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A Multi-Chart Approach for Mean Shift Detection *

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Abstract

In this paper we consider a multi-chart for detecting a unknown shift in the mean of an identically distributed process. It is shown that the multi-chart has usually two advantages: one is in that it can much reduce computational complexity compared to the GLR (generalized likelihood ratio) and GEWMA (generalized exponentially weighted moving average) control charts when the in-control ARL (average run length) is large; the other is that it can quickly detect the size of the mean shift. Moreover, the numerical simulations show that the multi-chart can not only perform better than its constituent charts which consist of the multi-chart in the sense that the average of the ARLs of the constituent charts is large than that of the multi-chart, but also be superior on the whole to a single CUSUM, EWMA, EWMA multi-chart and GLR control charts in detecting the various mean shifts when the in-control ARL is not large.

Keywords: Statistical process control, change point detection, average run length.

AMS Subject Classification: Primary 62L10; secondary 62N10.

§1. Introduction

Statistical process control (SPC) techniques such as the cumulative sum (CUSUM) charts and exponentially weighted moving average (EWMA) charts have been studied extensively in statistics and engineering (see Lai (1995) and references therein). It has been shown by Moustakides (1986) and Ritov (1990) that the performance in detecting the mean shift of the one-sided CUSUM control chart with the reference value δ is optimal if the real mean shift is δ in terms of the average run length (ARL). In fact, we rarely know the exact shift value of a process before we detect the mean shift. That is to say,

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the performance of the CUSUM chart in detecting the mean shift depends heavily on the given reference value which is the magnitude of a mean shift to be detected quickly. By the same reason the detecting performance of many control charts such as the EWMA, the optimal EWMA and cumulative score (Cuscore) is closely related to the given reference value or reference pattern. Many studies on the three control charts have been done by Crowder (1987, 1989), Lucas and Saccucci (1990), Montgomery and Mastrangelo (1991), Baxley (1995), Mastrangelo and Montgomery (1995), Reynolds (1996a and 1996b), Box and Luceno (1997), Ramirez (1998), Hawkins and Olwell (1998), Luceno (1999), Mastrangelo and Brown (2000), Jiang, Tsui and Woodall (2000), Jones, Champ and Rigdon (2001), and Shu, Apley and Tsung (2002).

To solve the problem of detecting the unknown mean shift of the process, Siegmund and Venkatraman (1995) presented a CUSUM-like control chart, called the GLR chart, and Han and Tsung (2004) proposed a generalized EWMA (GEWMA) control chart, these charts do not depend the reference value. However, the two charts require a complex computing and thus it not easy to use them in the real on-line problems in practice.

Another method to solve the problem of detecting the unknown magnitude of the change is to use a family of control charts. The pioneering work on this issue was done by Lorden (1971) and Lorden and Eisenberger (1973) on charting a set of CUSUM statistics. Dragalin (1993, 1997) investigated the design and analysis of a combination of two CUSUM charts. Sparks (2000) further explored this idea and studied a combination of three CUSUM charts in particular via simulation. On the other hand, Willsky and Jones (1976) introduced the window-limited GLR scheme, which was theoretically investigated by Lai (1995, 1998) and by Lai and Shan (1999). Although the window-limited GLR scheme and the GLR control chart have good performance in detecting the unknown magnitude of the changes, their computational complexity and lack of a capability in diagnosing the possible magnitude of the changes restrict their application in real on-line problems. To make the GLR scheme practicable, Nikiforov (2000) proposed a suboptimal recursive approach that is based on a collection of L parallel recursive χ^2-CUSUM charts and established a direct relation between the efficiency of the detection scheme and its computational complexity. However, the charts mentioned above are all consist of likelihood ratio statistics.

In this paper we will consider a general multi-chart and prove that the ARL of the multi-chart over a range is smaller than the average of ARLs of its constituent charts.

In the next section, we present a definition of the multi-chart and discuss its properties related to the CUSUM, EWMA and GLR control chart. Theoretical comparison of the multi-chart with its constituent charts is given in Section 3. Section 4 shows the simulation results of ARL's of the two-sided CUSUM, EWMA, CUSUM multi-chart, EWMA multi-chart and the GLR charts. Conclusions and problems for further study are discussed in Section 5.

