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Estimation for Partially Linear Errors-in-Variable Models *

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Abstract: In this paper, we consider the estimation problem for partially linear models with additive measurement errors in the nonparametric part. Two kinds of estimators are proposed. The first one is an integral moment-based estimator with deconvolution kernel techniques, associated with the strong consistency for the estimator. Another one is a simulation-based estimator to avoid the integrals involved in the integral moment-based estimator. Simulation studies are conducted to examine the performance of the proposed estimators.

Keywords: deconvolution; errors-in-variables; partial linear models; replicated measurement error data

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§1. Introduction

A partially linear model is modeling the scalar response Y and covariates $(\boldsymbol{X}, Z)$ as

$$Y = \boldsymbol{X}^{\mathsf{T}}\boldsymbol{\beta} + g(Z) + \varepsilon, \tag{1}$$

where $(\boldsymbol{X}, Z) \in \mathbb{R}^p \times \mathbb{R}^1$, $\boldsymbol{X} = (X_1, X_2, \ldots, X_p)^{\mathsf{T}}$, $g(\cdot)$ is an unknown link function, and $\boldsymbol{\beta}$ is an unknown vector in $\mathbb{R}^p$. Model error ε is independent with $(\boldsymbol{X}, Z)$ satisfies $\mathsf{E}(\varepsilon) = 0$ and $\mathsf{E}(\varepsilon^2) < \infty$. This semiparametric regression model (1) has the simplicity of linear regression model but also has the flexibility of allowing nonparametric regression affects, it has been widely used in many applications. Two comprehensive references of statistical methodologies and applications for partially linear models are [1] and [2]. The goal of this

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paper is to develop a new estimation procedure for $\boldsymbol{\beta}$ and $g(\cdot)$ when covariate Z in the nonparametric component is measured with additive errors:

$$T = Z + U,$$

where the measurement error U, independent of $(Y, Z, \boldsymbol{X})$, is a random error with known or unknown density function $f_U(u)$. To avoid the "curse of dimensionality" in nonparametric regression, we only consider that Z is univariate in this paper.

Measurement error data or errors-in-variables data is common in practice, due to the measuring mechanism or the nature of the environment in medical or health related studies and others. Statistical analysis may cause serious bias in estimation if we substitute the observed surrogates for the unobserved error-prone variables, see for examples [3, 4]. To eliminate the effect of the measurement error, various statistical procedures have been developed for statistical inference in measurement error models. [5] is a comprehensive book containing many classical references and techniques for linear models. [6] and [7] are comprehensive references containing recent research developments of nonlinear and semiparametric regressions. Recently, Cheng et al.[8] presented a method for checking the goodness of fit in the restricted measurement error model, which is employed when certain study variables are not observable by direct measurement and if some information about the unknown regression coefficients is available a priori. Sørensen et al.[9] studied the impact of measurement error on linear regression with the lasso penalty. Yang et al.[10] proposed a corrected empirical likelihood method to make statistical inference for a class of generalized linear measurement error models based on the moment identities of the corrected score function.

For estimating $\boldsymbol{\beta}$, the "Pesudo-$\boldsymbol{\beta}$" method proposed by Liang and Wang[11] cannot be directly used, because it is challenging to estimate $\mathsf{E}(Y_i \mid Z_i)$, $\mathsf{E}(\boldsymbol{X}_i \mid Z_i)$ as $\{Z_i, i = 1, 2, \ldots, n\}$ are unobserved. To solve this problem, Zhu and Cui[12] proposed an integral moment-based estimation procedure with deconvolution techniques to estimate $\boldsymbol{\beta}$ and $g(\cdot)$ by involving an integration on $\mathbb{R}^{p+2}$ space, which is computational burden. In this paper, we propose a new integral moment-based estimator which only needs integration on $\mathbb{R}^1$ space. Moreover, a simulation-based estimator also considered to reduce the integration used in the integral moment-based estimator. We first consider the estimation procedures when density function $f_U(u)$ of U is known, then we develop our estimators by using replicated data when $f_U(u)$ is unknown. A simulation study is conducted to evaluate the performance of our proposed estimators. The reminder of the paper is organized as follows: In Section 2, we proposed our estimation procedures. In Section 3, we provide a simulation study to examine the performance of the estimation.

§2. Methodology Development

2.1 Constructing an Integral Moment-Based Deconvolution Estimator

Suppose that $\{\boldsymbol{X}_l, T_l, Y_l\}_{l=1}^n$ are i.i.d. observations and generated from the following model

$$\begin{cases} Y_l = \boldsymbol{X}_l^{\mathsf{T}}\boldsymbol{\beta} + g(Z_l) + \varepsilon_l, \\ T_l = Z_l + U_l, \end{cases}$$

where U_l is the measurement error, independent of $\{\boldsymbol{X}_l, Z_l, \varepsilon_l\}$. It follows that $\mathsf{E}(Y \mid Z) = g(Z) + [\mathsf{E}(\boldsymbol{X} \mid Z)]^{\mathsf{T}}\boldsymbol{\beta}$, and then we have

$$Y - \mathsf{E}(Y \mid Z) = [\boldsymbol{X} - \mathsf{E}(\boldsymbol{X} \mid Z)]^{\mathsf{T}}\boldsymbol{\beta} + \varepsilon. \tag{2}$$

