respect to the natural filtration generated by $\{X(t)\}_{t\geqslant 0}$.

Let $\{r(t) : t \geqslant 0\}$ be the risk-free short rate process, which is defined by

$$r(t) =< \mathbf{r}, X(t) >, \tag{2}$$

where $\mathbf{r} = (r_1, r_2, \cdots, r_N)' \in R^N$ with $r_i > 0$ for each $i = 1, 2, \cdots, N$, and $< \cdot, \cdot >$ denotes the inner product in R^N.

Let $\{S(t) : t \geqslant 0\}$ be the share price process of the issuing company, $\{L(t) : t \geqslant 0\}$ be the loss process of the the issuing company, and $\{V(t) : t \geqslant 0\}$ be the price process of the asset of the issuing company. We assume that the dynamics of $S(t), L(t)$ and $V(t)$ are given by

$$S(t) = S(0)\exp\left\{\int_0^t \left[\mu_1(u) - \frac{1}{2}\sigma_1^2(u)\right] \mathrm{d}u + \int_0^t \sigma_1(u)\mathrm{d}\overline{W}_1(u) - \alpha L(t) + \beta(t)\right\}, \tag{3}$$

$$L(t) = \sum_{j=1}^{N(t)} Y_j, \quad \beta(t) = \int_0^t \int_0^\infty (1 - \mathrm{e}^{-\alpha y})\lambda(u) f_L(y)\mathrm{d}y\mathrm{d}u,$$
$$V(t) = V(0)\exp\left\{\int_0^t \left[\mu_2(u) - \frac{1}{2}\sigma_2^2(u)\right] \mathrm{d}u + \int_0^t \sigma_2(u)\mathrm{d}\overline{W}_2(u)\right\}, \tag{4}$$

where $\overline{W}_1(t)$ and $\overline{W}_2(t)$ are standard Brownian motions under P with

$$\mathsf{Cov}\left(\mathrm{d}W_1(t), \mathrm{d}\overline{W}_2(t)\right) = \rho \mathrm{d}t, \rho \in R;$$

the appreciation rate $\mu_1(t), \mu_2(t)$ and the volatility $\sigma_1(t), \sigma_2(t)$ depend on $\{X(t)\}_{t \geqslant 0}$, which are defined by

$$\mu_1(t) =< \boldsymbol{\mu}_1, X(t) >, \sigma_1(t) =< \boldsymbol{\sigma}_1, X(t) >,$$

$$\mu_2(t) =< \boldsymbol{\mu}_2, X(t) >, \sigma_2(t) =< \boldsymbol{\sigma}_2, X(t) >,$$

where

$$\boldsymbol{\mu}_1 = (\mu_{11}, \mu_{12}, \cdots, \mu_{1N})' \in R^N, \ \ \boldsymbol{\sigma}_1 = (\sigma_{11}, \sigma_{12}, \cdots, \sigma_{1N})' \in R^N,$$

$$\boldsymbol{\mu}_2 = (\mu_{21}, \mu_{22}, \cdots, \mu_{2N})' \in R^N, \ \ \boldsymbol{\sigma}_2 = (\sigma_{21}, \sigma_{22}, \cdots, \sigma_{2N})' \in R^N,$$

with $\mu_{1i} > 0, \mu_{2i} > 0, \sigma_{2i} > 0$ and $\sigma_{2i} > 0$ for each $i = 1, 2, \cdots, N$; $\{Y_j : j = 1, 2, \cdots\}$ are i.i.d. random variables representing the size of the i-th loss with p.d.f. $f_L(y)$ and mean l, and $\{N(t) : t \geqslant 0\}$ is a doubly stochastic Poisson process with arrival rate $\lambda(t)$, which is defined by

$$\lambda(t) =< \boldsymbol{\lambda}, X(t) >,$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_N)' \in R^N$ with $\lambda_i > 0$ for each $i = 1, 2, \cdots, N$. Since $N(t)$ is a doubly stochastic Poisson process, there exists a standard Poisson process $\overline{N}(t)$ which is independent of the intensity process $\lambda(t)$, such that $N(t) = \overline{N}(\Lambda_t)$, where

$$\Lambda_t = \int_0^t \lambda(s)\mathrm{d}s.$$

See Grandell[10]. We also suppose that $X(t)$ is independent of $\overline{W}_1(t), \overline{W}_2(t)$ and $\overline{N}(t)$.

Denote

$$\mathcal{F}_t = \mathcal{F}_t^X \bigvee \mathcal{F}_t^{\bar{W}_1} \bigvee \mathcal{F}_t^{\bar{W}_2} \bigvee \mathcal{F}_t^L, \tag{5}$$

where

$$\begin{aligned} \mathcal{F}_t^X &= \sigma(X(s) : 0 \leqslant s \leqslant t), \\ \mathcal{F}_t^{\overline{W}_1} &= \sigma\left(\overline{W}_1(s) : 0 \leqslant s \leqslant t\right), \\ \mathcal{F}_t^{\overline{W}_2} &= \sigma\left(\overline{W}_2(s) : 0 \leqslant s \leqslant t\right), \\ \mathcal{F}_t^L &= \sigma(L(s) : 0 \leqslant s \leqslant t) \end{aligned}$$

are the natural filtrations generated by $X(t), \overline{W}_1(t), \overline{W}_2(t)$ and $L(t)$, respectively.

3 Equivalent martingale probability measure

In this section, we will illustrate an equivalent martingale probability measure. As in Cox et al.[2], if a liquid market for catastrophe options exists, then an equivalent martingale probability measure Q exists by standard derivative pricing theory, not necessarily unique, under which the discounted relative price processes $\left\{\mathrm{e}^{-\int_0^t r(s)\mathrm{d}s}\psi(t) : t \geqslant 0\right\}$ are martingales, for all contingent capitals $\psi(t)$. We follow Cox et al.[2], Elliott et al.[6], and adopt Merton's[11] assumption that the jumps are systematic and non-diversifiable. So the arrival rate and distribution of catastrophic events are not altered by measure changes.