§2. A Multi-Chart

Let X_i, $i = 1, 2, \cdots$ be random variables with a known common probability distribution P_{μ_0}, where μ_0 is the mean of X_i. Suppose that at some time period, τ, which is usually called a change point, the probability distribution of X_i changes from P_{μ_0} to P_{μ}, in other words, from time period τ onwards X_i has the common distribution P_{μ}, that is, the mean of X_i undergoes a persistent shift of size $\mu - \mu_0$, where μ_0 and the standard deviation of X_i, σ are known and without loss of generality, it is assumed that $\mu_0 = 0$ and $\sigma = 1$. Now we give the definition of a multi-chart in the following.

Definition 2.1 Let $\Delta_m = \{\delta_k : 1 \le k \le m\}$, $C_m = \{c_k : 1 \le k \le m\}$ be two sets of numbers, where $m \ge 2$, δ_k is the known reference value and $c_k > 0$ is a control limit which depends usually on δ_k, and $f_n(\delta, x_1, \cdots, x_n)$, $n \ge 1$, a series continuous functions on the set of real numbers, $\delta, x_1, \cdots, x_n$. Then the following minimum stopping time

$$T^*(\Delta_m, C_m) = \min_{\delta_k \in \Delta_m} \{\min\{n > 0, |f_n(\delta_k, X_1, \cdots, X_n)| > c_k\}\} \tag{2.1}$$

is called a multi-chart.

Obviously, the $T^*(\Delta_m, C_m)$ can be rewritten as

$$T^*(\Delta_m, C_m) = \min(T(\delta_1, c_1), T(\delta_2, c_2), \cdots, T(\delta_m, c_m)), \tag{2.2}$$

where $T(\delta_k, c_k) = \min\{n > 0, |f_n(\delta_k, X_1, \cdots, X_n)| > c_k\}$.

From the definition we know that the multi-chart is different not only from the multivariate chart (Hotelling (1947), Lowry, Woodall, Champ and Rigdon (1992), Mason, Champ, Tracy, Wierda and Young (1997), and Apley and Tsung (2002)) but also from the multihypothesis test (Chernoff (1959), Baum and Veeravalli (1994), Dragalin, Tartakovsky and Veeravalli (1999), and Lai (2000)). Note that the multi-chart can be constituted by different forms of the test statistic functions, $f_n(\delta, x_1, \cdots, x_n)$. In this paper we only consider such a multi-chart which consists of a certain number of control charts with identically form.

When $\{X_n\}$ is a process of independent observations distributed normally and the test statistic functions $f_n(\cdot; X_1, \cdots, X_n)$ are taken in the following form

$$f_n(\delta, X_1, \cdots, X_n) = \max_{1 \le k \le n} \delta[X_n + \cdots + X_{n-k+1} - \delta k/2]$$

or equally

$$f_0 = 0, \qquad f_n(\delta, X_1, \cdots, X_n) = \max\{f_{n-1}(\delta, X_1, \cdots, X_{n-1}) + \delta(X_n - \delta/2), 0\}$$

and

$$f_0 = 0, \qquad f_n(\delta, X_1, \cdots, X_n) = \max\{f_{n-1}(\delta, X_1, \cdots, X_{n-1}) - \delta(X_n + \delta/2), 0\},$$

and

$$f_0 = 0, \qquad f_n(\delta, X_1, \cdots, X_n) = rX_n + (1-r)f_{n-1}(r, X_1, \cdots, X_{n-1})$$

where the reference values $\delta > 0$ and $0 < r \leq 1$, we can obtain the one-side CUSUM and EWMA multi-chart, $T_{\rm C}^*(\Delta_m, C_m)$ and $T_{\rm E}^*(R_m, D_m)$ respectively, as follows

$$T_{\rm C}^*(\Delta_m, C_m) = \min_{\delta_i \in \Delta_m} \Big\{ \min \Big\{ n : \max_{1 \leq k \leq n} \delta_i [X_n + \cdots + X_{n-k+1} - \delta_i k/2] > c_i \Big\} \Big\}$$

and

$$T_{\rm E}^*(R_m, D_m) = \min_{r_i \in R_m} \Big\{ \min \Big\{ n : \sum_{k=0}^{n-1} r_i(1-r_i)^k X_{n-k} > d_i \Big\} \Big\}.$$

