应用概率统计 第十八卷
第一期 2002年2月

Chinese Journal of Applied Probability
and Statistics Vol. 18 No. 1 Feb. 2002

Logarithmic Sobolev Inequalities for Birth–Death Process and Diffusion Process on the Line*

MAO YONGHUA
(Department of Mathematics. Beijing Normal University, Beijing, 100875)

Abstract

In this paper we give. by using Hardy–type inequalities, characterizations of the logarithmic Sobolev inequalities for birth–death process and diffusion process on the line.

Keywords: Logarithmic Sobolev inequalities, birth–death process, diffusion process, Hardy–type inequality.

AMS Subject Classification: 60J80, 39A12.

§1. Introduction

Logarithmic Sobolev inequalities are an essential tool in the study of many problems (Sec, for example, [1][2][3]). Recent years, many efforts have been made to verify whether a Markov process to satisfy logarithmic Sobolev inequality. Especially, [4][5][6][7] gave some sufficient conditions for general symmetric forms. The main purpose of the present paper is to give a characterization for the logarithmic Sobolev inequalities for birth–death process and diffusion process on the line. Our main tools are Hardy–type inequalities with weights, and an idea, due to Bobkov and Götze([8]), which transforms logarithmic Sobolev inequalities into Poincaré–type inequalities in an Orlicz space.

Let (E,π) denote a probability space, and $L_p(E,\pi)$ the usual L_p–space with norm $\|\cdot\|_p$. Assume on it there is a Dirichlet form $(D,\mathcal{D}(D))$ with Markov generator L. We will say that (E,π,L) satisfies the logarithmic Sobolev inequality if for all $f\in\mathcal{D}(D)$,

$$\mathrm{Ent}(f^2)\le\frac{2}{c}D(f,f),\tag{1.1}$$

where $\mathrm{Ent}(f)=\int f\log f d\pi-\int f d\pi\log\int f d\pi$ for $f\ge 0$. Denote α the greatest number in (1.1), which is called the log–Sobolev constant.

Suppose $\Psi,N:\mathbb{R}\longrightarrow\mathbb{R}_+$ the Young functions $\Psi(x)=|x|\log(1+|x|)$ and $N(x)=\Psi(x^2)$; the Orlicz space $L_N=L_N(E,\pi)$ consists of all measurable functions with norm

$$\|f\|_N=\inf\left\{\lambda>0:\int N(f/\lambda)d\pi\le 1\right\}<+\infty.$$

Define similarly the Orlicz space $L_\Psi=L_\Psi(E,\pi)$. Then $\|f^2\|_\Psi=\|f\|_N^2$.

In [8], (1.1) is connected to a Poincaré–type inequality in the Orlicz space L_N. Precisely, (1.1) is equivalent to

$$\left\|f-\int f d\pi\right\|_N^2\le\frac{2}{d}D(f,f),\tag{1.2}$$

with $\frac{2}{5}d\le\alpha\le\frac{3}{2}d$.

*Research supported in part by RFDP (No. 96002704), NSFC (No. 19771008) and Fok Ying-Tung Youth Foundation.
Received 2000.11.27. Revised 2000.12.25.

In what follows, $E = \mathbb{R}$ or $E = \mathbb{Z}_+$. To understand the method better, we begin with the birth–death process.

§2. Birth–death process

Consider a birth–death process $a_i > 0$ $(i \geq 1)$, $b_i > 0$ $(i \geq 0)$ on $E = \mathbb{Z}_+$. Suppose that the process is positively recurrent, i.e. $\sum_{n\geq 0} \mu_n \sum_{i\geq n} (\mu_i b_i)^{-1} = \infty$ and $\mu := \sum_{n=0}^{\infty} \mu_n < \infty$, where $\mu_0 = 1$, $\mu_n = b_0 b_1 \cdots b_{n-1}/a_1 a_2 \cdots a_n$. Denote $\pi_n = \mu_n/\mu$, the symmetric probability. Define

$$\delta = \sup_{n\geq 1} \sum_{i=n}^{\infty} \pi_i \Big(-\log \sum_{i=n}^{\infty} \pi_i \Big) \sum_{i=1}^{n} \frac{1}{\pi_i a_i},$$

and

$$\overline{\delta} = \limsup_{n\to\infty} \sum_{i=n}^{\infty} \mu_i \Big(-\log \sum_{i=n}^{\infty} \mu_i \Big) \sum_{i=1}^{n} \frac{1}{\mu_i a_i}.$$

Theorem 2.1 The logarithmic Sobolev constant α for the above process satisfies

$$c_1 \delta^{-1} \leq \alpha \leq c_2 \delta^{-1}$$

with $c_1 = \frac{1}{80}(1-\pi_0)[-\log(1-\pi_0)]$, $c_2 = 6(1-\sqrt{1-\pi_0})^{-2}$.