Proposition 1 Let η_T^Q be the Radon-Nikodym process

$$\begin{aligned} \eta_T^Q = \frac{\mathrm{d}Q}{\mathrm{d}P} =\exp\bigg\{ &\int_0^T l_1(u)\sigma_1(u)\mathrm{d}\overline{W}_1(u) + \int_0^T l_2(u)\sigma_2(u)\mathrm{d}\overline{W}_2(u) \\ &- \frac{1}{2}\int_0^T [l_1^2(u)\sigma_1^2(u) + l_2^2(u)\sigma_2^2(u) + 2\rho l_1(u)\sigma_1(u)l_2(u)\sigma_2(u)]\mathrm{d}u\bigg\}, \end{aligned} \tag{6}$$

where

$$l_1(u) = \frac{\rho[\mu_2(u) - r(u)]\sigma_1(u) - [\mu_1(u) - r(u)]\sigma_2(u)}{(1-\rho^2)\sigma_1^2(u)\sigma_1(u)},$$

$$l_2(u) = \frac{\rho[\mu_1(u) - r(u)]\sigma_2(u) - [\mu_2(u) - r(u)]\sigma_1(u)}{(1-\rho^2)\sigma_1(u)\sigma_1^2(u)}.$$

Then $W_1(t)$ and $W_2(t)$ defined by

$$W_1(t) = \overline{W}_1(t) - \int_0^t l_1(u)\sigma_1(u)\mathrm{d}u - \rho\int_0^t l_2(u)\sigma_2(u)\mathrm{d}u \tag{7}$$

and

$$W_2(t) = \overline{W}_2(t) - \rho\int_0^t l_1(u)\sigma_1(u)\mathrm{d}u - \int_0^t l_2(u)\sigma_2(u)\mathrm{d}u \tag{8}$$

are standard Brownian motions under Q with instantaneous correlation ρ, i.e.,

$$\mathsf{Cov}(\mathrm{d}W_1(t), \mathrm{d}W_2(t)) = \rho\mathrm{d}t.$$

Finally, let $D(t,T) = \exp\{-\int_t^T r(u)\mathrm{d}u\}$, and then $\{D(0,t)S(t) : t \geqslant 0\}$ and $\{D(0,t)V(t) : t \geqslant 0\}$ are martingales under Q, respectively.

Proof It can be easily seen that $\eta_T^Q > 0$, P-a.s. and η_T^Q is a Radon-Nikodym process which induces the measure change from P to Q. From Girsanov's theorem, we know that $W_1(t)$ and $W_2(t)$ are standard Brownian motions under Q with $\mathsf{Cov}(\mathrm{d}W_1(t), \mathrm{d}W_2(t)) = \rho\mathrm{d}t$.

By Bayes' rule, for all $t \geqslant u$, we have

$$\begin{aligned}
\mathsf{E}^Q[D(0,t)S(t)|\mathcal{F}_u] &= \frac{\mathsf{E}^P\left[D(0,t)S(t)\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_u\right]}{\mathsf{E}^P\left[\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_u\right]} \\
=&\frac{\mathsf{E}^P\left[D(0,t)S(t)\mathsf{E}^P(\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_t)|\mathcal{F}_u\right]}{\mathsf{E}^P\left[\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_u\right]} = \mathsf{E}^P\left[D(0,t)S(t)\frac{\eta_t^Q}{\eta_u^Q}|\mathcal{F}_u\right] \\
=&D(0,u)S(u)\mathsf{E}^P\bigg\{\exp\bigg\{\int_u^t [l_1(s)+1]\sigma_1(s)\mathrm{d}\overline{W}_1(s) + \int_u^t l_2(s)\sigma_2(s)\mathrm{d}\overline{W}_2(s) - A(u,t) \\
&+ \int_u^t \bigg[\mu_1(s) - r(s) - \frac{1}{2}\sigma_1^2(s) - \frac{1}{2}l_1^2(s)\sigma_1^2(s) - \frac{1}{2}l_2^2(s)\sigma_2^2(s) \\
&- \rho l_1(s)\sigma_1(s)l_2(s)\sigma_2(s)\bigg]\mathrm{d}s\bigg\}|\mathcal{F}_u\bigg\} \\
=&D(0,u)S(u)\exp\left\{\int_u^t [\mu_1(s) - r(s) + l_1(s)\sigma_1^2(s) + \rho\sigma_1(s)l_2(s)\sigma_2(s)]\mathrm{d}s\right\} \\
=&D(0,u)S(u),
\end{aligned} \tag{9}$$

where $A(u,t) = \alpha[L(t) - L(u)] - [\beta(t) - \beta(u)]$, and

$$\mathsf{E}^Q[D(0,t)V(t)|\mathcal{F}_u] = \frac{\mathsf{E}^P\left[D(0,t)V(t)\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_u\right]}{E^P\left[\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_u\right]}$$

$$= \frac{\mathsf{E}^P\left[D(0,t)V(t)\mathsf{E}^P\left(\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_t\right)|\mathcal{F}_u\right]}{\mathsf{E}^P\left[\dfrac{\mathrm{d}Q}{\mathrm{d}P}|\mathcal{F}_u\right]} = \mathsf{E}^P\left[D(0,t)V(t)\frac{\eta_t^Q}{\eta_u^Q}|\mathcal{F}_u\right]$$

$$= D(0,u)V(u)\mathsf{E}^P\Big\{\exp\Big\{\int_u^t l_1(s)\sigma_1(s)\mathrm{d}\overline{W}_1(s) + \int_u^t [l_2(s)+1]\sigma_2(s)\mathrm{d}\overline{W}_2(s)$$

$$+ \int_u^t \Big[\mu_2(s) - r(s) - \frac{1}{2}\sigma_2^2(s) - \frac{1}{2}l_1^2(s)\sigma_1^2(s) - \frac{1}{2}l_2^2(s)\sigma_2^2(s)$$

$$- \rho l_1(s)\sigma_1(s)l_2(s)\sigma_2(s)\Big]\mathrm{d}s\Big\}|\mathcal{F}_u\Big\}$$

$$= D(0,u)V(u)\exp\left\{\int_u^t (\mu_2(s) - r(s) + l_2(s)\sigma_2^2(s) + \rho l_1(s)\sigma_1(s)\sigma_2(s))\mathrm{d}s\right\}$$

$$= D(0,u)V(u). \tag{10}$$

So $\{D(0,t)S(t) : t \geqslant 0\}$ and $\{D(0,t)V(t) : t \geqslant 0\}$ are martingales under Q, respectively.