From (2), in the population level, a least squares estimator is obtained as

$$\boldsymbol{\beta} = [\mathsf{E}(\boldsymbol{X}^{\otimes 2}) - \mathsf{E}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]]^{-1}[\mathsf{E}(\boldsymbol{X}Y) - \mathsf{E}\{\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)\}], \tag{3}$$

where $A^{\otimes 2} = AA^{\mathsf{T}}$ for any vector or matrix A. If Z is observable and error-free, the estimator $\boldsymbol{\beta}$ in (3) is implemented as usual, see for example [1, 2, 4, 13]. In the scenario considered in this paper, the covariate Z is unobserved and measured with errors, then the deconvolution techniques are used to take into account for solving the estimation problems in (3). Generally, the deconvolution problem is notoriously very hard. For estimating $\boldsymbol{\beta}$, Zhu and Cui [12] proposed a moment based deconvolution estimator by using integration on $\mathbb{R}^{p+2}$ space, which is computational burden. In fact, (3) motivates us to propose an estimator of $\boldsymbol{\beta}$ by directly estimating $\mathsf{E}(\boldsymbol{X}^{\otimes 2})$, $\mathsf{E}(\boldsymbol{X}Y)$, $\mathsf{E}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]$ and $\mathsf{E}\{\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)\}$. Among these arguments, the later two are most concerned, because Z is unobserved. In the following, we propose the estimators of $\mathsf{E}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]$, $\mathsf{E}\{\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)\}$ and illustrate the advantage that these estimators are only needed to integrated on $\mathbb{R}^1$ space, not on $\mathbb{R}^{p+2}$ space.

Throughout this section we first assume that the density function $f_U(u)$ of measurement error U is known. To estimate $\mathsf{E}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]$ and $\mathsf{E}[\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)]$, we should estimate $\mathsf{E}(\boldsymbol{X} \mid Z = z)$ and $\mathsf{E}(Y \mid Z = z)$ first, and this is accomplished by using local linear smoothers proposed by Delaigle et al. [14]. The main idea of [14] is to find an *unbiased score* function $L_k(\cdot)$, such that, for $k = 0, 1, 2$,

$$\mathsf{E}[(T_l - z)^k L_{k,h}(T_l - z) \mid Z_l] = (Z_l - z)^k K_h(Z_l - z), \tag{4}$$

where $L_{k,h}(\cdot) = h^{-1}L_k(\cdot/h)$. However, it is very difficult to directly work out the explicit expression $L_{k,h}(\cdot)$ in the integral equation (4). To solve this problem, Delaigle et al. [14]

developed a remarkable methodology to find a solution by using Fourier transform as follows.

Denote ϕ_m as the Fourier transform of function $m(\cdot)$ and denote ϕ_S as the characteristic function of a random variable S. The closed form of $L_{k,h}(\cdot)$ satisfying equation (4) is solved by the following Fourier version equation,

$$\mathsf{E}\{\phi_{\{(T_l-z)^k L_{k,h}(T_l-z)\}}(t) \mid Z_l\} = \phi_{(Z_l-z)^k K_h(Z_l-z)}(t). \tag{5}$$

The Appendix A.1 in [14] shows the solution of (5) is

$$L_{k,h}(u) = u^{-k} K_{U,k}(u) \qquad \text{and} \qquad K_{U,k}(u) = i^{-k}\frac{1}{2\pi}\int \mathrm{e}^{-itu}\frac{\phi_K^{(k)}(t)}{\phi_U(-t/h)}\mathrm{d}t, \tag{6}$$

where i is the imaginary unit, and $\phi_K^{(k)}(t)$ is the k-th derivative of $\phi_K(t)$. Define $m_x(z) = \mathsf{E}(\boldsymbol{X} \mid Z = z) = (m_{x_1}(z), m_{x_2}(z), \ldots, m_{x_p}(z))^\mathsf{T}$ with $m_{x_s}(z) = \mathsf{E}(X_s \mid Z = z)$ for $s = 1, 2, \ldots, p$ and $m_y(z) = \mathsf{E}(Y \mid Z = z)$. Using $L_{k,h}(u)$ obtained in (6), the local linear smoothing estimators of $m_y(z)$ and $m_{x_s}(z)$ are constructed as

$$\widehat{m}_y(z) = \sum_{l=l}^n \omega_l(z) Y_l \Big/ \sum_{l=l}^n \omega_l(z), \qquad \widehat{m}_{x_s}(z) = \sum_{l=l}^n \omega_l(z) X_{sl} \Big/ \sum_{l=l}^n \omega_l(z), \tag{7}$$

where $\omega_l(z) = K_{U,k,h}(T_l - z)\{S_{n,2}(z) - (T_l - z)S_{n,1}(z)\}$ with $K_{U,k,h}(u) = h^{-1}K_{U,k}(u/h)$, and $S_{n,k}(z) = \sum_{l=l}^n (T_l - z)^k K_{U,k,h}(T_l - z)$ for $k = 1, 2$. Moreover, the density function $f_Z(z)$ of Z is estimated as

$$\widehat{f}_Z(z) = \frac{1}{n}\sum_{l=1}^n K_{U,0,h}(T_l - z). \tag{8}$$

As $\mathsf{E}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}] = \int m_x^{\otimes 2}(z) f_Z(z)\mathrm{d}z$ and $\mathsf{E}[\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)] = \int m_x(z) m_y(z) f_Z(z)\mathrm{d}z$, together with (7) and (8), the related integral moment-based deconvolution estimators are obtained as

$$\widehat{\mathsf{E}}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}] = \int_{a_0}^{a_1} \widehat{m}_x^{\otimes 2}(z)\widehat{f}_Z(z)\mathrm{d}z, \tag{9}$$

$$\widehat{\mathsf{E}}[\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)] = \int_{a_0}^{a_1} \widehat{m}_x(z)\widehat{m}_y(z)\widehat{f}_Z(z)\mathrm{d}z, \tag{10}$$

where $\widehat{m}_x(z) = (\widehat{m}_{x_1}(z), \widehat{m}_{x_2}(z), \ldots, \widehat{m}_{x_p}(z))^\mathsf{T}$. As a consequence, the final estimator of $\boldsymbol{\beta}$ is proposed as

$$\widehat{\boldsymbol{\beta}} = \Big[\frac{1}{n}\sum_{l=1}^n \boldsymbol{X}_l^{\otimes 2} - \widehat{\mathsf{E}}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]\Big]^{-1}\Big[\frac{1}{n}\sum_{l=1}^n \boldsymbol{X}_l Y_l - \widehat{\mathsf{E}}\{\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)\}\Big]. \tag{11}$$

It is easily seen that the estimators proposed in (9) – (11) need one-dimensional integration, which is more computational efficient than [12]. Moreover, the estimator of nonparametric function $g(\cdot)$ is obtained as

$$\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z) = \sum_{l=l}^{n} \omega_l(z)[\,Y_l - \boldsymbol{X}_l^{\mathsf{T}}\widehat{\boldsymbol{\beta}}\,] \Big/ \sum_{l=l}^{n} \omega_l(z). \tag{12}$$

In the following, we present the strong consistency properties of $\widehat{\boldsymbol{\beta}}$ and $\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z)$. To establish asymptotic property of estimators, we need to impose some regularity conditions. As the common assumptions imposed in the deconvolution problems, the tail behavior of the characteristic function of measurement error U will impact on the convergence rate of $\widehat{m}_y(z)$, $\widehat{m}_{x_s}(z)$ and $\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z)$.