As can be seen that, for the observations, $X_1, X_2, \cdots, X_n$, it needs only mn times of calculation for the CUSUM or EWMA multi-chart in detecting mean shift, while the GLR and GEWMA tests (see Siegmund and Venkatraman (1995), Han and Tsung (2004)) in the following

$$T_{\rm GL}(c) = \min \Big\{ n : \max_{1 \leq k \leq n} |[X_n + \cdots + X_{n-k+1}]/k^{1/2}| > c \Big\}$$

and

$$T_{\rm GE}(c) = \inf \Big\{ n \geq 1 : \max_{1 \leq k \leq n} \Big| \overline{W}_n \Big(\frac{1}{k}\Big) \Big| \geq c \Big\},$$

where

$$\overline{W}_n \Big(\frac{1}{k}\Big) = \frac{\sqrt{2 - 1/k}}{\sqrt{(1/k) \cdot [1 - (1 - 1/k)^{2n}]}} \sum_{i=0}^{n-1} \frac{1}{k} \Big(1 - \frac{1}{k}\Big)^i X_{n-i},$$

need $n(n+1)/2$ times. Especially, when n is large, e.g., 1000, the computational burden for the GLR and GEWMA charts become very heavy. Thus, the multi-chart has usually an advantage in the computational issue. Moreover, once a mean shift is out of the control limit we can diagnose the possible size of the mean shift if the chosen reference values have some relation with the sizes of the mean shift, e.g. the CUSUM multi-chart. That is to say, the multi-chart can overcome the two weaknesses of the GLR and GEWMA control charts. Another important problem we concern in the present paper is that how is the detecting performance of the multi-chart. That is, can it be quickly in detecting the mean shift? The following section will discuss the problem.

§3. Comparison of the Multi-Chart with Its Constituent Charts

For the convenience of discussion, we use the standard quality control terminology. Let $\mathsf{P}_0(\cdot)$ and $\mathsf{E}_0(\cdot)$ denote the probability and expectation when there is no change in the mean. Denote $\mathsf{P}_\mu(\cdot)$ and $\mathsf{E}_\mu(\cdot)$ as the probability and expectation when the change point is at $\tau = 1$, and the true mean shift value is μ. For a stopping time T as the alarm time with a detecting procedure, two most frequently used operating characteristics are the in-control average run length (ARL_0) and the out-of-control average run length (ARL_μ), defined by $\mathrm{ARL}_0(T) = \mathsf{E}_0(T)$ and $\mathrm{ARL}_\mu(T) = \mathsf{E}_\mu(T)$. Usually, comparisons of the control charts' performance are made by designing the common ARL_0 and comparing the ARL_μ's of the control charts for a given shift μ. The chart with the smaller ARL_μ is considered to have better performance.

Without loss of generality we assume in the following that the mean $\mu \geq 0$ and the parameters $\{\delta\}$ satisfy $0 < \delta_1 < \delta_2 < \cdots < \delta_m$. We first compare the multi-chart with its constituent charts. It is obvious that

$$\mathrm{ARL}_\mu(T(\delta_k, c_k)) \geq \mathrm{ARL}_\mu(T^*(\Delta_m, C_m))$$

for $1 \leq k \leq m$. But the above inequality usually doesn't hold if they have the common ARL_0. That is, when we take the control limits $c'_1, c'_2, \cdots, c'_m$, such that $c'_k > c_k$, $1 \leq k \leq m$, and $\mathrm{ARL}_0(T^*(\Delta_m, C'_m)) = \mathrm{ARL}_0(T(\delta_1, c_1)) = \mathrm{ARL}_0(T(\delta_2, c_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c_m))$, the following inequality

$$\mathrm{ARL}_\mu(T(\delta_k, c_k)) \geq \mathrm{ARL}_\mu(T^*(\Delta_m, C'_m))$$

usually doesn't holds, where $C'_m = \{c'_k, 1 \leq k \leq m\}$. In fact, the $\mathrm{ARL}_\mu(T(\delta_k, c_k))$ can attain at the minimum value for the CUSUM chart when the parameter δ_k is just equal to the mean shift μ. But the following inequality can hold under some conditions,

$$\frac{\sum\limits_{k=1}^{m} \mathrm{ARL}_\mu(T(\delta_k, c_k))}{m} \geq \mathrm{ARL}_\mu(T^*(\Delta_m, C'_m)). \tag{3.1}$$

This means that the multi-chart can perform better than its constituent charts in the sense that the average of the ARLs of the constituent charts is large than that of the multi-chart. The following theorem can be obtained from (3.1).

