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Credibility Premium Under Relative Loss Function

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Abstract

The classical Bühlmann credibility formula estimates the hypothetical mean of a particular insured, or risk by square error loss. However, the ratio of the charged premium and the true premium is more appropriate to measure equity of the premium than the absolute value of their difference. Regarding to this case, we propose two alternative loss functions to calculate the credibility premium in this paper. The one combines the squared error loss and the relative loss ratio is called as the relative mean square error loss. The other one mixes the relative entropy loss function instead of the squared error loss is called as the relative entropy loss function. The estimation method of credibility factors and their properties are investigated.

Keywords: Mean square error loss, entropy loss, relative loss, credibility.
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§ 1. Introduction

Credibility theory is a common approach to calculate insurance premium based on the policyholder's past experience and the experience of the entire group of policyholders. This method is widely used in commercial property or liability insurance and group health or life insurance. The popular formulas in credibility theory take premium as a weighted sum of the average experience of the policyholder and the average of the entire collection of policyholders. These formulas are easy to understand and simple to apply due to their linear properties.

Let X_i, $i = 1, 2, \cdots$, denote total claims of a policyholder in the $i^{\rm th}$ policy period. The distribution of X_i depends on the parameter θ, where θ varies across policyholders and maybe vector valued. If θ is given, X_i's are independent and identically distributed. Therefore, the value of θ completely determines the claim distribution of the policyholder. Since θ is generally unknown, the probability (density) function of θ is denoted by $\pi(\theta)$, which is called as the structure function in Bühlmann (1970) and the prior distribution by Bayesians.

One purpose of credibility theory is to calculate a premium for the $(n+1)^{\rm th}$ period of a policyholder, given the policyholder's claim experience in the first n periods, which is denoted as $\mathbf{X_n} = (X_1, X_2, \cdots, X_n) \in \mathbb{R}^n$. Generally, credibility estimators Y is a real valued function of the given information, i.e. $Y(\mathbf{X_n})$. If we constrain Y to be a liner function of the prior claim data, we can use L to present Y.

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Given the value of θ, $\mathsf{E}(X_{n+1}|\theta)$, or more simply $\mathsf{E}(X|\theta)$, would be the most equitable premium for the $(n+1)^{\text{th}}$ period. Let $\mu(\theta)$ denote the most equitable premium, which is $\mathsf{E}(X|\theta)$ and μ denote the overall or grand mean, which is $\mathsf{E}[\mu(\theta)]$. The inequity of any other premium $Y(\mathbf{X_n})$ is measured relative to $\mu(\theta)$. A general procedure is to minimize the risk function $\mathsf{E}U[Y(\mathbf{X_n}),\mu(\theta)]$, where U is the loss function.

In Bühlmann's classical credibility theory (1967, 1970), the loss function U is taken to be the traditional squared error loss function. Then $U[Y(\mathbf{X_n}),\mu(\theta)] = (Y(\mathbf{X_n}) - \mu(\theta))^2$. The resulting credibility premium $Y(\mathbf{X_n}) = \mathsf{E}[\mu(\theta)|\mathbf{X_n}]$, which is the posterior expected value of $\mu(\theta)$. Because the claims X_i's, are conditionally independent given θ, the credibility premium $Y(\mathbf{X_n})$ can also be expressed as $\mathsf{E}[X_{n+1}|\mathbf{X_n}]$, which is called the predictive mean. Restricting the credibility premium to be a liner combination of the prior claims, $L(\mathbf{X_n})$ is given as

$$L(\mathbf{X_n}) = Z\overline{X} + (1-Z)\mu, \qquad Z = \frac{n}{n+k}, \tag{1}$$

where $\overline{X} = \sum\limits_{i=1}^{n} X_i/n$ is the sample mean, $k = v/a$, $v = \mathsf{E}[v(\theta)]$, $v(\theta) = \mathsf{Var}\,(X|\theta)$, and $a = \mathsf{Var}\,[\mu(\theta)] = \mathsf{E}[(\mu(\theta)-\mu)^2]$. Therefore, k is the ratio of the expected value of conditional variance to the variance of conditional means. Generally, Z is called as the credibility factor.