Condition O Ordinary smooth error of order η: the characteristic function of $\phi_U(\cdot)$ satisfies $\phi_U(t) \neq 0$ for all t, and

$$\lim_{t\to+\infty} t^{\eta}\phi_U(t) = c \qquad \text{and} \qquad \lim_{t\to+\infty} t^{\eta+1}\phi_U'(t) = -c\eta$$

for some positive constants c and η. Moreover, $\|\phi_U'(t)\|_\infty < \infty$; For $0 \leqslant k \leqslant 3$, $\|\phi_K^{(k)}(t)\|_\infty < \infty$, $\int [|t|^{\eta} + |t|^{\eta-1}]|\phi_K^{(k)}(t)|\mathrm{d}t < \infty$; For $0 \leqslant k, k' \leqslant 2$, $\int |t|^{2\eta}|\phi_K^{(k)}(t)| \cdot |\phi_K^{(k')}(t)|\mathrm{d}t < \infty$.

Condition S Supersmooth error of order η: the characteristic function of $\phi_U(\cdot)$ satisfies $\phi_U(t) \neq 0$ for all t, and

$$d_1|t|^{\eta_1}\exp\{-|t|^{\eta}/\gamma\} \leqslant |\phi_U(t)| \leqslant d_2|t|^{\eta_2}\exp\{-|t|^{\eta}/\gamma\}, \qquad \text{as } |t| \to \infty,$$

for some positive constants d_1, d_2, η_1, η_2, γ and η. Moreover, $\phi_K(t)$ is supported in $[-1, 1]$ and $\sup\limits_{t\in[-1,1]} |\phi_K^{(k)}(t)| < \infty$ for $0 \leqslant k \leqslant 2$.

The above ordinary smooth error conditions are followed from [3, 14–18]. For example, the Condition O for ordinary errors contains the Gamma distribution, double exponential distribution and the Condition S for supersmooth errors contains Cauchy errors, Gaussian errors and their convolutions. Moreover, the extra conditions on the Fourier transformation of kernel function $K(t)$ are also needed to establish the asymptotic property of estimators. See more details in [14]. We now list some conditions for our asymptotic results.

(A1) The characteristic functions $\phi_Z(t)$ and $\phi_U(t)$ satisfy $\int_{-\infty}^{\infty} |\phi_Z(t)|\mathrm{d}t < \infty$, and $\phi_U(t) \neq 0$ for all t; $\phi_K^{(k)}(t)$ is not identically zero, furthermore, for all $h > 0$ and $0 \leqslant k \leqslant 2$, $\int_{-\infty}^{\infty} \big|\phi_K^{(k)}(t)/\phi_U(t/h)\big|\mathrm{d}t < \infty$.

(A2) The kernel function $K(\cdot)$ is symmetric about 0 and supported on a compact interval, moreover, satisfying m-th order kernel:

$$\int K(u)\mathrm{d}u = 1, \qquad \int |u|^m K(u)\mathrm{d}u < \infty, \quad \text{for } 0 \leqslant m \leqslant 5.$$

(A3) $f_Z(\cdot)$, $m_y(\cdot)$ and $m_{x_s}(\cdot)$ have three bounded and continuous derivatives on $\mathscr{Z}$, and $\mathscr{Z}$ is the bounded support of Z. Moreover, the density function $f_Z(\cdot) > 0$ on the support of Z.

(A4) $\sup\limits_{z\in\mathscr{Z}} |m_y^{(j)}(z)| < \infty$, $\sup\limits_{z\in\mathscr{Z}} |m_{x_s}^{(j)}(\cdot)| < \infty$ for $j = 0, 1, 2$ and $s = 1, 2, \ldots, p$. Furthermore, $\mathsf{E}(Y^2 \mid Z = z)$ and $\mathsf{E}(X_s^2 \mid Z = z)$ are both bounded on $\mathscr{Z}$ for $s = 1, 2, \ldots, p$.

The following lemma establishes the convergence rate of $n^{-1} \sum\limits_{l=1}^{n} [(T_l - z)/h]^k L_{k,h}(T_l - z)$, which is used to establishes the strong consistency of the estimator $\widehat{\boldsymbol{\beta}}$ and $\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z)$. The following lemma is proved in analogous to Theorem 2.1 in [19], we omit the details.

Lemma 1 Under conditions (A1) − (A4), we have

$$\sup_{z\in\mathscr{Z}} \Big| \frac{1}{n} \sum_{l=1}^{n} \Big(\frac{T_l - z}{h}\Big)^k L_{k,h}(T_l - z) - \mathsf{E}\Big[\Big(\frac{T - z}{h}\Big)^k L_{k,h}(T - z)\Big] \Big| = O\big(\gamma_n^{1/2}\big), \qquad \text{a.s.},$$

$\gamma_{n,h} = \ln n/(nh^{1+2\eta})$ for ordinary error case, and $\gamma_{n,h} = \ln(n\delta_n)/(nh)$, $\delta_n = \exp\{2h^{-\eta}/\gamma\}$ for supersmooth error case.