Remark 1 From (3), (4), (7) and (8), we know

$$S(t) = S(0)\exp\left\{\int_0^t \left[r(u) - \frac{1}{2}\sigma_1^2(s)\right]\mathrm{d}u + \int_0^t \sigma_1(s)\mathrm{d}W_1(u) - \alpha L(t) + \beta(t)\right\}, \tag{11}$$

$$V(t) = V(0)\exp\left\{\int_0^t \left[r(u) - \frac{1}{2}\sigma_2^2(s)\right]\mathrm{d}u + \int_0^t \sigma_2(s)\mathrm{d}W_2(u)\right\}. \tag{12}$$

4 Pricing catastrophe put options

To reflect the influence of the macroeconomic states on the price of a catastrophe put option with credit risk in a regime-switching model, We assume that the catastrophe put options whose promised payoff is $(K - S(T))^+$ and actual payoff is $(K - S(T))^+\frac{(1-\omega)V(T)}{D}$ in the event of a default, where ω denotes the value of the deadweight costs associated with bankruptcy and is expressed as a percentage of the value of the asset of the issuing company, and D denotes the total amount of claims. Hence, letting $p(0,T)$ denote the price of the catastrophe put option with credit risk at time 0, which matures at time T, we have

$$p(0,T) = \mathsf{E}^Q\left[D(0,T)I_{\{L(T)-L(0)>L\}}(K - S(T))^+ I_{\{V(T)\geqslant D^*\}}\right]$$
$$+ \mathsf{E}^Q\left[D(0,T)I_{\{L(T)-L(0)>L\}}(K - S(T))^+\frac{(1-\omega)V(T)}{D}I_{\{V(T)<D^*\}}\right], \tag{13}$$

where the parameter L is the trigger level of losses above which the catastrophe put option becomes in-the-money, K is the strike price at which the issuing company is obligated to purchase unit share if losses exceed L, and D^* is the default boundary of the issuing company.

Let $J_i = \int_0^T <\mathbf{e}_i, X(u)> \mathrm{d}u$ be the amount of time $\{X(t)\}_{t\geqslant 0}$ has spent in state i $(i = 1, 2, \cdots, N)$ over the time interval $[0, T]$, and then $J_1 + J_2 + \cdots + J_N = T$. So we only consider $J_1, J_2, \cdots, J_{N-1}$. Define

$$\mathbf{J} = (J_1, J_2, \cdots, J_{N-1}), \tag{14}$$

$$R_T(\mathbf{J}) = \int_0^T r(u)\mathrm{d}u = \sum_{i=1}^{N-1}(r_i - r_N)J_i + r_N T, \tag{15}$$

$$\lambda_T(\mathbf{J}) = \int_0^T \lambda(u)\mathrm{d}u = \sum_{i=1}^{N-1}(\lambda_i - \lambda_N)J_i + \lambda_N T, \tag{16}$$

$$U_T^1(\mathbf{J}) = \int_0^T \sigma_1^2(u)\mathrm{d}u = \sum_{i=1}^{N-1}(\sigma_{1i}^2 - \sigma_{1N}^2)J_i + \sigma_{1N}^2 T, \tag{17}$$

$$U_T^2(\mathbf{J}) = \int_0^T \sigma_2^2(u)\mathrm{d}u = \sum_{i=1}^{N-1}(\sigma_{2i}^2 - \sigma_{2N}^2)J_i + \sigma_{2N}^2 T, \tag{18}$$

$$U_T^{12}(\mathbf{J}) = \int_0^T \sigma_1(u)\sigma_2(u)\mathrm{d}u = \sum_{i=1}^{N-1}(\sigma_{1i}\sigma_{2i} - \sigma_{1N}\sigma_{2N})J_i + \sigma_{1N}\sigma_{2N} T, \tag{19}$$

and

$$\begin{aligned} p(0,T,\mathbf{J}) =& \mathsf{E}^Q[D(0,T)I_{\{L(T)-L(0)>L\}}(K - S(T))^+ I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X] + \\ &+ \mathsf{E}^Q\left[D(0,T)I_{\{L(T)-L(0)>L\}}(K - S(T))^+ \frac{(1-\omega)V(T)}{D} I_{\{V(T)<D^*\}}\right]|\mathcal{F}_T^X]. \end{aligned} \tag{20}$$

Then we can rewrite $p(0, T)$ as

$$p(0,T) = \mathsf{E}^Q[p(0,T,\mathbf{J})] = \mathsf{E}^Q[\mathsf{E}_1(\mathbf{J}) - \mathsf{E}_2(\mathbf{J}) + \mathsf{E}_3(\mathbf{J}) - \mathsf{E}_4(\mathbf{J})], \tag{21}$$

where

$$\mathsf{E}_1(\mathbf{J}) = \mathsf{E}^Q[D(0,T)I_{\{L(T)-L(0)>L\}}KI_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X], \tag{22}$$

$$\mathsf{E}_2(\mathbf{J}) = \mathsf{E}^Q[D(0,T)I_{\{L(T)-L(0)>L\}}S(T)I_{\{K\geqslant S(T)\}}I_{V(T)\geqslant D^*}|\mathcal{F}_T^X], \tag{23}$$

$$\mathsf{E}_3(\mathbf{J}) = \mathsf{E}^Q\left[D(0,T)I_{\{L(T)-L(0)>L\}}K\frac{(1-\omega)V(T)}{D}I_{\{K\geqslant S(T)\}}I_{\{V(T)<D^*\}}|\mathcal{F}_T^X\right], \tag{24}$$

$$\mathsf{E}_4(\mathbf{J}) = \mathsf{E}^Q\left[D(0,T)I_{\{L(T)-L(0)>L\}}S(T)\frac{(1-\omega)V(T)}{D}I_{\{K\geqslant S(T)\}}I_{\{V(T)<D^*\}}|\mathcal{F}_T^X\right]. \tag{25}$$