Promislow (1987) and Promislow and Young (2000) pointed out that it is inappropriate for measuring the equity of the premium by the difference between the charged premium $Y(\mathbf{X_n})$ and the true premium $\mu(\theta)$. They proposed to use the loss ratio $r = Y(\mathbf{X_n})/\mu(\theta)$ to measure the equity instead of difference. A class of loss functions with the form $U[Y(\mathbf{X_n}),\mu(\theta)] = \mu(\theta)g(r)$ was proposed, where

$$g(r) = \begin{cases} (r^c-1)/[c(c-1)], & c \neq 0,1; \\ -\ln(r), & c = 0; \\ r\ln(r), & c = 1. \end{cases} \tag{2}$$

Based on $g(r)$, the equitable credibility premium and the linear equitable credibility premium are obtained. Specially, when $c=2$, the linear equitable credibility premium is given as

$$L(\mathbf{X_n}) = Z_y\overline{X} + (1-Z_y)\mu, \qquad Z_y = \frac{n}{n+k_y}, \tag{3}$$

where $k_y = J/\mu W$ with $J = \mathsf{E}[\mathsf{Var}\,(X|\theta)/\mu(\theta)]$ and $W(\mu(\theta)) = \mathsf{E}(\mu(\theta))\mathsf{E}(\mu(\theta)^{-1}) - 1$.

In this paper, we focus on the relative mean square error loss function and the relative entropy loss function. We study the optimal premium and the optimal linear premium under each loss function separately in section 2 and section 3. The parametric and semi-parametric estimators are studied in section 4.

§ 2. Relative Mean Square Error Loss

The relative mean square error loss function of the order p is $U_p[Y(\mathbf{X_n}),\mu(\theta)] = \mu^p(\theta)h(r)$, where $h(r) = (r-1)^2$. That is, the loss is expressed as a function of the relative difference, weighted by the pth order of the most equitable premium.

Theorem 1 For the loss function $U_p[Y(\mathbf{X_n}),\mu(\theta)]$, the optimal premium $Y_p(\mathbf{X_n})$ is given by

$$Y_p(\mathbf{X_n})=\frac{\mathsf{E}[\mu^{p-1}(\theta)|\mathbf{X_n}]}{\mathsf{E}[\mu^{p-2}(\theta)|\mathbf{X_n}]}.$$

In particular, when $p=2$, $Y_2(\mathbf{X_n})$ is the posterior expectation of $\mu(\theta)$.

Proof Minimizing the expectation of the joint distribution of $\mathbf{X_n}$ is equivalent to minimize

$$\mathsf{E}\Big[\mu^p(\theta)\Big(\frac{Y_p(\mathbf{X_n})}{\mu(\theta)}-1\Big)^2\Big|\mathbf{X_n}\Big].$$

Differentiating with respect to $Y_p(\mathbf{X_n})$ and letting the score function equal 0, we obtain

$$\mathsf{E}\Big[\mu^p(\theta)\Big(\frac{Y_p(\mathbf{X_n})}{\mu(\theta)}-1\Big)\frac{1}{\mu(\theta)}\Big|\mathbf{X_n}\Big]=0.$$

Thus, we have $Y_p(\mathbf{X_n})=\mathsf{E}[\mu^{p-1}(\theta)|\mathbf{X_n}]/\mathsf{E}[\mu^{p-2}(\theta)|\mathbf{X_n}]$. #

Theorem 2 The optimum premium $Y_p(\mathbf{X_n})$ is an increasing function of the order p.

To prove Theorem 2, we consider the following lemma first.

Lemma 3 $\mathsf{E}|X|^{p+1}/\mathsf{E}|X|^p$ is an increasing function of the parameter p for any variable X.

