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带有随机干扰和确定投资回报风险过程下的一些分布 *

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摘 要

我们考虑既带有随机干扰又带有确定投资回报的风险过程, 得到了破产前瞬间盈余的分布$F_\delta(u,x)$及破产前瞬间盈余和破产时赤字的联合分布$H_\delta(u,x,y)$所满足的积分表达, 连续性及二次连续可微性和积分–微分方程. 同时, 只有随机干扰的风险模型下的破产前瞬间盈余的分布及破产前瞬间盈余和破产时赤字的联合分布所满足的性质也被得到. 已有文献中的诸多有关结果均可以通过令我们结论中的某些参数特殊化为零而得到.

关键词: 风险过程, 破产前瞬间盈余, 破产时赤字, 连续性及二次连续可微性, 积分–微分方程.

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§1. 引 言

风险理论是当今随机过程应用研究的热点之一, 在保险、投资、金融监管及其他与风险管理相关的领域都有重要应用. 自从1903年Lundberg作出了一些开创性工作以来, 国内外越来越多的学者关注并为其理论体系和应用研究的发展、完善作出了巨大的贡献. 令$(\Omega,\mathcal{F},\mathsf{P})$是一个包括下文中出现的所有变量的完备概率空间. Gerber (1970)引进带随机干扰的风险过程是

$$U(t)=u+ct+\sigma W_t-\sum_{i=1}^{N_t}Z_i, \tag{1.1}$$

确定投资回报产生过程为:

$$I(t)=\delta t. \tag{1.2}$$

这里, δ为常数, 表示投资回报的常数率, 于是由Paulsen, J.等(1997)知, 既有随机干扰又有确定投资回报风险过程是:

$$U_\delta(t)=e^{\delta t}\Big(u+\int_0^t e^{-\delta s}\mathrm{d}U(s)\Big). \tag{1.3}$$

(1.3)为强马氏过程, 且可以被写作:

$$U_\delta(t)=ue^{\delta t}+\frac{c(e^{\delta t}-1)}{\delta}+\sigma\int_0^t e^{\delta(t-s)}\mathrm{d}W(s)-\sum_{i=1}^{N_t}e^{-\delta(T_i-t)}Z_i. \tag{1.4}$$

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自然地, 模型(1.4)可看作是模型(1.1)的推广.

在(1.4)中, u, c, δ, σ是正常数, u表示保险公司的初始盈余, c是保费收取速率; N_t是带有正参数λ的齐次普哇松过程, 它表示$(0,t]$上的总理赔次数; Z_i, $i=1,2,\cdots$为带有分布函数$F(x)$的独立同分布随机变量序列且$F(0)=0$, $\mathsf{E}Z_i=\mu$; $\{W_t,t\geq 0\}$是一个标准布朗运动. 随机干扰项$\sigma W(t)$既可被解释为在总理赔量上的一个附加的不确定因素, 也可理解为在保费收入上附加的不确定因素. 进一步, 我们假定$\{W_t,t\geq 0\}$, $\{N_t,t\geq 0\}$, $\{Z_i,i\geq 1\}$是相互独立的. Wang (1999)讨论了模型(1.4)并且得到了生存概率, 破产前上界分布及破产时赤字的一些性质; Wang (2001)考虑了破产概率的分解; Zhao等(2005)讨论了破产时罚金折现期望, 得到了其积分表达, 连续性及二次连续可微性和积分–微分方程; 并考虑了破产时罚金折现期望的分解. 在本文里, 我们将继续讨论一些在风险模型(1.4)中还未被注意的问题, 主要内容有:

(1) 破产前瞬间盈余$U_\delta(T_u-)$的分布$F_\delta(u;x)$的积分表达, 连续性, 二次连续可微性及积分–微分方程.

(2) 破产前瞬间盈余$U_\delta(T_u-)$和破产时赤字$|U_\delta(T_u)|$的联合分布$H_\delta(u;x,y)$积分表达, 连续性, 二次连续可微性及积分–微分方程.