Especially, $\alpha > 0$ if and only if $\overline{\delta} < \infty$.

To prove the theorem, we need some preparations.

Lemma 2.2 Let (E, π) be an arbitrary probability space, for f in $L_N(E, \pi)$,

$$\Big\| f - \int_E f d\pi \Big\|_N \leq 2\|f\|_N. \tag{2.1}$$

If $f|_A = 0$ for a measurable subset A with $\pi(A) > 0$, then

$$\|f\|_N \leq \frac{1}{1-\sqrt{1-\pi(A)}} \Big\| f - \int_E f d\pi \Big\|_N. \tag{2.2}$$

Proof Denote Ψ^{-1} the inverse function of Ψ, by the definition, we get $\|1\|_N^2 = \|1\|_\Psi = (\Psi^{-1}(1))^{-1} < 1$. It follows from the facts $\|f\|_1 \leq \|f\|_2 \leq \Psi^{-1}(1)\|f\|_N$ that $\|f - \int f d\pi\|_N \leq \|f\|_N + \|f\|_1 \cdot \|1\|_N \leq 2\|f\|_N$. This prove (2.1). To prove (2.2), using the Cauchy–Schwarz inequality and the facts above, we have for $f|_A = 0$.

$$\Big| \int f d\pi \Big| = \Big| \int f I_{A^c} d\pi \Big| \leq \sqrt{1-\pi(A)}\|f\|_2 \leq \sqrt{(1-\pi(A))\Psi^{-1}(1)}\|f\|_N.$$

Hence,

$$\|f\|_N \leq \Big\| f - \int f d\pi \Big\|_N + \Big| \int f d\pi \Big| \|1\|_N \leq \Big\| f - \int f d\pi \Big\|_N + \sqrt{1-\pi(A)}\|f\|_N.$$

Therefore,

$$\|f\|_N \leq \frac{1}{1-\sqrt{1-\pi(A)}} \Big\| f - \int f d\pi \Big\|_N. \qquad \square$$

Proposition 2.3 Define

$$\tilde{\alpha} = \inf_{f\in\mathcal{F}} \frac{D(f,f)}{\|f\|_N^2},$$

where $\mathcal{F} = \{f \in \mathcal{D}(D) : f(0) = 0\}\backslash\{0\}$. Then we have

$$\frac{1}{5}\tilde{\alpha} \leq \alpha \leq 3\tilde{\alpha}(1-\sqrt{1-\pi_0})^{-2}.$$

Proof Suppose (1.1) is fulfilled with log–Sobolev constant $\alpha > 0$. So by (1.2), we get

$$\left\|f - \int f d\pi\right\|_N^2 \le \frac{3}{\alpha} D(f,f). \tag{2.3}$$

Thus for any $f \in \mathcal{F}$, it follows from Lemma 2.2 that

$$\|f\|_N^2 \le \frac{3}{\alpha(1-\sqrt{1-\pi_0})^2} D(f,f).$$

Conversely, suppose that $\tilde{\alpha} > 0$. $\forall f \in \mathcal{D}(D)$, set $\overline{f} = f - f(0)$, then $f \in \mathcal{F}$ and $D(\overline{f},\overline{f}) = D(f,f)$, $\overline{f} - \int \overline{f} d\pi = f - \int f d\pi$. So by Lemma 2.2, we have

$$\left\|f - \int f d\pi\right\|_N^2 \le 4\|\overline{f}\|_N^2 \le \frac{4}{\tilde{\alpha}} D(f,f).$$

Hence by (1.2), we get $\alpha \ge 5\tilde{\alpha}$, which completes the proof. □

In light of Proposition 2.3, we need only to estimate $\tilde{\alpha}$. To this aim, the following Hardy–type inequality in the discrete setting is useful (cf.[9] Appendix). Let u_i, v_i be two positive sequences on E, define, for $f(0) = 0$,

$$A = \sup_{f \in L_2(v)} \frac{\|f\|_{L_2(u)}^2}{\|\Delta f\|_{L_2(v)}^2},$$

where $\Delta f(i) = f(i) - f(i-1)$ $(i \ge 1)$ and for $w = u$ or v, $\|g\|_{L_2(w)}^2 = \sum_{n \ge 1} w_n g^2(n)$, $L_2(w) = \{g : \|g\|_{L_2(w)} < \infty\}$; and

$$B = \sup_{n \ge 1} \|I_{[n,\infty)}\|_{L_2(u)} \sum_{i=1}^{n} \frac{1}{v_i}.$$

Then $B \le A \le 16B$.