Proof Assume that the cummulative probability density function of X is $F(x)$, then a new probability function based on $F(x)$ is $|x|^p\mathrm{d}F(x)\Big/\displaystyle\int|x|^p\mathrm{d}F(x)$. Therefore, we have the following inequity

$$\Big(\int|x|\frac{|x|^p\mathrm{d}F(x)}{\displaystyle\int|x|^p\mathrm{d}F(x)}\Big)^2\le\int|x|^2\frac{|x|^p\mathrm{d}F(x)}{\displaystyle\int|x|^p\mathrm{d}F(x)}.$$

Simplifying the inequity, we obtain

$$\frac{\mathsf{E}|X|^{p+1}}{\mathsf{E}|X|^p}\le\frac{\mathsf{E}|X|^{p+2}}{\mathsf{E}|X|^{p+1}},$$

which means $\mathsf{E}|X|^{p+1}/\mathsf{E}|X|^p$ is an increasing function of the parameter p. #

Corollary 4 If X is a non-negative random variable, $\mathsf{E}X^{p+1}/\mathsf{E}X^p$ is an increasing function of the parameter p.

Theorem 2 can be obtained from Corollary 4 directly.

Therefore, the optimal premium $Y_p(\mathbf{X_n})$ decided by relative mean square error loss function of the order p has the following properties:

$$Y_p(\mathbf{X_n})\begin{cases}\le Y(\mathbf{X_n}) & \text{while } p<2,\\ =Y(\mathbf{X_n}) & \text{while } p=2,\\ \ge Y(\mathbf{X_n}) & \text{while } p>2,\end{cases}\qquad \mathsf{E}Y_p(\mathbf{X_n})\begin{cases}\le\mu & \text{while } p<2,\\ =\mu & \text{while } p=2,\\ \ge\mu & \text{while } p>2,\end{cases}$$

where $Y(\mathbf{X_n})$ is decided by the square error loss function and $\mathsf{E}Y(\mathbf{X_n})=\mu$.

According to Theorem 2, we know that the optimal premium $Y_p(\mathbf{X_n})$ will increase with the order p. And also does the expectation of $Y_p(\mathbf{X_n})$. While $p=2$, the expectation of the premium is equal to the grand mean, which means the total loss equals the total premium. While $p<2$, the expectation of the premium is less than the grand mean and the total premium collected by the insurance company will not cover the total claim. While $p>2$, the expectation of the premium is larger than the grand mean and the premium collected is relatively higher than the expected claims of insureds.

If $Y_p(\mathbf{X_n})$ is a linear function of $\mathbf{X_n}$, we denote it as $L_p(\mathbf{X_n})$, whose expectation can be assumed as μ. $L_p(\mathbf{X_n})$ can be written as $L_p(\mathbf{X_n})=\mu+\overline{\Delta}Z_s^{(p)}$, where $\overline{\Delta}=\overline{X}-\mu$ and $Z_s^{(p)}$ is the credibility factor. $Z_s^{(p)}$ can be obtained by minimizing

$$\mathsf{E}\mu^p(\theta)\Big(\frac{\mu+\overline{\Delta}Z_s^{(p)}}{\mu(\theta)}-1\Big)^2.$$

Differentiating with respect to $Z_s^{(p)}$ and letting the score function equal 0, we get

$$\mathsf{E}\Big[\mu^p(\theta)\Big(\frac{\mu+\overline{\Delta}Z_s^{(p)}}{\mu(\theta)}-1\Big)\frac{\overline{\Delta}}{\mu(\theta)}\Big]=0.$$

Therefore,

$$Z_s^{(p)}=\Big(1+\frac{\mathsf{E}[\mu^{p-2}(\theta)\overline{\Delta X}]-\mathsf{E}[\overline{\Delta}\mu^{p-1}(\theta)]}{\mathsf{E}[\overline{\Delta}\mu^{p-1}(\theta)]-\mu\mathsf{E}[\mu^{p-2}(\theta)\overline{\Delta}]}\Big)^{-1}.$$

It is easy to prove that for any p, $\mathsf{E}(\mu^p(\theta)\overline{X})=\mathsf{E}(\mu^{p+1}(\theta))$. Thus, we can further show that

$$\mathsf{E}[\mu^{p-2}(\theta)\overline{\Delta X}]-\mathsf{E}[\overline{\Delta}\mu^{p-1}(\theta)]=\mathsf{E}[\mu^{p-2}(\theta)\mathsf{Var}\,(\overline{X}|\theta)],$$
$$\mathsf{E}[\overline{\Delta}\mu^{p-1}(\theta)]-\mu\mathsf{E}[\mu^{p-2}(\theta)\overline{\Delta}]=\mathsf{E}[\mu^{p-2}(\theta)(\mu(\theta)-\mu)^2].$$