Theorem 2 Under conditions (A1) − (A4),

(i) if the Condition O holds, as $h \to 0$ and $\ln n/(nh^{1+2\eta}) \to 0$, we have $\widehat{\boldsymbol{\beta}} \xrightarrow{\text{a.s.}} \boldsymbol{\beta}$, and $\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z) \xrightarrow{\text{a.s.}} g(z)$.

(ii) if the Condition S holds, as $h \to 0$ and $\ln(n\delta_n)/(nh) \to 0$, $\delta_n = \exp\{2h^{-\eta}/\gamma\}$, we have $\widehat{\boldsymbol{\beta}} \xrightarrow{\text{a.s.}} \boldsymbol{\beta}$, and $\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z) \xrightarrow{\text{a.s.}} g(z)$.

Proof Let $\tau_{n,h} = h^2 + \gamma_{n,h}^{1/2}$. According the Lemma, it is easily seen that $\widehat{m}_{x_s}(z) = m_{x_s}(z) + O(\tau_{n,h})$, a.s., $\widehat{m}_y(z) = m_y(z) + O(\tau_{n,h})$, a.s. and $\widehat{f}_Z(z) = f_Z(z) + O(\tau_{n,h})$, a.s.. Thus,

$$\begin{aligned}
\widehat{\mathsf{E}}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}] &= \int_{a_0}^{a_1} \widehat{m}_x^{\otimes 2}(z) \widehat{f}_Z(z) \mathrm{d}z \\
&= \int_{a_0}^{a_1} [m_x(z) + O(\tau_{n,h})]^{\otimes 2} [f_Z(z) + O(\tau_{n,h})] \mathrm{d}z \\
&= \mathsf{E}[m_x^{\otimes 2}(Z)] + O(\tau_{n,h}), \qquad \text{a.s.}.
\end{aligned}$$

Similarly, $\widehat{\mathsf{E}}[\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z) = \mathsf{E}[m_x(Z) m_y(Z)] + O(\tau_{n,h})$, a.s.. Thus,

$$\begin{aligned}
\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta} = {} & \Big[\frac{1}{n} \sum_{l=1}^{n} \boldsymbol{X}_l^{\otimes 2} - \widehat{\mathsf{E}}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]\Big]^{-1} \\
& \times \Big[\frac{1}{n} \sum_{l=1}^{n} \boldsymbol{X}_l [Y_l - \boldsymbol{X}_l^{\mathsf{T}} \boldsymbol{\beta}] - \widehat{\mathsf{E}}\{\mathsf{E}(\boldsymbol{X} \mid Z)\mathsf{E}(Y \mid Z)\} + \widehat{\mathsf{E}}[\{\mathsf{E}(\boldsymbol{X} \mid Z)\}^{\otimes 2}]\boldsymbol{\beta}\Big]. \qquad (13)
\end{aligned}$$

Note that $\mathsf{E}[m_x(Z)m_y(Z)] = \mathsf{E}[m_x^{\otimes 2}(Z)]\boldsymbol{\beta} + \mathsf{E}[m_x(Z)g(Z)]$, (13) is represented as

$$\begin{aligned}\widehat{\boldsymbol{\beta}} - \boldsymbol{\beta} = &\Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l^{\otimes 2} - \mathsf{E}[m_x^{\otimes 2}(Z)] + O(\tau_{n,h})\Big]^{-1} \\ &\times \Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l\varepsilon_l + \Big\{\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l g(Z_l) - \mathsf{E}[m_x(Z)g(Z)]\Big\} + O(\tau_{n,h})\Big].\end{aligned}$$

By the strong law of large numbers, $n^{-1}\sum_{l=1}^{n}\boldsymbol{X}_l\varepsilon_l \xrightarrow{\text{a.s.}} 0$ and $n^{-1}\sum_{l=1}^{n}\boldsymbol{X}_l g(Z_l) - \mathsf{E}[m_x(Z)g(Z)] \xrightarrow{\text{a.s.}} 0$, together with $\tau_{n,h}\to 0$, we obtain that $\widehat{\boldsymbol{\beta}} \xrightarrow{\text{a.s.}} \boldsymbol{\beta}$. The consistency of $\widehat{g}_{\boldsymbol{\beta}}(z)$ is completed by directly using Lemma and $\widehat{\boldsymbol{\beta}} \xrightarrow{\text{a.s.}} \boldsymbol{\beta}$. □

A simulation-based estimator. To obtain estimators (9) – (11), the numerical integration techniques are used, for example, the quadrature method. However, some computational difficulties may occur when the objective integral functions are complex. In this section, we consider a simulation-based approach by using Monte Carlo methods to simulate the integrals, namely, the importance sampling techniques, see for example, [20–22].

We choose a known density function $f_W(w)$ supported on $[a_0, a_1]$ and generate an i.i.d. sample $\{W_r, 1 \leqslant r \leqslant N\}$ from $f_W(w)$, then (9) – (10) are approximated by the Monte Carlo simulators

$$\begin{aligned}\widehat{\widehat{\mathsf{E}}}[\{\mathsf{E}(\boldsymbol{X}\,|\,Z)\}^{\otimes 2}] &= \frac{1}{N}\sum_{r=1}^{N}\frac{\widehat{m}_x^{\otimes 2}(W_r)\widehat{f}_Z(W_r)}{f_W(W_r)}, \\ \widehat{\widehat{\mathsf{E}}}[\mathsf{E}(\boldsymbol{X}\,|\,Z)\mathsf{E}(Y\,|\,Z)] &= \frac{1}{N}\sum_{r=1}^{N}\frac{\widehat{m}_x(W_r)\widehat{m}_y(W_r)\widehat{f}_Z(W_r)}{f_W(W_r)}.\end{aligned}$$