(3) 作为上二结论的推论, 得到了带随机干扰的风险过程(1.1)下破产前瞬间盈余的分布$F_0(u;x)$及破产前瞬间盈余和破产时赤字的联合分布$H_0(u;x,y)$的积分表达, 连续性, 二次连续可微性及积分–微分方程.

易知, (a) 在我们的结果中令$\sigma=0$, 便得到Yang等(2001a, 2001b, 2001c)对于带确定投资回报模型下的相应结果; (b) 令$\delta=\sigma=0$, 关于经典风险模型的结果被得到, 见Dickson (1992); (c) 在结论(1)中令$x\to+\infty$或在结论(2)中令$x\to+\infty$且$y\to+\infty$, 则可得到关于破产(或生存)概率的对应结果; (d) 如果在结论(2)中令$x\to+\infty$, 则得到破产时赤字的分布$G_\delta(u;y)$所满足的方程可被得到, 见Wang等(2000)); (e) 在结论(2)中$\to+\infty$, 则得到破产前瞬间盈余$U_\delta(T_u-)$的分布$F_\delta(u;x)$的相应结果.

为方便讨论, 我们首先作如下说明:

(1) 令$T_u=\inf\{t:U_\delta(t)<0\}$表示破产时间, $\Psi_\delta(u)$表示给定初始值$u\geq 0$时的破产概率. 易知:

$$\mathsf{E}U_\delta(t)=e^{\delta t}\Big\{u+\frac{1}{\delta}(1-e^{-\delta t})(c-\lambda\mu)\Big\}, \tag{1.5}$$

于是, 为使$\Psi_\delta(u)<1$, 我们假定$c-\lambda\mu>0$.

(2) 沿用Wang (2001)的记法, 我们令

$$B_t=\sigma\int_0^t e^{-\delta s}\mathrm{d}W(s). \tag{1.6}$$

B_t是一个局部连续鞅, 其方差过程为$\langle B_t\rangle=[\sigma^2/(2\delta)]\cdot(1-e^{-2\delta t})$. 令$\nu(s)=\inf(t:\langle B_t\rangle>s)$,

则
$$\nu(s)=\frac{1}{2\delta}\ln\Big(\frac{\sigma^2}{\sigma^2-2\delta s}\Big). \tag{1.7}$$
对$s<\sigma^2/(2\delta)$, 设$W_s=B_{\nu(s)}$, 易知W_s, $s<\sigma^2/(2\delta)$是一个从原点开始的局部布朗运动.

(3) 对于$a>0$, 定义$\tau_a=\inf\{s:|W_s|=a\}$, 令

$$\begin{aligned}H(a,t,x) &= (2\pi t)^{-1/2}\sum_{k=-\infty}^{+\infty}\Big\{\exp\Big[-\frac{1}{2t}(x+4ka)^2\Big]\\ &\quad -\exp\Big[-\frac{1}{2t}(x-2a+4ka)^2\Big]\Big\}, \qquad (1.8)\\ h(a,t) &= \frac{1}{2\sqrt{2\pi}}at^{-3/2}\sum_{k=-\infty}^{+\infty}\Big\{(4k+1)\exp\Big[-\frac{a^2}{2t}(4k+1)^2\Big]\\ &\quad +(4k-3)\exp\Big[-\frac{a^2}{2t}(4k-3)^2\Big]+(4k-1)\exp\Big[-\frac{a^2}{2t}(4k-1)^2\Big]\Big\}. \qquad (1.9)\end{aligned}$$

由Revuz and Yor (1991, $\mathrm{P}_{105-106}$), 可知:
$$\mathsf{P}(\tau_a>t,W_t\in\mathrm{d}x)=H(a,t,x)\mathrm{d}x,\qquad \mathsf{P}(\tau_a\in\mathrm{d}t)=h(a,t)\mathrm{d}t.$$

§2. 破产前瞬间盈余的分布

令$F_\delta(u;x)$表示初始盈余为u条件下破产前瞬间盈余$U_\delta(T_u-)$的分布, 即
$$F_\delta(u;x)=\mathsf{P}(T_u<+\infty,U_\delta(T_u-)\le x|U_\delta(0)=u). \tag{2.1}$$