Next we give some conditions under which (3.1) holds.

Lemma 3.1 Let $H = \max\limits_{\delta} \max\limits_{c>0} \{\mathrm{ARL}_0(T(\delta, c))\}$. Then for every $m \geq 2$ and positive number $M < H$, there exist numbers $c'_k > c_k$, $1 \leq k \leq m$ such that

$$\mathrm{ARL}_0(T(\delta_k, c'_k)) > \mathrm{ARL}_0(T(\delta_k, c_k))$$

for $1 \leq k \leq m$,

$$\mathrm{ARL}_0(T(\delta_1, c'_1)) = \mathrm{ARL}_0(T(\delta_2, c'_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c'_m))$$

and

$$\begin{aligned} M &= \mathrm{ARL}_0(T^*(\Delta_m, C'_m)) \\ &= \mathrm{ARL}_0(T(\delta_1, c_1)) = \mathrm{ARL}_0(T(\delta_2, c_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c_m)) \end{aligned}$$

where $C'_m = \{c'_k, 1 \leq k \leq m\}$.

Proof Since the test statistic function $f_n(\delta, x_1, \cdots, x_n)$ is continuous and strictly increasing, it follows that, for every δ, $\mathrm{ARL}_0(T(\delta, c))$ is also continuous and increasing on c $(c > 0)$ and so does $\mathrm{ARL}_0(T^*(\Delta_m, C_m))$ on C_m. Thus, for the positive constant M we can take the numbers c_k, $1 \le k \le m$ such that

$$M = \mathrm{ARL}_0(T(\delta_1, c_1)) = \mathrm{ARL}_0(T(\delta_2, c_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c_m)).$$

Similarly, we can choose the numbers c'_k, $1 \le k \le m$ such that $c'_k > c_k$, $\mathrm{ARL}_0(T(\delta_k, c'_k)) > \mathrm{ARL}_0(T(\delta_k, c_k))$ for $1 \le k \le m$ and

$$\mathrm{ARL}_0(T(\delta_1, c'_1)) = \mathrm{ARL}_0(T(\delta_2, c'_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c'_m)).$$

Note that

$$\mathrm{ARL}_0(T^*(\Delta_m, C'_m)) < \mathrm{ARL}_0(T(\delta_1, c'_1)) = \mathrm{ARL}_0(T(\delta_2, c'_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c'_m)).$$

If $\mathrm{ARL}_0(T^*(\Delta_m, C'_m)) < M$, we can take the numbers c''_k, $1 \le k \le m$ such that $c''_k > c'_k$, $\mathrm{ARL}_0(T(\delta_k, c''_k)) > \mathrm{ARL}_0(T(\delta_k, c'_k))$ for $1 \le k \le m$,

$$\mathrm{ARL}_0(T(\delta_1, c''_1)) = \mathrm{ARL}_0(T(\delta_2, c''_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c''_m))$$

and

$$\mathrm{ARL}_0(T^*(\Delta_m, C''_m)) = M,$$

where $C''_m = \{c''_k, 1 \le k \le m\}$, since $\mathrm{ARL}_0(T^*(\Delta_m, C_m))$ is continuous and increasing on C_m.

If $\mathrm{ARL}_0(T^*(\Delta_m, C'_m)) > M$, we can similarly take the number c^*_k, $1 \le k \le m$ such that $c'_k > c^*_k > c_k$, $\mathrm{ARL}_0(T(\delta_k, c^*_k)) > \mathrm{ARL}_0(T(\delta_k, c_k))$ for $1 \le k \le m$,

$$\mathrm{ARL}_0(T(\delta_1, c^*_1)) = \mathrm{ARL}_0(T(\delta_2, c^*_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c^*_m))$$

and

$$\mathrm{ARL}_0(T^*(\Delta_m, C^*_m)) = M,$$

where $C^*_m = \{c^*_k, 1 \le k \le m\}$. This completes the proof. 1cm□

Lemma 3.2 Suppose that $\mathrm{ARL}_0(T(\delta,c))$ is differentiable and its two partial derivatives are continuous with respect to δ and c, then, for every positive constant $M < H$, there exists $(\Delta_{(m+1)}, C_{(m+1)})$ such that