Proposition 2 Let $E_1(\mathbf{J})$ and $E_2(\mathbf{J})$ be determined by (22) and (23). Then we have

$$\mathsf{E}_1(\mathbf{J}) = K\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\frac{(\lambda_T(\mathbf{J}))^n}{n!}\int_L^{\infty} D(0,T,\mathbf{J})N_2(d_1(y,\mathbf{J}), d_2(\mathbf{J}), -\hat{\rho}(\mathbf{J}))f_L^{*n}(y)\mathrm{d}y, \tag{26}$$

$$\mathsf{E}_2(\mathbf{J}) = \mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\frac{(\lambda_T(\mathbf{J}))^n}{n!}\int_L^{\infty} S(0)\mathrm{e}^{-\alpha y+\beta(T)}N_2(d_3(y,\mathbf{J}),d_4(\mathbf{J}),-\hat{\rho}(\mathbf{J}))f_L^{*n}(y)\mathrm{d}y, \tag{27}$$

where

$$d_1(y,\mathbf{J}) = \frac{\ln\dfrac{K}{S(0)} - R_T(\mathbf{J}) + \dfrac{1}{2}U_T^1(\mathbf{J}) + \alpha y - \beta(T)}{\sqrt{U_T^1(\mathbf{J})}},$$

$$d_2(\mathbf{J}) = \frac{\ln\dfrac{V(0)}{D^*} + R_T(\mathbf{J}) - \dfrac{1}{2}U_T^2(\mathbf{J})}{\sqrt{U_T^2(\mathbf{J})}},\ \hat{\rho}(\mathbf{J}) = \frac{\rho U_T^{12}(\mathbf{J})}{\sqrt{U_T^1(\mathbf{J})U_T^2(\mathbf{J})}},$$

$$d_3(y,\mathbf{J}) = \frac{\ln\dfrac{K}{S(0)} - R_T(\mathbf{J}) - \dfrac{1}{2}U_T^1(\mathbf{J}) + \alpha y - \beta(T)}{\sqrt{U_T^1(\mathbf{J})}},$$

$$d_4(\mathbf{J}) = \frac{\ln\dfrac{V(0)}{D^*} + R_T(\mathbf{J}) - \dfrac{1}{2}U_T^2(\mathbf{J}) + \rho U_T^{12}(\mathbf{J})}{\sqrt{U_T^2(\mathbf{J})}}.$$

$N_2(\cdot,\cdot,\cdot)$ is the bivariate normal cumulative distribution function and $f_L^{*n}(y)$ represents the n -fold convolution of the probability density function $f_L(y)$.

Proof By the property of conditional expectation, we have

$$\mathsf{E}_1(\mathbf{J}) = K\mathsf{E}^Q\{D(0,T)I_{\{L(T)-L(0)>L\}}\mathsf{E}^Q[I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L]|\mathcal{F}_T^X\}. \tag{28}$$

From (11) and (12), we know

$$S(T) = S(0)\exp\left\{\int_0^T\left[r(u) - \frac{1}{2}\sigma_1^2(u)\right]\mathrm{d}u + \int_0^T\sigma_1(u)\mathrm{d}W_1(u) - \alpha L(T) + \beta(T)\right\}, \tag{29}$$

$$V(T) = V(0)\exp\left\{\int_0^T\left[r(u) - \frac{1}{2}\sigma_2^2(u)\right]\mathrm{d}u + \int_0^T\sigma_2(u)\mathrm{d}W_2(u)\right\}. \tag{30}$$

So

$$\begin{aligned}
&\mathsf{E}^Q\left[I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\\
=&Q\Bigg(\int_0^T\sigma_1(u)\mathrm{d}W_1(u)\leqslant\ln\frac{K}{S(0)} - R_T(\mathbf{J}) + \frac{1}{2}U_T^1(\mathbf{J}) + \alpha L(T) - \beta(T),\\
&\int_0^T\sigma_2(u)\mathrm{d}W_2(u)\geqslant\ln\frac{D^*}{V(0)} - R_T(\mathbf{J}) + \frac{1}{2}U_T^2(\mathbf{J})|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\Bigg)\\
=&N_2(d_1(L(T),\mathbf{J}),d_2(\mathbf{J}),-\hat{\rho}(\mathbf{J})).
\end{aligned}$$

By computing the expectation over the observed losses $L(T)$, we get

$$\begin{aligned}\mathsf{E}_1(\mathbf{J}) &= K\mathsf{E}^Q\{D(0,T)I_{\{L(T)>L+L(0)\}}N_2(d_1(L(T),\mathbf{J}),d_2(\mathbf{J}),-\hat{\rho}(\mathbf{J}))|\mathcal{F}_T^X\}\\ &=K\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\tfrac{(\lambda_T(\mathbf{J}))^n}{n!}\int_L^{\infty}D(0,T,\mathbf{J})N_2(d_1(y,\mathbf{J}),d_2(\mathbf{J}),-\hat{\rho}(\mathbf{J}))f_L^{*n}(y)\mathrm{d}y,\end{aligned}\tag{31}$$

where $D(0,T,\mathbf{J})=D(0,T)$.

Let

$$\eta_T^S=\frac{\mathrm{d}Q^S}{\mathrm{d}Q}=\exp\left\{\int_0^T\sigma_1(u)\mathrm{d}W_1(u)-\frac{1}{2}\int_0^T\sigma_1^2(u)\mathrm{d}u\right\}.\tag{32}$$

By Girsanov's theorem, $W_1^S(t)=W_1(t)-\int_0^t\sigma_1(u)\mathrm{d}u$ and $W_2^S(t)=W_2(t)-\rho\int_0^t\sigma_1(u)\mathrm{d}u$ are two standard Brownian motions with $\mathsf{Cov}(\mathrm{d}W_1^S(t),\mathrm{d}W_2^S(t))=\rho\mathrm{d}t$ under Q^S. So

$$S(T)=S(0)\exp\left\{\int_0^T\left[r(u)+\frac{1}{2}\sigma_1^2(u)\right]\mathrm{d}u+\int_0^T\sigma_1(u)\mathrm{d}W_1^S(u)-\alpha L(T)+\beta(T)\right\},\tag{33}$$

$$V(T)=V(0)\exp\left\{\int_0^T\left[r(u)-\frac{1}{2}\sigma_2^2(u)\right]\mathrm{d}u+\int_0^T\sigma_2(u)\mathrm{d}W_2^S(u)+\int_0^T\rho\sigma_1(u)\sigma_2(u)\mathrm{d}u\right\}.\tag{34}$$