定理 2.1 对于$x>u>0$, $F_\delta(u;x)$满足如下表达式:
$$\begin{aligned}&F_\delta(u;x)\\ =&\int_0^{+\infty}\lambda e^{-\lambda s}\mathrm{d}s\int_{-u}^{u}H(u,\nu^{-1}(s),y)\mathrm{d}y\Big\{\int_0^{\Delta_1}F_\delta(\Delta_1-z;x)\mathrm{d}F(z)\Big\}\\ &+\frac12\int_0^{\sigma^2/(2\delta)}\Big\{F_\delta\Big(2ue^{\delta\nu(s)}+\frac{c}{\delta}(e^{\delta\nu(s)}-1);x\Big)+F_\delta\Big(\frac{c}{\delta}(e^{\delta\nu(s)}-1);x\Big)\Big\}\\ &\cdot e^{-\lambda\nu(s)}h(u,s)\mathrm{d}s\\ &+\int_{-u}^{u}\int_0^{+\infty}\lambda e^{-\lambda s}(1-F(\Delta_1))I(\Delta_1\le x)H(u,\nu^{-1}(s),y)\mathrm{d}s\mathrm{d}y. \qquad (2.2)\end{aligned}$$
这里, $\Delta_1=(u+y)e^{\delta s}+(c/\delta)\cdot(e^{\delta s}-1)$.

证明: 定义$\tau_u^0=\inf\{\nu(s):|B_{\nu(s)}|=u\}$且$\tau_u=\inf\{s:|B_{\nu(s)}|=u\}$, 易知: $\tau_u^0=\nu(\tau_u)$. 令$M_t=\mathsf{E}\{I(T_u<+\infty,U_\delta(T_u-)\le x)|\mathcal{F}_t\}$, 其中$F=\{\mathcal{F}_t:t\ge0\}$ ($\mathcal{F}_t=\sigma(U_\delta(s):s\le t)$)是一个自然流. 易证$M_t$是一个$F$-鞅. 令$S=T_1\wedge\tau_u^0$ (T_1为首次理赔到达时刻), 由风险过

程$\{U_\delta(t)\}_{t\geq 0}$的强马氏性及有界停时定理, 可知

$$\begin{aligned}F_\delta(u;x) &= M_0 = \mathsf{E}(M_s) = \mathsf{E}\{\mathsf{E}[I(T_u<+\infty, U_\delta(T_u-)\leq x)|\mathcal{F}_s]\}\\ &= \mathsf{E}\{\mathsf{E}[I(T_u<+\infty, U_\delta(T_u-)\leq x)|U_\delta(S)]\} = \mathsf{E}\{F_\delta(U_\delta(S);x)\}\\ &= \mathsf{E}\{F_\delta(U_\delta(S);x)I(T_1>\tau_u^0)\} + \mathsf{E}\{F_\delta(U_\delta(S);x)I(T_1\leq\tau_u^0)\}\\ &= \mathsf{E}\Big\{F_\delta\Big(e^{\delta\tau_u^0}\Big(u+\frac{c}{\delta}(1-e^{-\delta\tau_u^0})+B_{\tau_u^0}\Big);x\Big)I(T_1>\tau_u^0)\Big\}\\ &\quad +\mathsf{E}\{F_\delta(\Delta_2)-Z_1;x)I(T_1\leq\tau_u^0, Z_1\leq\Delta_2)\}\\ &\quad +\mathsf{E}\{F_\delta(\Delta_2)-Z_1;x)I(T_1\leq\tau_u^0, Z_1>\Delta_2)\}\\ &= I_1+I_2+I_3.\end{aligned} \tag{2.3}$$

这里, $\Delta_2 = e^{\delta T_1}(u+(c/\delta)\cdot(1-e^{-\delta T_1})+B_{T_1})$.