Then the simulation-based estimators of $\boldsymbol{\beta}$ and $g(z)$ are proposed as

$$\begin{aligned}\widehat{\boldsymbol{\beta}}_S &= \Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l^{\otimes 2} - \widehat{\widehat{\mathsf{E}}}[\{\mathsf{E}(\boldsymbol{X}\,|\,Z)\}^{\otimes 2}]\Big]^{-1}\Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l Y_l - \widehat{\widehat{\mathsf{E}}}\{\mathsf{E}(\boldsymbol{X}\,|\,Z)\mathsf{E}(Y\,|\,Z)\}\Big], \\ \widehat{g}_{\widehat{\boldsymbol{\beta}}_S}(z) &= \sum_{l=l}^{n}\omega_l(z)[Y_l - \boldsymbol{X}_l^{\mathsf{T}}\widehat{\boldsymbol{\beta}}_S]\Big/\sum_{l=l}^{n}\omega_l(z).\end{aligned}$$

2.2 Generalization: Unknown Measurement Error Distribution

If the density function $f_U(u)$ is unknown, the characteristic function $\phi_U(t)$ is estimated by using replicated data. Suppose we further observe an external sample

$$T'_{vd} = Z'_v + U'_{vd} \qquad \text{for } 1 \leqslant d \leqslant N_v \text{ and } 1 \leqslant v \leqslant m,$$

where the random variables Z'_v's are i.i.d. and have the same distribution as Z, the U'_{vd}'s are i.i.d. and has the same distribution as U, and the Z'_v's and U'_{vd}'s are independent.

From [23], a consistent estimator of $\phi_U(t)$ is given by

$$\widehat{\phi}_U(t) = \Big|\frac{1}{N(m)}\sum_{v=1}^{m}\sum_{(k_1,k_2)\in\mathscr{D}_v}\cos\{t(T'_{vk_1} - T'_{vk_2})\}\Big|^{1/2},$$

where $\mathscr{D}_v$ denotes the set of $2^{-1}N_v(N_v-1)$ distinct pairs (k_1, k_2) with $1 \leqslant k_1 < k_2 \leqslant N_v$, $N(m) = 2^{-1}\sum_{v=1}^{m} N_v(N_v-1)$, and we ignore values of v for which $N_v = 1$. Thus, an estimator of $L_{k,h}(u)$ is given by

$$\widehat{L}_{k,h}(u) = u^{-k}\widehat{K}_{U,k}(u), \qquad \widehat{K}_{U,k}(u) = i^{-k}\frac{1}{2\pi}\int \mathrm{e}^{-itu}\frac{\phi_K^{(k)}(t)}{\widehat{\phi}_U(-t/h)+\rho}\mathrm{d}t, \tag{14}$$

where $\rho \geqslant 0$ is a ridge parameter. The ridge parameter entails the stability of the estimator $\widehat{K}_{U,k}(u)$ without concern for fluctuations of the denominator in this integral. The ridge parameter ρ could be taken as $N(m)^{-\kappa}$ for some positive constant κ. In other words, the ridge parameter ρ converges to zero and entails that the estimator $\widehat{K}_{U,k}(u)$ is a consistent estimator of $K_{U,k}(u)$ as $N(m)$ goes to infinity. See more details in [23].

Let $\widehat{K}_{U,k,h}(u) = h^{-1}\widehat{K}_{U,k}(u/h)$. Followed by [23], according to (8), the estimator of $f_Z(z)$ is constructed as

$$\widehat{f}_Z(z)_\rho = \frac{1}{M+n}\sum_{l=1}^{n}\widehat{K}_{U,0,h}(T_l - z) + \frac{1}{M+n}\sum_{v=1}^{m}\sum_{d=1}^{N_v}\widehat{K}_{U,0,h}(T'_{vd} - z), \tag{15}$$

where $M = \sum_{v=1}^{m} N_v$. Similar to (9) – (10), $m_y(z)$ and $m_{x_s}(z)$ are estimated as

$$\widehat{m}_y(z)_\rho = \sum_{l=1}^{n}\widehat{\omega}_l(z)Y_l\Big/\sum_{l=1}^{n}\widehat{\omega}_l(z), \qquad \widehat{m}_{x_s}(z)_\rho = \sum_{l=1}^{n}\widehat{\omega}_l(z)X_{sl}\Big/\sum_{l=1}^{n}\widehat{\omega}_l(z),$$

where $\widehat{\omega}_l(z) = \widehat{K}_{U,k,h}(T_l - z)\{\widehat{S}_{n,2}(z) - (T_l - z)\widehat{S}_{n,1}(z)\}$ and $\widehat{S}_{n,k}(z) = \sum_{l=1}^{n}(T_l - z)^k\widehat{K}_{U,k,h}(T_l - z)$ for $k = 1, 2$.

Let $\widehat{m}_x(z)_\rho = (\widehat{m}_{x_1}(z)_\rho, \widehat{m}_{x_2}(z)_\rho, \ldots, \widehat{m}_{x_p}(z)_\rho)^\mathsf{T}$,

$$\widehat{\mathsf{E}}_\rho[\{\mathsf{E}(\boldsymbol{X}\,|\,Z)\}^{\otimes 2}] = \int_{a_0}^{a_1}\widehat{m}_x^{\otimes 2}(z)_\rho\widehat{f}_Z(z)_\rho\mathrm{d}z$$

and

$$\widehat{\mathsf{E}}_\rho[\mathsf{E}(\boldsymbol{X}\,|\,Z)\mathsf{E}(Y\,|\,Z)] = \int_{a_0}^{a_1}\widehat{m}_x(z)_\rho\widehat{m}_y(z)_\rho\widehat{f}_Z(z)_\rho\mathrm{d}z,$$

the estimators of $\boldsymbol{\beta}$ and $g(z)$ are obtained as

$$\widehat{\boldsymbol{\beta}}_\rho = \Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l^{\otimes 2} - \widehat{\mathsf{E}}_\rho[\{\mathsf{E}(\boldsymbol{X}\,|\,Z)\}^{\otimes 2}]\Big]^{-1}\Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_lY_l - \widehat{\mathsf{E}}_\rho\{\mathsf{E}(\boldsymbol{X}\,|\,Z)\mathsf{E}(Y\,|\,Z)\}\Big], \tag{16}$$

$$\widehat{g}_{\widehat{\boldsymbol{\beta}}_\rho}(z)=\sum_{l=l}^{n}\widehat{\omega}_l(z)[Y_l-\boldsymbol{X}_l^{\mathsf{T}}\widehat{\boldsymbol{\beta}}_\rho]\Big/\sum_{l=l}^{n}\widehat{\omega}_l(z). \tag{17}$$