By the property of conditional expectation, we get

$$\mathsf{E}_2(\mathbf{J})=\mathsf{E}^Q\left\{D(0,T,\mathbf{J})I_{\{L(T)-L(0)>L\}}E^Q[S(T)I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L]|\mathcal{F}_T^X\right\}.\tag{35}$$

From Bayes' rule, we have

$$\begin{aligned}&\mathsf{E}^Q\left[S(T)I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\\ =&\mathsf{E}^Q\left[\eta_T^S|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\mathsf{E}^{Q^S}\left[\frac{S(T)}{\eta_T^S}I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\\ =&\mathsf{E}\left[\exp\left\{\int_0^T\sigma_1(u)\mathrm{d}W_1(u)-\frac{1}{2}\int_0^T\sigma_1^2(u)\mathrm{d}u\right\}\right]\mathsf{E}^{Q^S}\left[\frac{S(T)}{\eta_T^S}I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\\ =&S(0)D(0,T,\mathbf{J})\mathrm{e}^{-A(0,T)}\mathsf{E}^{Q^S}\left[I_{\{K\geqslant S(T)\}}I_{\{V(T)\geqslant D^*\}}|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\\ =&S(0)D(0,T,\mathbf{J})\mathrm{e}^{-A(0,T)}Q^S\left[\int_0^T\sigma_1(u)\mathrm{d}W_1^S(u)\leqslant\ln\frac{K}{S(0)}-R_T(\mathbf{J})-\frac{1}{2}U_T^1(\mathbf{J})+A(0,T),\right.\\ &\left.\int_0^T\sigma_2(u)\mathrm{d}W_2^S(u)\geqslant\ln\frac{D^*}{V(0)}-R_T(\mathbf{J})+\frac{1}{2}U_T^2(\mathbf{J})-\rho U_T^{12}(\mathbf{J})|\mathcal{F}_T^X\bigvee\mathcal{F}_T^L\right]\\ =&S(0)D(0,T,\mathbf{J})\mathrm{e}^{-A(0,T)}N_2(d_3(L(T),\mathbf{J}),d_4(\mathbf{J}),-\hat{\rho}(\mathbf{J})).\end{aligned}\tag{36}$$

Substituting (36) into (35), we get

$$\mathsf{E}_2(\mathbf{J})=\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\tfrac{(\lambda_T(\mathbf{J}))^n}{n!}\int_L^{\infty}S(0)\mathrm{e}^{-\alpha y+\beta(T)}N_2(d_3(y,\mathbf{J}),d_4(\mathbf{J}),-\hat{\rho}(\mathbf{J}))f_L^{*n}(y)\mathrm{d}y.\tag{37}$$

The proof is completed. □

Proposition 3 Let $\mathsf{E}_3(\mathbf{J})$ and $\mathsf{E}_4(\mathbf{J})$ be determined by (24) and (25). Then we have

$$\mathsf{E}_3(\mathbf{J}) = K\frac{(1-\omega)}{D}\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\frac{(\lambda_T(\mathbf{J}))^n}{n!}\int_L^{\infty} V(0)f_L^{*n}(y)N_2(d_5(y,\mathbf{J}),d_6(\mathbf{J}),\hat{\rho}(\mathbf{J}))\mathrm{d}y, \tag{38}$$

$$\begin{aligned}\mathsf{E}_4(\mathbf{J}) =&\frac{(1-\omega)}{D}\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\frac{(\lambda_T(\mathbf{J}))^n}{n!}\\ &\times\int_L^{\infty}\frac{S(0)V(0)}{D(0,T,\mathbf{J})}\mathrm{e}^{-\alpha y+\beta(T)}f_L^{*n}(y)N_2(d_7(y,\mathbf{J}),d_8(\mathbf{J}),\hat{\rho}(\mathbf{J}))\mathrm{d}y,\end{aligned} \tag{39}$$

where

$$d_5(y,\mathbf{J}) = \frac{\ln\dfrac{K}{S(0)} - R_T(\mathbf{J}) + \dfrac{1}{2}U_T^1(\mathbf{J}) - \rho U_T^{12}(\mathbf{J}) + \alpha y - \beta(T)}{\sqrt{U_T^1(\mathbf{J})}},$$

$$d_6(\mathbf{J}) = \frac{\ln\dfrac{D^*}{V(0)} - R_T(\mathbf{J}) - \dfrac{1}{2}U_T^1(\mathbf{J})}{\sqrt{U_T^2(\mathbf{J})}},$$

$$d_7(y,\mathbf{J}) = \frac{\ln\dfrac{K}{S(0)} - R_T(\mathbf{J}) - \dfrac{1}{2}U_T^1(\mathbf{J}) - \rho U_T^{12}(\mathbf{J}) + \alpha y - \beta(T)}{\sqrt{U_T^1(\mathbf{J})}},$$

$$d_8(\mathbf{J}) = \frac{\ln\dfrac{D^*}{V(0)} - R_T(\mathbf{J}) - \dfrac{1}{2}U_T^2(\mathbf{J}) - \rho U_T^{12}(\mathbf{J})}{\sqrt{U_T^2(\mathbf{J})}}.$$

Proof Define

$$\eta_T^V = \frac{\mathrm{d}Q^V}{\mathrm{d}Q} = \exp\left\{\int_0^T \sigma_2(u)\mathrm{d}W_2(u) - \frac{1}{2}\int_0^T \sigma_2^2(u)\mathrm{d}u\right\}. \tag{40}$$