Since $\mathsf{Var}\,(\overline{X}|\theta)=v(\theta)/n$, we can obtain

$$Z_s^{(p)}=\frac{n}{n+k_{p-2}},\qquad k_{p-2}=\frac{v_{p-2}}{a_{p-2}},\tag{4}$$

where $v_\omega=\mathsf{E}[\mu^\omega(\theta)v(\theta)]$, $a_\omega=\mathsf{E}[\mu^\omega(\theta)(\mu(\theta)-\mu)^2]$. Note that the notations v_ω, a_ω and $k_\omega=v_\omega/a_\omega$ will be used throughout this paper. And it is easy to prove that $0\le Z_s^{(p)}\le 1$. Obviously, $v_0=v$, $a_0=a$, and $Z_s^{(0)}$ is the same as (1). It is because that loss functions are equivalent under these two models. $Z_s^{(1)}$ is the same as (3) and the expectation of these two loss functions are same. Therefore, the credibility premium from the relative mean square error loss function includes the credibility premiums from Bühlmann's (1967, 1970) and Promislow and Young (2000).

§ 3. Relative Entropy Loss

We incorporate $g(x)=x-\ln x-1$, $x\geq 0$, which is called the entropy loss function in this section. Thus the relative entropy loss function of the order λ is given by $\mu^\lambda(\theta)[r-\ln r-1]$.

Theorem 5 For the relative entropy loss function, the credibility premium $Y_\lambda(\mathbf{X_n})$ is

$$Y_\lambda(\mathbf{X_n})=\frac{\mathsf{E}[\mu^\lambda(\theta)|\mathbf{X_n}]}{\mathsf{E}[\mu^{\lambda-1}(\theta)|\mathbf{X_n}]}.$$

The proof is similar as the proof in Theorem 1 and the credibility premium $Y_\lambda(\mathbf{X_n})$ is an increasing function of the order λ from Corollary 4.

Similar to $Y_p(\mathbf{X_n})$, the credibility premium $Y_\lambda(\mathbf{X_n})$ has the following properties:

$$Y_\lambda(\mathbf{X_n})\begin{cases}\le Y(\mathbf{X_n}) & \text{while } \lambda<1,\\ = Y(\mathbf{X_n}) & \text{while } \lambda=1,\\ \ge Y(\mathbf{X_n}) & \text{while } \lambda>1,\end{cases}\qquad \mathsf{E}Y_\lambda(\mathbf{X_n})\begin{cases}\le \mu & \text{while } \lambda<1,\\ = \mu & \text{while } \lambda=1,\\ \ge \mu & \text{while } \lambda>1.\end{cases}$$

When the premium $Y_\lambda(\mathbf{X_n})$ is restricted to be a linear combination of $\mathbf{X_n}$ with $\mathsf{E}(L_\lambda(\mathbf{X_n}))=\mu$ we can denote $L_\lambda(\mathbf{X_n})=\mu+\overline{\Delta}Z_e^{(\lambda)}$, where $Z_e^{(\lambda)}$ is the credibility factor. $Z_e^{(\lambda)}$ can be calculated by minimizing

$$\mu^\lambda(\theta)\Big[\frac{\mu+\overline{\Delta}Z_e^{(\lambda)}}{\mu(\theta)}-\ln\frac{\mu+\overline{\Delta}Z_e^{(\lambda)}}{\mu(\theta)}-1\Big].$$

Differentiating with respect to $Z_e^{(\lambda)}$ and letting the score function equal 0, we get

$$\mathsf{E}\Big[\mu^\lambda(\theta)\Big(\frac{\overline{\Delta}}{\mu(\theta)}-\frac{\mu(\theta)}{\mu+\overline{\Delta}Z_e^{(\lambda)}}\frac{\overline{\Delta}}{\mu(\theta)}\Big)\Big]=0.$$

Using Taylor Series, we get the asymptotic expansion

$$\Big(1+\frac{\overline{\Delta}}{\mu}Z_e^{(\lambda)}\Big)^{-1}\approx 1-\frac{\overline{\Delta}}{\mu}Z_e^{(\lambda)}+\Big(\frac{\overline{\Delta}}{\mu}Z_e^{(\lambda)}\Big)^2.$$