其中, 借鉴Wang等(2000)中式(3.6)及(3.7)的推导方法, 可得:

$$\begin{aligned}&I_2+I_1\\ =&\int_0^{+\infty}\lambda e^{-\lambda s}\mathrm{d}s\int_{-u}^{u}H(u,\nu^{-1}(s),y)\mathrm{d}y\int_0^{\Delta_1}F_\delta(\Delta_1-z;x)\mathrm{d}F(z)\\ &+\frac{1}{2}\int_0^{\sigma^2/(2\delta)}\Big\{F_\delta\Big(2ue^{\delta\nu(s)}+\frac{c}{\delta}(e^{\delta\nu(s)}-1);x\Big)+F_\delta\Big(\frac{c}{\delta}(e^{\delta\nu(s)}-1);x\Big)\Big\}\\ &\cdot e^{-\lambda\nu(s)}h(u,s)\mathrm{d}s.\end{aligned} \tag{2.4}$$

因为$x>u>0$, (2.3)中的I_3可被改写为:

$$\begin{aligned}I_3 &= \mathsf{E}\{I(T_1\leq\tau_u^0, Z_1>\Delta_2, \Delta_2\leq x)\}\\ &= \int I(s\leq\nu(t), z>\Delta_1, \Delta_1\leq x)\mathsf{P}(T_1\in\mathrm{d}s, Z_1\in\mathrm{d}z, \tau_u\in\mathrm{d}t, B_s\in\mathrm{d}y)\\ &= \int I(t\geq\nu^{-1}(s))I(z>\Delta_1)I(\Delta_1\leq x)\mathsf{P}(T_1\in\mathrm{d}s)\mathsf{P}(Z_1\in\mathrm{d}z)\mathsf{P}(\tau_u\in\mathrm{d}t, B_s\in\mathrm{d}y)\\ &= \int I(\Delta_1\leq x)\lambda e^{-\lambda s}\mathrm{d}s\int I(z>\Delta_1)\mathrm{d}F(z)\int I(t\leq\nu^{-1}(s))\mathsf{P}(\tau_u\in\mathrm{d}t, B_s\in\mathrm{d}y)\\ &= \int_{-u}^{u}\int_0^{+\infty}\lambda e^{-\lambda s}(1-F(\Delta_1))I(\Delta_1\leq x)H(u,\nu^{-1}(s),y)\mathrm{d}s\mathrm{d}y.\end{aligned} \tag{2.5}$$

结合式(2.3), (2.4)及(2.5), 定理得证. □

定理 2.2 对于$x>u>0$, 设$F(z)$在$[0,+\infty)$上有连续概率密度函数, 则:

(i) $F_\delta(u;x)$关于u在$[0,x]$上连续;

(ii) $F_\delta(u;x)$关于u在$(0,x)$上二次连续可微.

证明: 对任意$u_0\in(0,x)$, 选择$0<\varepsilon_0<u_0$, 使$(1/2)\cdot\varepsilon_0+u\leq x$, 在定理2.1的证明中

分别以$\tau^0_{\varepsilon_0/2}$和$\tau_{\varepsilon_0/2}$代替τ^0_u和τ_u, 于是

$$\begin{aligned}
F_\delta(u;x) &= \int_0^{+\infty}\lambda e^{-\lambda s}\mathrm{d}s\int_{-\varepsilon_0/2}^{\varepsilon_0/2}H\Big(\frac{\varepsilon_0}{2},\nu^{-1}(s),y\Big)\mathrm{d}y\int_0^{\Delta_1}F_\delta(\Delta_1-z;x)\mathrm{d}F(z)\\
&\quad+\frac{1}{2}\int_0^{\sigma^2/(2\delta)}\Big\{F_\delta\Big(\Big(u-\frac{\varepsilon_0}{2}\Big)e^{\delta\nu(s)}+\frac{c}{\delta}(e^{\delta\nu(s)}-1);x\Big)\\
&\quad+F_\delta\Big(\Big(u+\frac{\varepsilon_0}{2}\Big)e^{\delta\nu(s)}+\frac{c}{\delta}(e^{\delta\nu(s)}-1);x\Big)\Big\}e^{-\lambda\nu(s)}h\Big(\frac{\varepsilon_0}{2},s\Big)\mathrm{d}s\\
&\quad+\int_{-\varepsilon_0/2}^{\varepsilon_0/2}\int_0^{+\infty}\lambda e^{-\lambda s}(1-F(\Delta_1))I(\Delta_1\le x)H\Big(\frac{\varepsilon_0}{2},\nu^{-1}(s),y\Big)\mathrm{d}s\mathrm{d}y\\
&= I_1^0+I_2^0+I_3^0.
\end{aligned}\tag{2.6}$$