Using the same argument on $\mathbb{R}$ as in [8], we can have the following natural generalization to the Orlicz spaces for discrete settings. The proof is similar to that of the case on $\mathbb{R}$ ([8] Corollary 5.2), so we omit it.

Proposition 2.4 Let $L_N(E,\pi)$ the Orlicz space defined as above. Define A and B with $\|\cdot\|_{L_2(u)}$ replaced by the Orlicz norm $\|\cdot\|_N$. Then $B \le A \le 16B$.

Proof of Theorem 2.1 Since $\|f^2\|_\Psi = \|f\|_N^2$, we have

$$\beta(n) := \|I_{[n,\infty)}\|_\Psi = 1/\Psi^{-1}(\pi([n,\infty))^{-1}).$$

Set, for $i \ge 0$, $u_i = \pi_i$, and $v_i = \pi_i a_i$. Then for $f \in \mathcal{F}$, $D(f,f) = \|\Delta f\|_{L_2(v)}^2$ and $\|f^2\|_\Psi = \|f\|_N^2$. Hence by Proposition 2.4 and the definition of $\tilde{\alpha}$, we get

$$\sup_{n\ge1} \beta(n) \sum_{i=1}^{n} \frac{1}{\pi_i a_i} \le \tilde{\alpha}^{-1} \le 16 \sup_{n \ge 1} \beta(n) \sum_{i=1}^{n} \frac{1}{\pi_i a_i}.$$

Thus by Proposition 2.3 and Lemma 2.5 below, we arrive at the first assertion. Then the second assertion follows easily. □

Lemma 2.5 For $t \ge t_0 > 1$, then

$$\frac{ct}{\log t} \le \Psi^{-1}(t) \le \frac{2t}{\log t} \tag{2.4}$$

for $c = \log t_0 / t_0$.

Proof The first inequality in (2.4), i.e.

$$\Psi\Big(\frac{ct}{\log t}\Big) = \frac{ct}{\log t} \log\Big(1 + \frac{ct}{\log t}\Big) \le t,$$
$$\log\Big(1 + \frac{ct}{\log t}\Big) \le c^{-1} \log t$$

will follow, for $t \geq t_0$, from (denote $\gamma = c^{-1}$)

$$t^{\gamma} - \frac{t}{\gamma \log(t_0)} - 1 \geq 0.$$

If set $\phi(t) = t^{\gamma} - \dfrac{t}{\gamma \log(t_0)} - 1$, then $\phi'(t) = \gamma t^{\gamma - 1} - \dfrac{1}{\gamma \log(t_0)}$, so we can choose $\gamma = t_0/\log t_0 > 1$ such that $\phi(t_0) \geq 0$, $\phi'(t) \geq 0$. This proves the first inequality. The second inequality in (2.4)

$$\Psi\Big(\frac{2t}{\log t}\Big) = \frac{2t}{\log t} \log\Big(1 + \frac{2t}{\log t}\Big) \geq t$$

can be read as

$$1 + \frac{2t}{\log t} \geq t^{1/2}$$

and follows from $2t^{1/2} \geq \log t$, which, however, is always true for $t > 1$. Thus Lemma 2.5 follows. □

Corolllary 2.6 Set

$$\gamma(n) = \mu_n \Big/ \sum_{i \geq n+1} \mu_i,$$

suppose that the limit $\gamma := \lim_{n\to\infty} \gamma_n$ exists and the limits involved below exist.