Solving the equation by using the Taylor expansion, we obtain

$$Z_e^{(\lambda)}=\frac{\mathsf{E}[\mu^\lambda(\theta)\overline{\Delta}/\mu]-\mathsf{E}[\mu^{\lambda-1}(\theta)\overline{\Delta}]}{\mathsf{E}[\mu^\lambda(\theta)(\overline{\Delta}/\mu)^2]}.$$

Since

$$\mathsf{E}\Big[\mu^\lambda(\theta)\frac{\overline{\Delta}}{\mu}\Big]-\mathsf{E}[\mu^{\lambda-1}(\theta)\overline{\Delta}]=\frac{1}{\mu}(\mathsf{E}[\mu^{\lambda-1}(\theta)(\mu(\theta)-\mu)^2]),$$
$$\mathsf{E}\Big[\mu^\lambda(\theta)\Big(\frac{\overline{\Delta}}{\mu}\Big)^2\Big]=\frac{1}{\mu^2}\mathsf{E}[\mu^\lambda(\theta)\mathsf{E}(\overline{X}-\mu)^2],$$

we get the credibility factor

$$Z_e^{(\lambda)}=\frac{\mu a_{\lambda-1}}{a_\lambda}\frac{n}{n+k_\lambda}. \tag{5}$$

Obviously, $Z_e^{(\lambda)}\ge 0$. If $Z_e^{(\lambda)}>1$, we can take $Z_e^{(\lambda)}=1$ instead.

While $\lambda=0$, using (5), we can obtain the value of the credibility factor $Z_e^{(0)}=(\mu a_{-1}/a_0)Z$, where Z is the same as Bühlmann credibility factor (1). Different from (1), there is a weight $\mu a_{-1}/a_0$ before it. We know that $a_0=\mathsf{E}(\mu(\theta)-\mu)^2)$ is the variance of conditional mean $\mu(\theta)$, while we can think $\mu a_{-1}=\mathsf{E}\mu\mu^{-1}(\theta)(\mu(\theta)-\mu)^2)$ as a weighted variance of conditional mean $\mu(\theta)$ with weight $\mu\mu^{-1}(\theta)$. When some policyholder's conditional mean $\mu(\theta)$ is large, the variance of conditional mean $\mu(\theta)$ increases rapidly, while the weighted variance of conditional mean $\mu(\theta)$ with weight $\mu\mu^{-1}(\theta)$ increases slowly. Then the ratio will less than 1. Therefore, we may see that the credibility factor of the relative entropy loss of $\lambda = 0$ will be less than the Bühlmann credibility factor, which means the premium decided by relative entropy loss function will decrease the penalty of larger claim. Thus, the insurance may not charge higher premium just for one large claim based on relative entropy loss function of $\lambda = 0$ and the competitive of the insurance policy and equity of premium may be kept. So, we can use the relative entropy loss of $\lambda = 0$ when the distribution of $\mu(\theta)$ have a heavy right tail.

While $\lambda = 1$, the credibility factor $Z_e^{(1)} = (\mu a_0/a_1)Z$. We may think $a_1/\mu = \mathsf{E}\mu^{-1}\mu(\theta)(\mu(\theta)-\mu)^2)$ as a weighted variance of conditional mean $\mu(\theta)$ with weight $\mu^{-1}\mu(\theta)$. Similar to the relative entropy loss function of $\lambda = 0$, the premium decided by a unlucky policyholder's past experience based on Bülhmann method may be larger than that based on relative entropy loss function of $\lambda = 1$. It means the premium decided by relative entropy loss function of $\lambda = 1$ may decrease the penalty of larger claim. We also can use the relative entropy loss of $\lambda = 1$ when the distribution of $\mu(\theta)$ have a heavy right tail. As to the heavy left tail, in section 4 we point out that the relative entropy loss of $\lambda = 1$ order can also be used.