Analogous to (16) – (17), the simulation-based estimators of $\boldsymbol{\beta}$ and $g(z)$ are obtained as

$$\widehat{\boldsymbol{\beta}}_{S\rho}=\Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_l^{\otimes 2}-\widehat{\widehat{\mathsf{E}}}_\rho[\{\mathsf{E}(\boldsymbol{X}\mid Z)\}^{\otimes 2}]\Big]^{-1}\Big[\frac{1}{n}\sum_{l=1}^{n}\boldsymbol{X}_lY_l-\widehat{\widehat{\mathsf{E}}}_\rho\{\mathsf{E}(\boldsymbol{X}\mid Z)\mathsf{E}(Y\mid Z)\}\Big], \tag{18}$$

$$\widehat{g}_{\widehat{\boldsymbol{\beta}}_{S\rho}}(z)=\sum_{l=l}^{n}\widehat{\omega}_l(z)[Y_l-\boldsymbol{X}_l^{\mathsf{T}}\widehat{\boldsymbol{\beta}}_{S\rho}]\Big/\sum_{l=l}^{n}\widehat{\omega}_l(z), \tag{19}$$

where

$$\widehat{\widehat{\mathsf{E}}}_\rho[\{\mathsf{E}(\boldsymbol{X}\mid Z)\}^{\otimes 2}]=\frac{1}{N}\sum_{r=1}^{N}\frac{\widehat{m}_x^{\otimes 2}(W_r)_\rho\widehat{f}_Z(W_r)_\rho}{f_W(W_r)},$$

$$\widehat{\widehat{\mathsf{E}}}_\rho[\mathsf{E}(\boldsymbol{X}\mid Z)\mathsf{E}(Y\mid Z)]=\frac{1}{N}\sum_{r=1}^{N}\frac{\widehat{m}_x(W_r)_\rho\widehat{m}_y(W_r)_\rho\widehat{f}_Z(W_r)_\rho}{f_W(W_r)}.$$

§3. A Simulation Study

In this section, we run 500 simulations to investigate the performance of our proposed methods. Considering the following model:

$$Y=X_1\beta_1+X_2\beta_2+X_3\beta_3+0.2Z^3+0.3\cos(Z)+\varepsilon, \tag{20}$$

$\boldsymbol{\beta}=(\beta_1,\beta_2,\beta_3)^{\mathsf{T}}=(4,-4,1)^{\mathsf{T}}$, X_i independently follows from $\mathrm{N}(0,1)$ for $i=1,2,3$, $Z\sim\mathrm{N}(0,1.56^2)$ independently with $\boldsymbol{X}=(X_1,X_2,X_3)^{\mathsf{T}}$, and model error $\varepsilon\sim\mathrm{N}(0,0.5^2)$, independent with $(\boldsymbol{X}^{\mathsf{T}},Z)^{\mathsf{T}}$. Moreover, the following two cases for the error distribution are considered.

Case 1 Ordinary smooth error: U is generated from a double exponential distribution $f_U(u)=\sqrt{2}\exp\{-2\sqrt{2}|u|\}$. The reliability ratio [5] $\mathsf{Var}\,(Z)/(\mathsf{Var}\,(U)+\mathsf{Var}\,(Z))$ equals to 90.5%.

Case 2 Supersmooth error: U is generated from a normal distribution $\mathrm{N}(0,0.5^2)$. The reliability ratio $\mathsf{Var}\,(Z)/(\mathsf{Var}\,(U)+\mathsf{Var}\,(Z))$ equals to 90.7%.

The Fourier transform of kernel function $K(\cdot)$ is given by $\phi_K(t)=(1-t^2)^3 1\{|t|\leqslant 1\}$ [14]. From (10), we have that

$$K_{U,0}(u)=\frac{1}{\pi}\int_0^1\cos(tu)(1-t^2)^3\Big\{1+\frac{t^2}{h^2}\Big\}\mathrm{d}t,\qquad \text{Ordinary smooth error},$$

$$K_{U,0}(u)=\frac{1}{\pi}\int_0^1\cos(tu)(1-t^2)^3\exp\Big\{\frac{t^2}{2h^2}\Big\}\mathrm{d}t,\qquad \text{Supersmooth error}.$$

The expressions of $K_{U,1}(u)$ and $K_{U,2}(u)$ are obtained similarly under these two measurement error settings. When the density function $f_U(u)$ is known, the sample size n is set to be $n = 500, 1\,000$; When the density function $f_U(u)$ is unknown, the extra replicated sample to estimate $\phi_U(u)$ is set to be $T'_{vd} = Z'_v + U'_{vd}$, $d = 1, 2$. The sample size of v for replicated data equals to n. Furthermore, Z'_vs are generated the same as Z, and U'_{vd}s are generated from the same distributions designed in Case 1 and Case 2, respectively. The numerical integration method is used to obtain integral moment-based deconvolution estimators $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\beta}}_\rho$. For simulation-based estimators $\widehat{\boldsymbol{\beta}}_S$ and $\widehat{\boldsymbol{\beta}}_{S\rho}$, the important sampling density function $f_W(w)$ is chosen as $\mathrm{N}(0,1)$ for Case 1 and Case 2. The important sampling sample size N equals to 200. Moreover, the ridge parameter ρ equals to v^{-1} in order to reduce the fluctuations of the denominator in the integral at (14).