(1) When $\gamma = 0$, then $\alpha > 0$ if and only if

$$\lim_{n\to\infty} \frac{\sum_{i\geq n} \mu_i \sqrt{- \log \sum_{i \geq n} \mu_i}}{\mu_n \sqrt{a_n}} < \infty; \tag{2.5}$$

(2) When $\gamma = \infty$, then $\alpha > 0$ if and only if

$$\lim_{n\to\infty} (-\mu_n \log \mu_n) \sum_{i=1}^{n} \frac{1}{\mu_i a_i} < \infty; \tag{2.6}$$

(3) When $\gamma \in (0, \infty)$, then $\alpha > 0$ if and only if

$$\lim_{n\to\infty} \mu_n \sum_{i=1}^{n} \frac{1}{\mu_i a_i} < \infty, \tag{2.7}$$

and

$$\lim_{n\to\infty} \frac{- \log \mu_n}{a_n} < \infty. \tag{2.8}$$

Proof Denote $\xi_n = \sum_{i\geq n} \mu_n$, then ξ_n decreases to zero. Hence by Stolz' theorem, $\overline{\delta} > 0$ if and only if the limit of the following expression is finite

$$\begin{aligned} &\frac{- \sum_{i=1}^{n+1} \frac{1}{\mu_i a_i} \log \sum_{i \geq n+1} \mu_i + \sum_{i=1}^{n} \frac{1}{\mu_i a_i} \log \sum_{i\geq n} \mu_i}{\xi_{n+1}^{-1} - \xi_n^{-1}} \\ =: &I_1(n) + I_2(n) + I_3(n) + I_4(n), \end{aligned}$$

where

$$I_1(n) = \frac{1}{\mu_n}\Big(\sum_{i\ge n+1}\mu_i\Big)^2\Big(\sum_{i=1}^n \frac{1}{\mu_i a_i}\Big)\log\Big(\frac{\sum_{i\ge n}\mu_i}{\sum_{i\ge n+1}\mu_i}\Big)$$

$$= \gamma_n^{-2}\mu_n\Big(\sum_{i=1}^n \frac{1}{\mu_i a_i}\Big)\log(1+\gamma_n),$$

$$I_2(n) = \Big(\sum_{i\ge n+1}\mu_i\Big)\Big(\sum_{i=1}^n \frac{1}{\mu_i a_i}\Big)\log\Big(\frac{\sum_{i\ge n}\mu_i}{\sum_{i\ge n+1}\mu_i}\Big) = \gamma_n I_1(n),$$

$$I_3(n) = \Big(\sum_{i\ge n+1}\mu_i\Big)^2\log\sum_{i\ge n+1}\mu_i = \gamma_n^{-2}\frac{-\mu_n\log\sum_{i\ge n+1}\mu_i}{\mu_{n+1}a_{n+1}},$$

$$I_4(n) = \frac{-\sum_{i\ge n+1}\mu_i\log\sum_{i\ge n+1}\mu_i}{\mu_{n+1}a_{n+1}} = \gamma_n I_3(n).$$

(1) When $\gamma = 0$, then it holds that $\mu_n/\mu_{n+1}\to 1$. Hence the limit for $I_3(n)$ may be writen as (2.5). Notice that $\frac{1}{t}\log(1+t)\to 1$ as $t\to 0$, the limit for $I_1(n)$ reduces to

$$\lim_{n\to\infty}\Big(\sum_{i\ge n+1}\mu_i\Big)\Big(\sum_{i=1}^n\frac{1}{\mu_i a_i}\Big),$$

which, by a criterion in [4], can be derived from (2.5).

Since $I_2(n)$ (resp. $I_1(n)$) is dominated by $I_4(n)$ (resp. $I_3(n)$), then $\alpha > 0$ if and only if (2.5) holds.

(2) When $\gamma = \infty$, then it holds that $\mu_n\big/\sum_{i\ge n}\mu_i \to 1$. Thus we can deduce (2.6) from Theorem 2.1 and the definition of $\overline{\delta}$ directly.

(3) When $\gamma\in(0,\infty)$, then $I_1(n)$ and $I_2(n)$ (resp. $I_1(n)$ and $I_1(n)$) dominate each other, whose limits are finite if only if (2.7) (resp. (2.8)) holds. □

Example 2.7 Consider the birth–death process $a_i = i$, $b_{i-1} = i\rho$ $(i\ge 1)$ with $\rho\in(0,1)$, then $\gamma=\rho$, and by Corollary 2.6 (3), this process admits the logarithmic Sobolev inequality.

Example 2.8 For $a_n = n$, $\mu_n = \frac{\lambda^n}{n!}$, then $\gamma = \infty$, by Corollary 2.6 (2), direct calculus shows that $-\mu_n\log\mu_n \ge const\cdot\lambda^n\log n/(n-1)!$ and $\sum_{k=1}^n\frac{1}{\mu_k a_k} = \sum_{k=1}^n (k-1)!/\lambda^k \ge (n-1)!/\lambda^n$, so the desired limit is infinite. Therefor the log-Sobolev constant $\alpha = 0$.