对于I_1^0及I_3^0, 施行变量代换, 有:

$$\begin{aligned}
&I_1^0+I_3^0\\
=\;&\int_{-\varepsilon_0/2}^{\varepsilon_0/2}\int_{u+y}^{+\infty}\int_0^t\lambda(\delta u+\delta y+c)^{\lambda/\delta}H\Big(\frac{\varepsilon_0}{2},\nu^{-1}\Big(\frac{1}{\delta}\ln\Big(\frac{\delta t+c}{\delta u+\delta y+c}\Big)\Big),y\Big)\\
&\cdot F_\delta(t-z;x)\mathrm{d}F(z)\mathrm{d}t\mathrm{d}y\\
&+\int_{-\varepsilon_0/2}^{\varepsilon_0/2}\lambda(\delta u+\delta y+c)^{\lambda/\delta}\int_0^x(c+\delta t)^{-\lambda/\delta-1}(1-F(t))\\
&\cdot H\Big(\frac{\varepsilon_0}{2},\nu^{-1}\Big(\frac{1}{\delta}\ln\Big(\frac{\delta t+c}{\delta u+\delta y+c}\Big)\Big),y\Big)\mathrm{d}t\mathrm{d}y.
\end{aligned}$$

对于I_2^0, 先拆成两个积分, 再分别实行变量代换, 得:

$$\begin{aligned}
I_2^0 &= \frac{1}{2}\int_{u-\varepsilon_0/2}^{+\infty}F_\delta(t;x)\Big(1-\frac{2}{\sigma^2}\ln\Big(\frac{\delta t+c}{\delta u+c-\frac{\varepsilon_0}{2}\delta}\Big)\Big)h\Big(\frac{\varepsilon_0}{2},\frac{1}{\delta}\ln\Big(\frac{\delta t+c}{\delta u+\delta y+c}\Big)\Big)\mathrm{d}t\\
&\quad+\frac{1}{2}\int_{u+\varepsilon_0/2}^{+\infty}F_\delta(t;x)\Big(1-\frac{2}{\sigma^2}\ln\Big(\frac{\delta t+c}{\delta u+c-\frac{\varepsilon_0}{2}\delta}\Big)\Big)h\Big(\frac{\varepsilon_0}{2},\frac{1}{\delta}\ln\Big(\frac{\delta t+c}{\delta u+\delta y+c}\Big)\Big)\mathrm{d}t.
\end{aligned}$$

利用H及h所具有的的良好性质(Revuz和Yor (1991, $\mathrm{P}_{105-106}$)), 容易证明I_1^0, I_2^0, I_3^0关于u在(u_0,x)上连续且二次连续可微; 又据王(1999, P_{14})性质2.5.2同样推导方法, 可以证明它们在0及x点连续, 从而定理得证. □

定理 2.3 对于$x>u>0$, 设$F(z)$在$[0,+\infty)$上有连续概率密度函数, 则$F_\delta(u;x)$满足如下积分–微分方程

$$\begin{aligned}
&(\delta u+c)\frac{\partial F_\delta(u;x)}{\partial u}+\frac{\sigma^2}{2}\frac{\partial^2F_\delta(u;x)}{\partial u^2}\\
=\;&\lambda F_\delta(u;x)-\lambda\int_0^uF_\delta(u-z;x)f(z)\mathrm{d}z-\lambda(1-F(u)).
\end{aligned}\tag{2.7}$$