§ 4. Estimators of Credibility Factors

If we have r group of data, each group has m_i data: $x_{i1}, \cdots, x_{im_i}$. Using the moment method of estimation, the estimators of credibility formulas under the relative mean square error loss function and the relative entropy loss function can be obtained respectively. Firstly, we give the moment estimators of μ, v_ω, a_ω and k_ω respectively as follows.

$$\widehat{\mu} = \overline{x} = \frac{1}{m}\sum_{i=1}^{r} m_i\overline{x}_i = \frac{1}{m}\sum_{i=1}^{r}\sum_{j=1}^{m_i} x_{ij},$$

$$\widehat{v}_\omega = \frac{1}{\sum\limits_{i=1}^{r}(m_i-1)}\sum_{i=1}^{r}\overline{x}_i^{\omega}\sum_{j=1}^{m_i}(x_{ij}-\overline{x}_i)^2,$$

$$\widehat{a}_\omega = \frac{1}{m}\sum_{i=1}^{r} m_i\overline{x}_i^{\omega}(\overline{x}_i-\overline{x})^2,$$

and $\widehat{k}_\omega = \widehat{v}_\omega/\widehat{a}_\omega$, where $m = \sum\limits_{i=1}^{r} m_i$, $\overline{x}_i = (1/m_i)\cdot\sum\limits_{j=1}^{m_i} x_{ij}$. The credibility factor $Z_s^{(p)}$ (see (4)) and $Z_e^{(\lambda)}$ (see (5)) can be estimated separately as follows,

$$\widehat{Z}_s^{(p)} = \frac{n}{n+\widehat{k}_{p-2}}, \qquad \widehat{Z}_e^{(\lambda)} = \frac{\widehat{\mu}\widehat{a}_{\lambda-1}}{\widehat{a}_\lambda}\frac{n}{n+\widehat{k}_\lambda}.$$

The model may have a parametric distribution for X given θ, but an unspecified non-parametric distribution for θ. In this case, we may be able to obtain the semi-parametric estimators of credibility factors.

Let X be the number of claims an insured has during one year, and be assumed to be Poisson distribution with an unknown mean $\theta > 0$ that varies among insureds. The experience for n insureds is $x_1, \cdots, x_n$. The conditional distribution of X given θ is Poisson distribution with parameter θ, so that $\mu(\theta)=\mathsf{E}[X|\theta]=\theta$, $v(\theta)=\mathsf{Var}[X|\theta]=\theta$. Moreover, $\mathsf{E}[(X)_k|\theta]=\theta^k$, where $(X)_k=X(X-1)\cdots(X-k+1)$. Thus, when $\omega\geq 0$ we have

$$\begin{aligned} v_\omega &= \mathsf{E}[\mu^\omega(\theta)v(\theta)]=\mathsf{E}[\theta^{\omega+1}]=\mathsf{E}[(X)_{\omega+1}],\\ a_\omega &= \mathsf{E}[(X)_{\omega+2}]-2\mathsf{E}[(X)_{\omega+1}]\mathsf{E}[X]+\mathsf{E}[(X)_\omega](\mathsf{E}[X])^2. \end{aligned}$$

Obviously, $\widehat{\mu}=\overline{x}$. When $\omega\geq 0$, the semi-parametric estimator of v_ω and a_ω are

$$\widehat{v}_\omega=\frac{\sum\limits_{i=1}^n (x_i)_{\omega+1}}{n}, \tag{6}$$

$$\widehat{a}_\omega=\frac{\sum\limits_{i=1}^n (x_i)_{\omega+2}}{n}-2\frac{\sum\limits_{i=1}^n (x_i)_{\omega+1}}{n}\cdot\frac{\sum\limits_{i=1}^n x_i}{n}+\frac{\sum\limits_{i=1}^n (x_i)_{\omega}}{n}\cdot\left(\frac{\sum\limits_{i=1}^n x_i}{n}\right)^2. \tag{7}$$

Obviously, $\widehat{v}_\omega$ is the unbiased estimator of v_ω, but $\widehat{a}_\omega$ is biased. Using U-statistics, the unbiased estimator of a_ω can be constructed as follows

$$\widehat{a}^*_\omega=\frac{\sum\limits_{i=1}^n (x_i)_{\omega+2}}{n}-2\frac{\sum\limits_{i\neq j}(x_i)_{\omega+1}\cdot x_j}{n(n-1)}+\frac{\sum\limits_{i\neq j,i\neq k,j\neq k}(x_i)_\omega\cdot x_j\cdot x_k}{n(n-1)(n-2)}. \tag{8}$$