But, if we change a_n but μ_n remain stable, a same calculus leads to that $\alpha > 0$ if and only if $\lim a_n/n > 0$ (provided the limit exists).

Example 2.9 For $\beta > 1$, let $\mu_i = i^{-\beta}$, a_i unposed, this is the $\gamma = 0$ case, if choose a_i bigger than $i^2\log(n+1)$, then $\alpha > 0$ for all $\beta > 1$.

§3. Diffusions on the line

Consider the diffusion operator $L = a(x)\frac{d^2}{dx^2}+b(x)\frac{d}{dx}$ on $\mathbb{R}$. Assume that $a(x)$ is strictly positive, and denote $Z_1 := \int_{-\infty}^0 \frac{dx}{a(x)}\exp[C(x)]$, $Z_2 := \int_0^\infty \frac{dx}{a(x)}\exp[C(x)]$, where $C(x) = \int_0^x \frac{b(t)}{a(t)}dt$. Suppose that the diffusion is non-explosive

$$\int_{\mathbb{R}} dx\, e^{-C(x)}\int_0^x \frac{e^{C(y)}}{a(y)}dy < \infty, \tag{3.1}$$

and positively recurrent i.e. $Z := Z_1 + Z_2 < \infty$. Set $\mu(dx) = \frac{dx}{Za(x)}\exp[C(x)]$.

Theorem 3.1 For the diffusion defined above, set, for $x > 0$,

$$\delta^+(x) = -\int_0^x e^{-C(y)}dy \int_x^\infty \frac{e^{C(u)}}{a(u)}du \log\Big(\int_x^\infty \frac{e^{C(u)}}{a(u)}du\Big/Z\Big),$$

and for $x < 0$,

$$\delta^-(x) = -\int_x^0 e^{-C(y)}dy \int_{-\infty}^x \frac{e^{C(u)}}{a(u)}du \log\Big(\int_x^\infty \frac{e^{C(u)}}{a(u)}du\Big/Z\Big).$$

Then L satisfies logarithmic Sobolev inequality (1.1) if and only if

$$\limsup_{x\to\pm\infty} \delta^\pm(x) < \infty. \tag{3.2}$$

More precisely, it holds that

$$c_1\Big(\sup_{x<0}\delta^-(x) + \sup_{x>0}\delta^+(x)\Big)^{-1} \le \alpha \le c_2\Big(\sup_{x<0}\delta^-(x) + \sup_{x>0}\delta^+(x)\Big)^{-1} \tag{3.3}$$

with $c_1 = \dfrac{1}{80}\Big(\dfrac{Z}{Z_2}\Big/\log\dfrac{Z}{Z_2} + \dfrac{Z}{Z_2}\Big/\log\dfrac{Z}{Z_2}\Big)^{-1}$, $c_2 = 12(1-\sqrt{(Z_1\vee Z_2)/Z})^{-2}$.

For the proof of the theorem, we quote the following Hardy-type inequality with weights in [10], which extended to Orlicz spaces by [8] (Corollary 5.2). Let ν is a Borel measure on $[0,\infty]$, denote by $p_\nu(x)$ the absolutely continuous component of ν w.r.t. Lebesgue measure.

Theorem 3.2 Let A be the optimal constant in the inequality

$$\|f^2\|_\Psi \le A\int_0^\infty f'(x)^2 d\nu(x), \qquad f\in C^1,\ f(0)=0.$$

Then $B \le A \le 4B$, where

$$B = \sup_{x>0}\|I_{[x,\infty)}\|_\Psi \int_0^x \frac{dt}{p_\nu(t)},$$

where by definition

$$\|I_{[x,\infty)}\|_\Psi = 1/\Psi^{-1}\Big(\frac{1}{\pi([x,\infty))}\Big).$$

Since $\pi((-\infty,0)) = Z_1/Z > 0$, then by Lemma 2.5, we have

$$\frac{1}{2}[-\pi([x,\infty))\log\pi([x,\infty))] \le \|I_{[x,\infty)}\|_\Psi \le c[-\pi([x,\infty))\log\pi([x,\infty))] \tag{3.4}$$

with $c = \dfrac{Z}{Z_2}\Big/\log\dfrac{Z}{Z_2}$.