证明: 我们考虑一个充分小时间区间$[0,\mathrm{d}t]$, $\mathrm{d}t$时刻的盈余$U_\delta(\mathrm{d}t)$将为:

(1) $ue^{\delta\mathrm{dt}}+(c/\delta)\cdot(e^{\delta\mathrm{dt}}-1)+\sigma W_{\mathrm{dt}}e^{\delta\mathrm{dt}}\approx u+(\delta u+c)\mathrm{dt}+\delta\sigma W_{\mathrm{dt}}\mathrm{dt}+\sigma W_{\mathrm{dt}}$

或

(2) $ue^{\delta\mathrm{dt}}+(c/\delta)\cdot(e^{\delta\mathrm{dt}}-1)+\sigma W_{\mathrm{dt}}e^{\delta\mathrm{dt}}-z\approx u+(\delta u+c)\mathrm{dt}+\delta\sigma W_{\mathrm{dt}}\mathrm{dt}+\sigma W_{\mathrm{dt}}-z.$

其中, 情况(1)表明$[0,\mathrm{dt}]$上无理赔产生($N_{\mathrm{dt}}=0$)其概率为$1-\lambda\mathrm{d}t$; 在情况(2)下, 产生一次理赔($N_{\mathrm{dt}}=1$), 其概率为$\lambda\mathrm{d}t$, 于是, 由风险过程(1.4)

$$\begin{aligned}
F_\delta(u;x) &= \mathsf{E}\{F_\delta(U_\delta(\mathrm{d}t);x)\}\\
&= (1-\lambda\mathrm{d}t)(F_\delta(\Delta_3;x))+\lambda\mathrm{d}t\mathsf{E}\Big\{\int_0^{\Delta_3}F_\delta(\Delta_3-z;x)\mathrm{d}F(z)\Big\}\\
&\quad +\lambda\mathrm{d}t\mathsf{E}\{I(z>\Delta_3\le x)\}\\
&= (1-\lambda\mathrm{dt})\Big(F_\delta(u;x)+(\delta u+c)\frac{\partial F_\delta(u;x)}{\partial u}\mathrm{dt}+\frac{\sigma^2}{2}\frac{\partial^2F_\delta(u;x)}{\partial u^2}\Big)\\
&\quad +\lambda\mathrm{dt}\int_0^u F_\delta(u-z;x)f(z+\Delta_3-u)\mathrm{d}z\\
&\quad +\lambda\mathrm{dt}(1-F(\Delta_3)\mathsf{P}(\Delta_3\le x).
\end{aligned}\tag{2.8}$$

其中, $\Delta_3=u+(\delta u+c)\mathrm{dt}+\sigma\delta W_{\mathrm{dt}}\mathrm{dt}+\sigma W_{\mathrm{dt}}$. 对式(2.8)化简并令$\mathrm{d}t\to 0$即得到式(2.7). □

注记 1 令$\sigma=0$, 我们得到Yang等(2001c)对于带确定投资回报模型下的相应结果; 令$\delta=\sigma=0$, 关于经典风险模型的结果被得到, 见Dickson (1992); 当我们令$x\to+\infty$, 则关于破产(或生存)概率的积分–微分方程被得到.

§3. 破产前瞬间盈余和破产时赤字的联合分布

现在我们来考虑如下概率:

$$H_\delta(u;x,y)=\mathsf{P}(T_u<+\infty,U_\delta(T_u-)\le x\ \text{且}\ |U_\delta(T_u)|\le y|U_\delta(0)=u).\tag{3.1}$$

其中x, y是正变量, $H_\delta(u;x,y)$恰是模型(1.2)中破产前瞬间盈余和破产时赤字的联合分布.