The simulation results for estimators $\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\beta}}_S$, $\widehat{\boldsymbol{\beta}}_\rho$ and $\widehat{\boldsymbol{\beta}}_{S\rho}$ are reported in Table 1. It is seen that the means are all close to the true values $\boldsymbol{\beta}$. As sample size n increases to 1 000, the values of MSEs decrease. Moreover, $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\beta}}_S$ are better than $\widehat{\boldsymbol{\beta}}_\rho$ and $\widehat{\boldsymbol{\beta}}_{S\rho}$, the latter two are impacted by a small price to pay for having to estimate $\phi_U(u)$.

Table 1 Simulation study for the example. The means, standard errors (SE) and mean squares errors (MSE) for integral moment-based deconvolution estimators $\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\beta}}_\rho$ and simulation-based estimators $\widehat{\boldsymbol{\beta}}_S$, $\widehat{\boldsymbol{\beta}}_{S\rho}$.

		$\widehat{\boldsymbol{\beta}}$			$\widehat{\boldsymbol{\beta}}_S$			$\widehat{\boldsymbol{\beta}}_\rho$			$\widehat{\boldsymbol{\beta}}_{S\rho}$		
	$\boldsymbol{\beta}$	Mean	SE	MSE	Mean	SE	MSE	Mean	SE	MSE	Mean	SE	MSE
ordinary smooth error													
	β_1	4.0078	0.1367	0.0186	3.9947	0.1286	0.0165	4.0157	0.1531	0.0236	4.0279	0.1605	0.0264
$n = 500$	β_2	-3.9947	0.1387	0.0192	-4.0059	0.1232	0.0151	-3.9967	0.1483	0.0219	-4.0239	0.1443	0.0231
	β_3	1.0135	0.1316	0.0174	1.0034	0.1253	0.0156	1.0060	0.1530	0.0233	0.9882	0.1449	0.0210
	β_1	4.0053	0.0923	0.0085	3.9995	0.0803	0.0064	3.9941	0.1059	0.0112	4.0017	0.1133	0.0128
$n = 1\,000$	β_2	-3.9962	0.0953	0.0090	-3.9996	0.0783	0.0061	-4.0057	0.1071	0.0114	-4.0037	0.1055	0.0111
	β_3	1.0001	0.0884	0.0078	1.0027	0.0729	0.0053	0.9886	0.1053	0.0112	1.0040	0.0912	0.0083
supersmooth error													
	β_1	4.0143	0.1374	0.0190	4.0003	0.1413	0.0210	4.0026	0.1409	0.0240	4.0011	0.1510	0.0227
$n = 500$	β_2	-4.0030	0.1340	0.0179	-3.9845	0.1386	0.0214	-4.0026	0.1398	0.0255	-4.0022	0.1524	0.0232
	β_3	0.9975	0.1409	0.0198	1.0147	0.1578	0.0225	0.9981	0.1386	0.0262	1.0071	0.1611	0.0229
	β_1	4.0008	0.0909	0.0082	3.9977	0.1149	0.0131	3.9958	0.1089	0.0219	4.0011	0.1398	0.0174
$n = 1\,000$	β_2	-4.0028	0.0990	0.0098	-3.9981	0.1225	0.0149	-3.9974	0.1043	0.0209	-4.0017	0.1183	0.0175
	β_3	0.9956	0.1038	0.0108	0.9980	0.1098	0.0120	0.9996	0.1098	0.0220	0.9877	0.1134	0.0156

To investigate the performance of estimators of $g(\cdot)$, we use the square root of average square error (RASE) criteria:

$$\mathrm{RASE} = \left\{ n_{\mathrm{grid}}^{-1} \sum_{k=1}^{n_{\mathrm{grid}}} [\widehat{g}(z_k) - g(z_k)]^2 \right\}^{1/2}, \tag{21}$$

where $\{z_k,\ k=1,2,\ldots,n_{\rm grid}\}$ are the grid points at which the estimate $\widehat{g}(z_k)$ is evaluated and $n_{\rm grid}=200$ is used. We compare the performances of estimators $\widehat{g}_{\widehat{\boldsymbol\beta}}(z)$, $\widehat{g}_{\widehat{\boldsymbol\beta}_S}(z)$, $\widehat{g}_{\widehat{\boldsymbol\beta}_\rho}(z)$ and $\widehat{g}_{\widehat{\boldsymbol\beta}_{S\rho}}(z)$ based on RASE. Numerical results are reported in Table 2. We can see that all these four estimators of $g(z)$ perform well. For each simulation, we compute the RASE defined in (21) by using a sequence of 10 bandwidths ranging from $[0.2,0.4]$. The optimal bandwidth is selected to minimize the RASE among these 10 candidates. We conduct one simulation and present the estimated curves of $\widehat{g}_{\widehat{\boldsymbol\beta}}(z)$, $\widehat{g}_{\widehat{\boldsymbol\beta}_S}(z)$, $\widehat{g}_{\widehat{\boldsymbol\beta}_\rho}(z)$ and $\widehat{g}_{\widehat{\boldsymbol\beta}_{S\rho}}(z)$ for samples of size $n=500$ in Figures 1−2. It is seen that our estimation procedures works well.

Table 2 Simulation study for the example. The means and standard errors (SE) of RASE for estimators $\widehat{g}_{\widehat{\boldsymbol\beta}}(z)$, $\widehat{g}_{\widehat{\boldsymbol\beta}_S}(z)$, $\widehat{g}_{\widehat{\boldsymbol\beta}_\rho}(z)$ and $\widehat{g}_{\widehat{\boldsymbol\beta}_{S\rho}}(z)$.