Proof of Theorem 3.1 Suppose $\alpha > 0$, or equally, (1.2) holds with $d \ge \dfrac{2}{3}\alpha$. Set $f_1 = fI_{(-\infty,0]}$, $f_2 = fI_{[0,\infty)}$, then apply (1.2) to f_1, f_2 to get

$$\Big\|f_1 - \int f_1 d\pi\Big\|_N^2 \le \frac{3}{\alpha}\int_{-\infty}^0 a(x)f'(x)^2 d\pi(x),$$
$$\Big\|f_2 - \int f_2 d\pi\Big\|_N^2 \le \frac{3}{\alpha}\int_0^\infty a(x)f'(x)^2 d\pi(x).$$

Hence by Lemma 2.2,

$$\|f_1\|_N^2 \le \frac{3}{\alpha}\Big(1-\sqrt{Z_2/Z}\Big)^{-2}\int_{-\infty}^0 a(x)f'(x)^2 d\pi(x),$$
$$\|f_2\|_N^2 \le \frac{3}{\alpha}\Big(1-\sqrt{Z_1/Z}\Big)^{-2}\int_0^\infty a(x)f'(x)^2 d\pi(x),$$

form which we can deduce the second inequality in (3.3) by Theorem 3.2 and (3.4).

Conversely, since (1.2) remains same if f is replaced by $f + constant$, we can assume $f(0) = 0$. Hence $f = f_1 + f_2$, and by Lemma 2.2,

$$\left\|f - \int f d\pi\right\|_N^2 \le \left(\left\|f_1 - \int f_1 d\pi\right\|_N + \left\|f_2 - \int f_2 d\pi\right\|_N\right)^2$$
$$\le 4\left(\left\|f_1\right\|_N + \left\|f_2\right\|_N\right)^2 \le 8\left(\left\|f_1\right\|_N^2 + \left\|f_2\right\|_N^2\right).$$

Thus by Theorem 3.2 and (3.4), we obtain the first inequality in (3.3). Then Theorem 3.1 is proved. □

Remark Recently, Miclo proved in [11] the logarithmic Sobolev inequality for the birth–death process by a similar method.

Acknowledgement The author would thank Prof. Chen Mu–Fa for many helps and constructive suggestions.

References

[1] Gross L., Logarithmic Sobolev inequalities and contractivity of semigrougs, In: Fabes E., ed. *Dirichlet Forms*, Varenna, 1992: *Lecture Notes in Mathematics*, **1563**(1993), Berlin: Springer–Verlag, 54–88.

[2] Stroock D., Logarithmic Sobolev inequalities for Gibbs states, In: Fabes E., ed. *Dirichlet Forms*, Varenna, 1992; *Lecture Notes in Mathematics*, **1563**(1993), Berlin: Springer–Verlag, 194–228.

[3] Diaconis P., Saloff–Coste L., Logarithmic Sobolev inequalities for finite Markov chains, *Ann. Appl. Prob.*, **6**(1996), 695–750.

[4] Chen Mu–Fa, Nash inequalities for general symmetric forms, *Acta. Math. Sinica, English Series*, **15(3)**(1999), 353–370.

[5] Chen Mu–Fa, Wang Feng–Yu, Estimates of logarithmic Sobolev constant: an improvement of Barky–Emery criterion, *J. Funct. Anal.*, **144(2)**(1997), 287–300.

[6] Chen Mu–Fa, Logarithmic Sobolev inequalities for symmetric forms, *Sci. in China (A)*, **30**(2000), 203–209 (Chinese Edition): **43**(2000), 601–608 (English Edition).

[7] Wang Feng–Yu, Sobolev type inequalities for general symmetric forms, *Proc. AMS.*, **128**(2000), 3675–3682.

[8] Bobkov S, Götze F., Exponential integrability and transportation cost related to logarithmic Sobolev inequalities, *J. Funct. Anal.*, **163(1)**(1999), 1–28.

[9] Miclo L., Relation entre isopérimétrie et trou spectral pour les chaînes de Markov finies, *Probability Theory and Related Fields*, **114(4)**(1999), 431–486.

[10] Muckenhoupt B., Hardy's inequality with weights, *Studia Math.*, **XLIV**(1972), 31–38.

[11] Miclo L., An example of application of discrete Hardy's inequalities, *Markov Process and Related Fields*, **5**(1999), 319–330.

生灭过程与一维扩散过程的对数Sobolev不等式

毛 永 华

(北京师范大学数学系, 北京, 100875)

本文运用加权的Hardy不等式的方法给出了生灭过程与一维扩散过程满足对数Sobolev不等式的显式判别准则.

关键词: 对数Sobolev不等式, 生灭过程, 扩散过程, 加权的Hardy不等式.

学科分类号: O211.62.