由公式(2.1), (3.1)及Wang等(2000)中关于破产时赤字的分布函数的定义, 易知

$$\lim_{x\to+\infty}H_\delta(u;x,y)=G_\delta(u;y)\qquad \text{且}\qquad \lim_{y\to+\infty}H_\delta(u;x,y)=F_\delta(u;x).$$

定理 3.1 对于$x > u > 0$, $H_\delta(u;x,y)$满足如下积分方程

$$
\begin{aligned}
& H_\delta(u;x,y) \\
= & \int_0^{+\infty} \lambda e^{-\lambda s}\mathrm{d}s \int_{-u}^{u} H(u,\nu^{-1}(s),y)\mathrm{d}y \Big\{ \int_0^{\Delta_4} H_\delta(\Delta_4 - z; x, y)\mathrm{d}F(z) \Big\} \\
& + \frac{1}{2}\int_0^{\sigma^2/(2\delta)} \Big\{ H_\delta\Big(2ue^{\delta\nu(s)} + \frac{c}{\delta}(e^{\delta\nu(s)}-1); x, y\Big) + H_\delta\Big(\frac{c}{\delta}(e^{\delta\nu(s)}-1); x, y\Big) \Big\} \\
& \cdot e^{-\lambda\nu(s)} h(u,s)\mathrm{d}s \\
& + \int_{-u}^{u}\int_0^{+\infty} \lambda e^{-\lambda s}(F(\Delta_4 + y) - F(\Delta_4))I(\Delta_4 \le x)H(u,\nu^{-1}(s),y)\mathrm{d}s\mathrm{d}r. \qquad (3.2)
\end{aligned}
$$

这里, $\Delta_4 = (u+r)e^{\delta s} + (c/\delta)\cdot(e^{\delta s}-1)$.

证明: 注意到破产时赤字$|U_\delta(T)| = z - \Delta_4 \le y$意味着引起破产的理赔量界于$\Delta_4$和$\Delta_4 + y$之间, 利用定理2.1同样推导方法可得证. □

与定理2.2及定理2.3类似, 我们有:

定理 3.2 对于$x > u > 0$, 设$F(z)$在$[0,+\infty)$上有连续概率密度函数, 则:

(i) $H_\delta(u;x,y)$关于u在$[0,x]$上连续;

(ii) $H_\delta(u;x,y)$关于u在$(0,x)$ 上二次连续可微.

定理 3.3 对于$x > u > 0$, 设$F(z)$在$[0,+\infty)$上有连续概率密度函数, 则$H_\delta(u;x,y)$满足如下积分–微分方程

$$
\begin{aligned}
& (\delta u + c)\frac{\partial H_\delta(u;x,y)}{\partial u} + \frac{\sigma^2}{2}\frac{\partial^2 H_\delta(u;x,y)}{\partial u^2} \\
= & \lambda H_\delta(u;x,y) - \lambda\int_0^u H_\delta(u-z;x,y)f(z)\mathrm{d}z - \lambda(1-F(u)). \qquad (3.3)
\end{aligned}
$$

注记 2 在公式(3.3)中令$y \to +\infty$则得到关于$F_\delta(u,x)$的相应结果(见定理2.3); 令$x \to +\infty$则得到$G_\delta(u,y)$满足的积分–微分方程, 见Wang等(2000). 如果令$x \to +\infty$且$y \to +\infty$, 则破产(或生存)概率$\Psi_\delta(u)$ $(\overline{\Psi}_\delta(u))$所满足的方程可被得到. 令$\sigma \to 0$, 我们得到Yang等(2001a)对于带确定投资回报模型下的相应结果.

§4. 一些推论

现在让我们回到风险过程(1.1), 尽管关于模型(1.1)已得到很多结果, 但关于破产前瞬间盈余的分布$F_0(u;x)$, 及破产前瞬间盈余和破产时赤字的联合分布$H_0(u;x,y)$的讨论一直没有. 显然关于$F_0(u;x)$和$H_0(u;x,y)$的结论该是本文定理3.1–定理3.3的特例.