$$\mathrm{ARL}_0(T^*(\Delta_{(m+1)}, C_{(m+1)})) = M$$

and

$$\mathrm{ARL}_0(T^*(\Delta_m^{(1)}, C_m^{(1)})) = \mathrm{ARL}_0(T(\Delta_m^{(2)}, C_m^{(2)})) = \cdots = \mathrm{ARL}_0(T^*(\Delta_m^{(m+1)}, C_m^{(m+1)}))$$

where $\Delta_m^{(k)} = \{\delta_i : 1 \le i \le m+1, i \ne k\}$ and $C_m^{(k)} = \{c_i : 1 \le i \le m+1, i \ne k\}$, $1 \le k \le m+1$.

Proof Since the partial derivatives of $\mathrm{ARL}_0(T(\delta,c))$ are continuous, it follows from (2.2), the definition of multi-chart that $\mathrm{ARL}_0(T^*(\Delta_m,C_m))$ is also differentiable and its 2m partial derivatives are continuous with respect to $\delta_1, c_1, \delta_2, c_2, \cdots, \delta_m, c_m$, respectively. Thus by the inversion and implicit theorem we obtain the lemma. 1cm□

Theorem 3.3 Let the numbers $c_1, c_2, \cdots, c_m$ satisfy $\mathrm{ARL}_0(T(\delta_k, c_k)) = M < H$ for $1 \le k \le m$ and the condition of Lemma 3.2 holds. Suppose that, for the stopping times $T_x = \min\limits_{1\le j\le k}\{T(\delta_{1j}, x)\}$ and $T'_x = \min\limits_{1\le j\le k}\{T(\delta_{2j}, x')\}$, where $1 \le k \le m-1$, $\{\delta_{ij}, 1 \le j \le k\} \in \Delta_k$ for $i = 1, 2$, and $x = a,\ b > 0$,

$$\frac{\mathrm{ARL}_\mu(T_a) + \mathrm{ARL}_\mu(T_b)}{2} \ge \mathrm{ARL}_\mu(\min\{T'_a, T'_b\}) \tag{3.2}$$

holds so long as

$$\mathrm{ARL}_0(\min\{T'_a, T'_b\}) = \mathrm{ARL}_0(T_a) = \mathrm{ARL}_0(T_b)$$

and

$$\mathrm{ARL}_0(T'_a) = \mathrm{ARL}_0(T'_b) > \mathrm{ARL}_0(T_a) = \mathrm{ARL}_0(T_b).$$

Then, for every $m \ge 2$, there exists the numbers $c'_k > c_k$, $1 \le k \le m$ such that $\mathrm{ARL}_0(T(\delta_k, c'_k)) > \mathrm{ARL}_0(T(\delta_k, c_k))$, $1 \le k \le m$, $\mathrm{ARL}_0(T^*(\Delta_m, C'_m)) = M$ and

$$\frac{\sum\limits_{k=1}^{m} \mathrm{ARL}_\mu(T(\delta_k, c_k))}{m} \ge \mathrm{ARL}_\mu(T^*(\Delta_m, C'_m)). \tag{3.3}$$

Proof Let $d_\mu(\delta, c) = \mathrm{ARL}_\mu(T(\delta, c))$ and $D_\mu(\Delta_m, C_m) = \mathrm{ARL}_\mu(T^*(\Delta_m, C_m))$. From Lemma 3.1 and (3.2) it follows that the theorem holds for $m = 2$. Assume that the theorem is true for $m = k \ge 2$. Then, for $m = k+1$ we have

$$\frac{d_\mu(\delta_1, c_1) + d_\mu(\delta_2, c_2) + \cdots + d_\mu(\delta_{k+1}, c_{k+1})}{k+1} = \frac{\sum\limits_{i=1}^{k+1} \sum\limits_{j\ne i} d_\mu(\delta_j, c_j)}{k(k+1)}. \tag{3.4}$$