By Girsanov's theorem, $W_1^V(t) = W_1(t) - \rho\int_0^t \sigma_2(u)\mathrm{d}u$ and $W_2^V(t) = W_2(t) - \int_0^t \sigma_2(u)\mathrm{d}u$ are two standard Brownian motions with $\mathsf{Cov}(\mathrm{d}W_1^V(t),\mathrm{d}W_2^V(t)) = \rho\mathrm{d}t$ under Q^V. Similar to the calculation of $\mathsf{E}_1(\mathbf{J})$ and $\mathsf{E}_2(\mathbf{J})$, we get

$$\mathsf{E}_3(\mathbf{J}) = K\frac{(1-\omega)}{D}\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\tfrac{(\lambda_T(\mathbf{J}))^n}{n!}\int_L^{\infty} V(0)N_2(d_5(y,\mathbf{J}),d_6(\mathbf{J}),\hat{\rho}(\mathbf{J}))f_L^{*n}(y)\mathrm{d}y. \tag{41}$$

Define

$$\eta_T^{SV} = \frac{\mathrm{d}Q^{SV}}{\mathrm{d}Q^V} = \exp\left\{\int_0^T \sigma_1(u)\mathrm{d}W_1^V(u) - \frac{1}{2}\int_0^T \sigma_1^2(u)\mathrm{d}u\right\}. \tag{42}$$

By Girsanov's theorem, $W_1^{SV}(t) = W_1^V(t) - \int_0^t \sigma_1(u)\mathrm{d}u$ and $W_2^{SV}(t) = W_2^V(t) - \rho\int_0^t \sigma_1(u)\mathrm{d}u$ are two standard Brownian motions with $\mathsf{Cov}(\mathrm{d}W_1^{SV}(t),\mathrm{d}W_2^{SV}(t)) = \rho\mathrm{d}t$ under Q^{SV}. Similar to the calculation of $\mathsf{E}_1(\mathbf{J})$, $\mathsf{E}_2(\mathbf{J})$ and $\mathsf{E}_3(\mathbf{J})$, we get

$$\begin{aligned}\mathsf{E}_4(\mathbf{J}) =& \frac{(1-\omega)}{D}\mathrm{e}^{-\lambda_T(\mathbf{J})}\sum_{n=1}^{\infty}\frac{(\lambda_T(\mathbf{J}))^n}{n!} \\ &\times\int_L^{\infty}\frac{S(0)V(0)}{D(0,T,\mathbf{J})}\mathrm{e}^{-\alpha y+\beta(T)}f_L^{*n}(y)N_2(d_7(y,\mathbf{J}),d_8(\mathbf{J}),\hat{\rho}(\mathbf{J}))\mathrm{d}y. \end{aligned} \tag{43}$$

The proof is completed. □

In order to deduce the closed-form expression for pricing the catastrophe put option with credit risk in a regime-switching model, we need to know the distribution of $\mathbf{J}$. In particular, we consider $N=2$, that is $X(t)$ switches between only 2 states, where state $\mathbf{e}_1$ and state $\mathbf{e}_2$ represent a "good" economy and a "bad" economy, respectively. Taylor[14] pointed out that a Markov chain with two states is sufficient to distinguish a normal market from the one experiencing a crisis. Let $f_1^k(s,T)$ be the probability density of J_1 given $X(0)=k$, for $k\in\{1,2\}$. As in Kovchegova et al.[12] and Yoon et al.[13], for all $0\leqslant s\leqslant T$, $f_1^k(s,T)$ satisfies

$$f_1^1(s,T) =\mathrm{e}^{-\lambda s-\nu(T-s)}[\delta(T-s)+\sqrt{\frac{\lambda\nu s}{T-s}}I_1(2\sqrt{\lambda\nu s(T-s)}+\lambda I_0(2\sqrt{\lambda\nu s(T-s)})],$$
$$f_1^2(s,T) =\mathrm{e}^{-\lambda s-\nu(T-s)}[\delta(s)+\sqrt{\frac{\lambda\nu(T-s)}{s}}I_1(2\sqrt{\lambda\nu s(T-s)}+\nu I_0(2\sqrt{\lambda\nu s(T-s)})],$$

where $I_\rho(z)$ is the modified Bessel function defined by

$$I_\rho(z)=\sum_{n=0}^{\infty}\frac{1}{n!\Gamma(\rho+n+1)}(\frac{z}{2})^{2n+\rho}.$$

Let $f_1(s,T)$ be the probability density of J_1, and then

$$f_1(s,T)=<(f_1^1(s,T),f_1^2(s,T))',X_0>.$$

Therefore, combining Proposition 2 and Proposition 3, the price of the catastrophe put option with credit risk in a regime-switching model is given by the following theorem.

Theorem 4 Let $p(0,T)$ be determined by (21), and then the price of the catastrophe put option with credit risk in a regime-switching model is

$$\begin{aligned} p(0,T) &= \int_0^T p(0,T,s)f_1(s,T)\mathrm{d}s \\ &= \int_0^T[\mathsf{E}_1(s)-\mathsf{E}_2(s)+\mathsf{E}_3(s)-\mathsf{E}_4(s)]f_1(s,T)\mathrm{d}s. \end{aligned} \tag{44}$$

5 Numerical analysis

In this section, using the explicit formulae obtained in the previous sections, we present some numerical analysis to examine how the parameters of the model affect the price of the catastrophe put option. We assume that the loss size conditional on a loss is fixed at l and that the trigger level is an integer multiple of the loss size, i.e., $L = Cl$. As explained in Jaimungal and Wang[2], and Jiang et al.[4], the parameter C, which coin the trigger ratio level, represents the ratio of the trigger level to the total expected loss amount. In this predetermined loss case, the probability density function of loss sizes is a Dirac density $f_L(y) = \delta(y - l)$, and then $\mathsf{E}_1(\mathbf{J}), \mathsf{E}_2(\mathbf{J}), \mathsf{E}_3(\mathbf{J})$ and $\mathsf{E}_4(\mathbf{J})$ can be expressed as follows:

$$\mathsf{E}_1(\mathbf{J}) = K\mathrm{e}^{-\lambda_T(\mathbf{J})} \sum_{n=C}^{\infty} \frac{(\lambda_T(\mathbf{J}))^n}{n!} D(0, T, \mathbf{J}) N_2(d_1(nl, \mathbf{J}), d_2(\mathbf{J}), -\hat{\rho}(\mathbf{J})), \tag{45}$$