The estimator of the entropy credibility factor $Z_e^{(1)}$ and Bülhmann credibility factor Z are

$$\widehat{Z}_e^{(1)}=\frac{\widehat{\mu}\widehat{a}_0}{(\widehat{a}_1+\widehat{v}_1/n)},\qquad \widehat{Z}=\frac{\widehat{a}_0}{\widehat{a}_0+\widehat{v}_0/n}.$$

$\widehat{Z}_e^{(1)}$ and $\widehat{Z}$ only have difference in denominator. Suppose m is added to the number of policyholders who have not any claim and $m\to\infty$, which means the distribution of $\mu(\theta)=\theta$ have a heavy left tail. Then $\widehat{\mu}$, $\widehat{v}_\omega$, $\widehat{a}_\omega$ and $\widehat{a}^*_\omega$ will be modified respectively as follows:

$$\begin{aligned} \widehat{\mu}&=\frac{\sum\limits_{i=1}^n x_i}{n+m},\qquad \widehat{v}_\omega=\frac{\sum\limits_{i=1}^n (x_i)_{\omega+1}}{n+m},\\ \widehat{a}_\omega&=\frac{\sum\limits_{i=1}^n (x_i)_{\omega+2}}{n+m}-2\frac{\sum\limits_{i=1}^n (x_i)_{\omega+1}}{n+m}\cdot\frac{\sum\limits_{i=1}^n x_i}{n+m}+\frac{\sum\limits_{i=1}^n (x_i)_{\omega}}{n+m}\cdot\left(\frac{\sum\limits_{i=1}^n x_i}{n+m}\right)^2,\\ \widehat{a}^*_\omega&=\frac{\sum\limits_{i=1}^n (x_i)_{\omega+2}}{n+m}-2\frac{\sum\limits_{i\neq j}(x_i)_{\omega+1}\cdot x_j}{(n+m)(n+m-1)}+\frac{\sum\limits_{i\neq j,i\neq k,j\neq k}(x_i)_\omega\cdot x_j\cdot x_k}{(n+m)(n+m-1)(n+m-2)}. \end{aligned}$$

Hence, when $m\to\infty$. we have

$$\widehat{Z}_e^{(1)}=\frac{\widehat{\mu}\widehat{a}_0}{\widehat{a}_1+\widehat{v}_1/n}\to 0,\qquad \widehat{Z}=\frac{\widehat{a}_0}{\widehat{a}_0+\widehat{v}_0/n}\to 1.$$

When some policyholders have no claims incurred, the credibility factor of the relative entropy loss of $\lambda=1$ order will be smaller than the Bühlmann credibility factor, which means the premium decided by relative entropy loss function will decrease the bonus of smaller claim. Therefore, the insurance may not decrease premium much just for one small claim based on relative entropy loss function of $\lambda = 1$. So, we can use the relative entropy loss of $\lambda = 1$ when the distribution of $\mu(\theta)$ have a heavy left tail.

The problem that calculating a policyholder's insurance premium is very important. To deal with this problem, many factors should be considered, for example, social, economical and marketing factors and so on. Here the one method of the quantitative analysis, credibility theory is discussed. Besides Bühlmann's classical credibility premium and the Promislow and Young's equitable credibility premium, the relative mean square error loss and entropy loss credibility premiums are proposed in this paper. We would say that each method has its own advantage. Basing on the actuary's experience and synthesizing with the factors of various aspects to select the appropriate method, you can make the reasonable premium for the $(n+1)^{\text{th}}$ period.

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相对损失函数下的倍度保费

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传统的倍度保费公式利用均方损失函数估计特定保人的风险．然而，索取保费与真实保费之间的比例比它们差的绝对值更适合于衡量保费的公平性．基于这一点，我们提出了两种计算保费的损失函数：均方相对损失函数和熵相对损失函数，并且给出了倍度因子的估计公式及它们的性质．

关 键 词: 均方损失，熵损失，相对损失，倍度.

学 科 分 类 号: O211.9, F840.69.