	$\widehat{g}_{\widehat{\boldsymbol\beta}}(z)$		$\widehat{g}_{\widehat{\boldsymbol\beta}_S}(z)$		$\widehat{g}_{\widehat{\boldsymbol\beta}_\rho}(z)$		$\widehat{g}_{\widehat{\boldsymbol\beta}_{S\rho}}(z)$	
	RASE	SE	RASE	SE	RASE	SE	RASE	SE
ordinary smooth error								
$n=500$	0.1011	0.0344	0.1096	0.0398	0.1023	0.0352	0.1081	0.0371
$n=1\,000$	0.0789	0.0249	0.0844	0.0270	0.0805	0.0253	0.0782	0.0277
supersmooth error								
$n=500$	0.1098	0.0372	0.1191	0.0419	0.1185	0.0430	0.1192	0.0455
$n=1\,000$	0.0832	0.0269	0.0848	0.0248	0.0848	0.0311	0.0870	0.0335

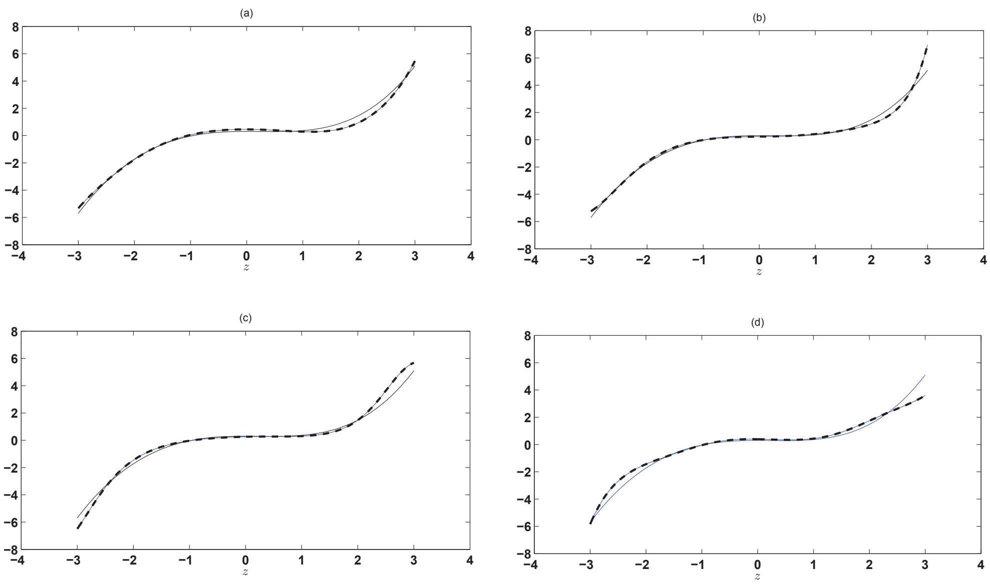

Figure 1 Ordinary error case: Plots of (a) $\widehat{g}_{\widehat{\boldsymbol\beta}}(z)$ (dotted lines) and $g(z)$ (solid line); (b) $\widehat{g}_{\widehat{\boldsymbol\beta}_S}(z)$ (dotted lines) and $g(z)$ (solid line); (c) $\widehat{g}_{\widehat{\boldsymbol\beta}_\rho}(z)$ (dotted lines) and $g(z)$ (solid line); (d) $\widehat{g}_{\widehat{\boldsymbol\beta}_{S\rho}}(z)$ (dotted lines) and $g(z)$ (solid line)

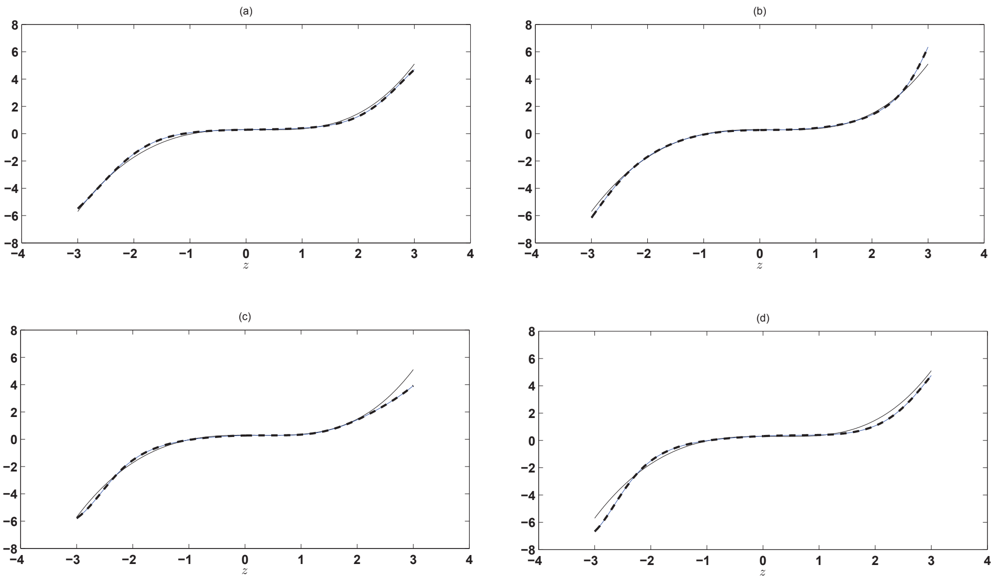

Figure 1 Supersmooth error case: Plots of (a) $\widehat{g}_{\widehat{\boldsymbol{\beta}}}(z)$ (dotted lines) and $g(z)$ (solid line); (b) $\widehat{g}_{\widehat{\boldsymbol{\beta}}_S}(z)$ (dotted lines) and $g(z)$ (solid line); (c) $\widehat{g}_{\widehat{\boldsymbol{\beta}}_\rho}(z)$ (dotted lines) and $g(z)$ (solid line); (d) $\widehat{g}_{\widehat{\boldsymbol{\beta}}_{S_\rho}}(z)$ (dotted lines) and $g(z)$ (solid line)

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部分线性测量误差模型中的估计问题

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摘　要: 本文考虑了部分线性模型中非参数部分带有可加测量误差的估计问题. 本文提出了两种估计方法, 第一种是基于逆卷积的积分矩估计方法, 给出该估计的强相合收敛性. 第二种是基于模拟的估计方法, 该方法避免了积分矩估计方法中的积分问题. 最后本文用一些数值结果来说明估计方法的估计效果.

关键词: 逆卷积; 测量误差模型; 部分线性模型; 重复测量误差数据

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