$$\mathsf{E}_2(\mathbf{J}) = \mathrm{e}^{-\lambda_T(\mathbf{J})} \sum_{n=C}^{\infty} \frac{(\lambda_T(\mathbf{J}))^n}{n!} S(0)\mathrm{e}^{-\alpha nl + \lambda_T(\mathbf{J})(1-\mathrm{e}^{-\alpha l})} N_2(d_3(nl, \mathbf{J}), d_4(\mathbf{J}), -\hat{\rho}(\mathbf{J})), \tag{46}$$

$$\mathsf{E}_3(\mathbf{J}) = K\frac{(1-\omega)}{D}\mathrm{e}^{-\lambda_T(\mathbf{J})} \sum_{n=C}^{\infty} \frac{(\lambda_T(\mathbf{J}))^n}{n!} V(0) N_2(d_5(nl, \mathbf{J}), d_6(\mathbf{J}), \hat{\rho}(\mathbf{J})), \tag{47}$$

$$\mathsf{E}_4(\mathbf{J}) = \frac{(1-\omega)}{D}\mathrm{e}^{-\lambda_T(\mathbf{J})} \sum_{n=C}^{\infty} \frac{(\lambda_T(\mathbf{J}))^n}{n!} \frac{S(0)V(0)}{D(0,T,\mathbf{J})} \mathrm{e}^{-\alpha nl + \lambda_T(\mathbf{J})(1-\mathrm{e}^{-\alpha l})} \cdot N_2(d_7(nl, \mathbf{J}), d_8(\mathbf{J}), \hat{\rho}(\mathbf{J})). \tag{48}$$

Unless otherwise specified, the following parameters are fixed: $T = 5, V = 9$, $D^* = 5, D = 8, C = 2, \alpha = 0.2, l = 3, \mathbf{r} = (0.05, 0.02)'$, $\boldsymbol{\sigma_1} = (0.1, 0.4)', \boldsymbol{\sigma_2} = (0.1, 0.4)', A = \begin{pmatrix} -0.4 & 0.4 \\ 0.5 & -0.5 \end{pmatrix}$.

Fig. 1 presents the relation between the strike price K and the catastrophe put option price p for different ω and X(0). Fixed $\omega = 0.4, \boldsymbol{\lambda} = (0.1, 0.3)', \rho = 0.1, S(0) = 80$, we see that the catastrophe put option price p increases as the strike price K increases. The catastrophe put option price p is higher when we start at the state $\mathbf{e_2}$. Fixed the strike price K, we see that the catastrophe put option price p decreases as ω increases. Fig. 2 presents the relation between the share price S and the catastrophe put option price p for different ω and X(0). Fixed $\omega = 0.4, \boldsymbol{\lambda} = (0.1, 0.3)', \rho = 0.1, K = 80$, we see that the catastrophe put option price p decreases as the share price S increases. The catastrophe put option price p is higher when we start at the state $\mathbf{e_2}$. Fixed the share price S, we see that the catastrophe put option price p decreases as ω increases.

Fig. 3 presents the relation between the strike price K and the catastrophe put option price p for different $\boldsymbol{\lambda}$ and X(0). Fixed $\omega = 0.4, \boldsymbol{\lambda} = (0.1, 0.3)', \rho = 0.1, S(0) = 80$, we see that the catastrophe put option price p increases as the strike price K increases. The catastrophe put option price p is higher when we start at the state $\mathbf{e_2}$. Fixed the strike price K, we see that the catastrophe put option price p increases as λ increases. Fig. 4 presents the relation between the share price S and the catastrophe put option price p for different $\boldsymbol{\lambda}$ and X(0). Fixed $\omega = 0.4, \boldsymbol{\lambda} = (0.1, 0.3)', \rho = 0.1, K = 80$, we see that the catastrophe put option price p decreases as the share price S increases. The catastrophe put option price p is higher when we start at the state $\mathbf{e_2}$. Fixed the share price S, we see that the catastrophe put option price p increases as $\boldsymbol{\lambda}$ increases.