在定理2.1–定理2.3及定理3.1–定理3.3的证明中令$\delta = 0$, 我们得到:

推论 4.1 对于$x>u>0$, 有

$$
\begin{aligned}
&F_0(u;x)\\
=&\int_0^{+\infty}\lambda e^{-\lambda s}\mathrm{d}s\int_{-u}^{u}H(u,\nu^{-1}(s),r)\mathrm{d}r\Big\{\int_0^{u+cs+r}F_0(u+cs+r-z;x)\mathrm{d}F(z)\Big\}\\
&+\frac{1}{2}\int_0^{+\infty}\{F_0(2u+c\nu(s)+r;x)+F_0(c\nu(s);x)\}e^{-\lambda\nu(s)}h(u,s)\mathrm{d}s\\
&+\int_{-u}^{u}\int_0^{+\infty}\lambda e^{-\lambda s}(1-F(u+cs+r))I(u+r+cs\le x)H(u,\nu^{-1}(s),r)\mathrm{d}s\mathrm{d}r,
\end{aligned}\tag{4.1}
$$

$$
\begin{aligned}
&H_0(u;x,y)\\
=&\int_0^{+\infty}\lambda e^{-\lambda s}\mathrm{d}s\int_{-u}^{u}H(u,\nu^{-1}(s),r)\mathrm{d}r\Big\{\int_0^{u+cs+r}H_0(u+cs+r-z;x,y)\mathrm{d}F(z)\Big\}\\
&+\frac{1}{2}\int_0^{+\infty}\{H_0(2u+c\nu(s)+r;x,y)+H_0(c\nu(s);x,y)\}e^{-\lambda\nu(s)}h(u,s)\mathrm{d}s\\
&+\int_{-u}^{u}\int_0^{+\infty}\lambda e^{-\lambda s}(F(u+cs+r+y)-F(u+cs+r))I(u+r+cs\le x)\\
&\cdot H(u,\nu^{-1}(s),r)\mathrm{d}s\mathrm{d}r,
\end{aligned}\tag{4.2}
$$

其中$\nu(s)=c/\sigma^2$.

推论 4.2 对于$x>u>0$, 设$F(z)$在$[0,+\infty)$上有连续概率密度函数$f(z)$, 则:

(i) $F_0(u;x)$和$H_0(u;x,y)$关于u在$[0,x]$上连续;

(ii) $F_0(u;x)$和$H_0(u;x,y)$关于u在$(0,x)$ 上二次连续可微.

推论 4.3 对于$x>u>0$, 设 $F(z)$ 在 $[0,+\infty)$ 上有连续概率密度函数, 则 $F_0(u;\ x)$ 和 $H_0(u;x,y)$满足如下积分–微分方程

$$
\begin{aligned}
&c\frac{\partial F_0(u;x)}{\partial u}+\frac{\sigma^2}{2}\frac{\partial^2F_0(u;x)}{\partial u^2}\\
=&\lambda F_0(u;x)-\lambda\int_0^u F_0(u-z;x)f(z)\mathrm{d}z-\lambda(1-F(u)),
\end{aligned}\tag{4.3}
$$

$$
\begin{aligned}
&c\frac{\partial H_0(u;x,y)}{\partial u}+\frac{\sigma^2}{2}\frac{\partial^2H_0(u;x,y)}{\partial u^2}\\
=&\lambda H_0(u;x,y)-\lambda\int_0^u H_0(u-z;x,y)f(z)\mathrm{d}z-\lambda(F(u+y)-F(u)).
\end{aligned}\tag{4.4}
$$

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Some Distributions for Risk Process Perturbed by Diffusion under Interest Force

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We consider the risk process perturbed by diffusion under interest force in this article. The integral expressions, continuities, twice continuous differentiability and integro-differential equations about $F_\delta(u;x)$, the distribution of the surplus immediately before ruin, and $H_\delta(u;x,y)$, the joint distribution of the surplus immediately before ruin and the deficit at ruin are obtained. As corollaries, some distributions for the classical risk process that is perturbed by diffusion are also considered. Certainly, it is seen that many results in references may be derived from our conclusions by letting some constant variable zero.

Keywords: Risk process, the surplus immediatly before ruin, the deficit at ruin, twice continuous differentiability, integro-differential equation.

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