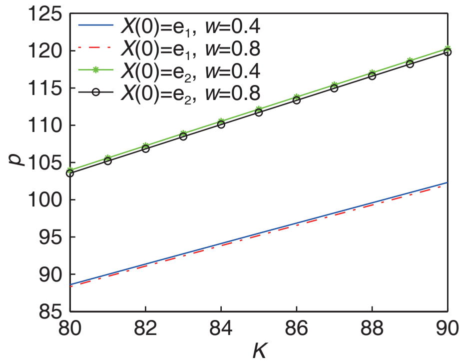

Figure 1 **The relation between K and p for different ω and $\mathbf{X}(0)$**

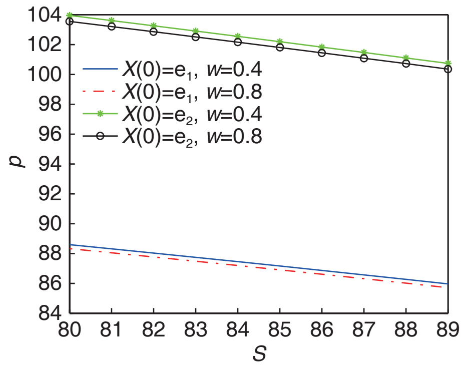

Figure 2 **The relation between S and p for different ω and $\mathbf{X}(0)$**

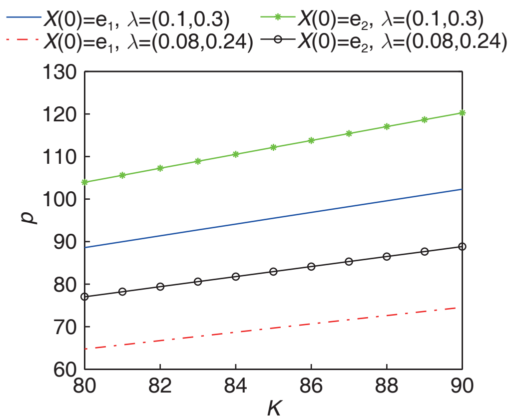

Figure 3 **The relation between K and p for different λ and $\mathbf{X}(0)$**

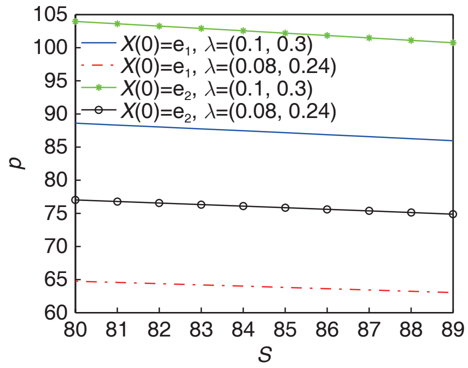

Figure 4 The relation between S and p for different λ and $\mathbf{X}(0)$

Fig. 5 presents the relation between the strike price K and the catastrophe put option price p for different ρ and X(0). Fixed $\omega = 0.4, \boldsymbol{\lambda} = (0.1, 0.3)', \rho = 0.1, S(0) = 80$, we see that the catastrophe put option price p increases as the strike price K increases. The catastrophe put option price p is higher when we start at the state $\mathbf{e_2}$. Fixed the strike price K, we see that the catastrophe put option price p decreases as ρ increases. Fig. 6 presents the relation between the share price S and the catastrophe put option price p for different ρ and X(0). Fixed $\omega = 0.4, \boldsymbol{\lambda} = (0.1, 0.3)', \rho = 0.1, K = 80$, we see that the catastrophe put option price p decreases as the share price S increases. The catastrophe put option price p is higher when we start at the state $\mathbf{e_2}$. Fixed the share price S, we see that the catastrophe put option price p decreases as ρ increases.

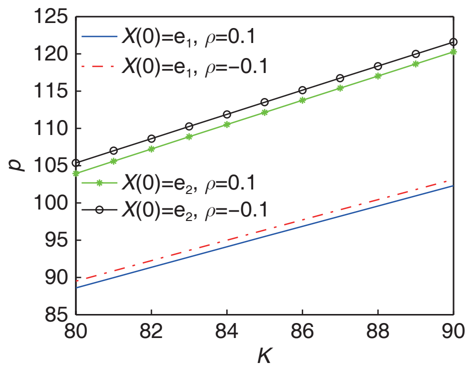

Figure 5 The relation between K and p for different ρ and $\mathbf{X}(0)$

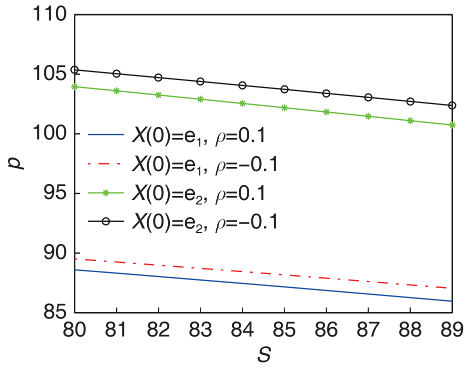

Figure 6 The relation between S and p for different ρ and $\mathbf{X}(0)$

Acknowledgements The authors are very grateful to the referees for valuable comments and criticisms to improve the earlier version of the paper.

References

[1] COX S H, PEDERSEN H W. Catastrophe risk bonds [J]. *N Am Actuar J*, 2000, **4(4)**: 56–82.

[2] COX S H, FAIRCHILD J R, PEDERSEN H W. Valuation of structured risk management products [J]. *Insur Math Econ*, 2004, **34(2)**: 259–272.

[3] JAIMUNGAL S, WANG T. Catastrophe options with stochastic interest rates and compound Poisson losses [J]. *Insur Math Econ*, 2006, **38(3)**: 469–483.

[4] JIANG I M, YANG S Y, LIU Y H, et al. Valuation of double trigger catastrophe options with counterparty risk [J]. *N Am J Econ Financ*, 2013, **25(3)**: 226–242.

[5] XU Y J, WANG G J. Pricing catastrophe options with counterparty credit risk in a reduced form model [J]. *Acta Math Sci Engl Ser*, 2018, **38(1)**: 347–360.

[6] ELLIOTT R J, CHAN L, SIU T K. Option pricing and Esscher transform under regime switching [J]. *Ann Finance*, 2005, **1(4)**: 423–432.

[7] WANG W, WANG W S. Pricing vulnerable options under a Markov-modulated regime switching model [J]. *Commun Stat-theor M*, 2010, **39(19)**: 3421–3433.

[8] WANG W, SU X N, GAN S B, et al. Pricing vulnerable European options under a Markov-modulated jump diffusion process [J]. *WSEAS Trans Math*, 2017, **16**: 123–132.

[9] ELLIOTT R J, AGGOUN L, MOORE J B. *Hidden Markov Models: Estimation and Control* [M]. New York: Springer, 1994.

[10] GRANDELL J. *Aspects of Risk Theory* [M]. New York: Springer-Verlag, 1991.

[11] MERTON R C. Option pricing when underlying stock returns are discontinuous [J]. *J Fin Econ*, 1976, **3(1-2)**: 125–144.

[12] KOVCHEGOVA Y, MEREDITH N, NIR E. Occupation times and Bessel densities [J]. *Stat Probabil Lett*, 2010, **80(2)**: 104–110.

[13] YOON J H, JANG B G, ROH K H. An analytic valuation method for multivariate contingent

claims with regime-switching volatilities [J]. *Oper Res Lett*, 2011, **39(3)**: 180–187.

[14] TAYLOR S J. *Asset Price Dynamics, Volatility and Prediction* [M]. New Jersey: Princeton University Press, 2005.

状态转移模型中带有信用风险的巨灾期权的定价

徐亚娟[1] 王过京[2]

[1] 苏州市职业大学数理部, 苏州, 215104

[2] 苏州大学金融工程中心和数学科学学院, 苏州, 215006

摘 要: 在本文中, 我们考虑了状态转移模型中带有信用风险的巨灾期权的定价问题. 我们假设宏观经济状态由具有有限状态空间的连续时间的马尔可夫链描述. 通过测度变换技术,我们导出了巨灾看跌期权的定价表达式. 此外, 我们通过数值分析展示了模型的参数变化对巨灾看跌期权价格的影响.

关键词: 定价; 巨灾期权; 信用风险; 状态转移; 测度变换

中图分类号: O211.6