According to the assumption that (3.2) holds for $m = k$, thus there exist numbers $c_j^{(i)}$, $1 \le i \le k+1$, $1 \le j \le k$ such that $c_j^{(i)} > c_j$ for $1 \le j \le k$, $D_0(\triangle_k^{(i)}, C_k^{(i)}) = M$ and

$$\frac{\sum\limits_{j=1, \ne i}^{k+1} d_\mu(\delta_j, c_j^{(i)})}{k} \ge D_\mu(\triangle_k^{(i)}, C_k^{(i)}) \tag{3.5}$$

for $1 \le i \le k+1$, where $\triangle_k^{(i)} = \{\delta_j : 1 \le j \le k+1, j \ne i\}$ and $C_k^{(i)} = \{c_j^{(i)}, 1 \le j \le k\}$. On the other hand, it follows form Lemma 3.2 that there exist numbers $c'_i > c_i$, $1 \le i \le k+1$ such that $D_0(\triangle_{k+1}, C'_{k+1}) = M$, and

$$D(\triangle_k^{(1)}, C_k^{(1)'}) = D(\triangle_k^{(2)}, C_k^{(2)'}) = \cdots = D(\triangle_k^{(k+1)}, C_k^{(k+1)'})$$

where $C'_{k+1} = \{c'_i : 1 \le i \le k+1\}$, $C_k^{(i)'} = \{c'_j : 1 \le j \le k+1, j \ne i\}$ for $1 \le i \le k+1$. Note that $D_0(\triangle_k^{(i)}, C_k^{(i)'}) > D_0(\triangle_k^{(i)}, C_k^{(i)})$, since $D_0(\triangle_{k+1}, C'_{k+1}) = M = D_0(\triangle_k^{(i)}, C_k^{(i)})$. Hence, by using (3.2) we have

$$\frac{D_\mu(\triangle_k^{(i)}, C_k^{(i)}) + D_\mu(\triangle_k^{(j)}, C_k^{(j)})}{2} \ge \mathrm{ARL}_\mu(\min\{T_i, T_j\}) \tag{3.6}$$

for $1 \le i \ne j \le k+1$, where $T_i = \min\limits_{1 \le l \le k+1,\, l \ne i} \{T(\delta_l, c'_l)\}$, $1 \le i \le k+1$. Obviously, $\min\{T_i, T_j\} = T^*(\Delta_{k+1}, C'_{k+1})$ for $1 \le i \ne j \le k+1$. Thus, by (3.4), (3.5) and (3.6),

$$\begin{aligned}
& \frac{d_\mu(\delta_1, c_1) + d_\mu(\delta_2, c_2) + \cdots + d\mu(\delta_{k+1}, c_{k+1})}{k+1} \\
\ge\ & \frac{\sum\limits_{i+1}^{k+1} D_\mu(\triangle_k^{(i)}, C_k^{(i)})}{k+1} = \frac{2\sum\limits_{i=1}^{k+1} D_\mu(\triangle_k^{(i)}, C_k^{(i)})}{2(k+1)} \\
\ge\ & \mathrm{ARL}_\mu(T^*(\triangle_{k+1}, C'_{k+1})).
\end{aligned}$$

By mathematical inductive method, the theorem is proved. 1cm□

Remark 1 The result of Theorem 3.3 is comparatively general since it does not depend on the specific information about the observations $\{X_n\}$.

For the CUSUM multi-chart

$$T_C^*(\Delta_m, C_m) = \min_{1 \le k \le m} \{T(\delta_k, c_k)\}$$

where

$$T(\delta_k, c_k) = \min\left\{n : \max_{1 \le i \le n} \delta_k[X_n + \cdots + X_{n-i+1} - \delta_k i/2] > c_k\right\}$$

and $0 < \delta_1 < \delta_2 < \cdots < \delta_m$, we have the following corollary which say that (3.3) of Theorem 3.3 hold for large ARL_0.

Corollary 3.4 Let the distribution of the observations $\{X_n\}$ be normal and $M = \mathrm{ARL}_0(T(\delta_k, c_k))$ for $1 \le k \le m$, then

$$\frac{1}{m}\sum_{k=1}^{m} \mathrm{ARL}_\mu(T(\delta_k, c_k)) > \mathrm{ARL}_\mu(T_C^*(\triangle_m, C'_m)) \tag{3.7}$$

holds for large M, so long as $\mathrm{ARL}_0(T^*(\triangle_m, C'_m)) = M$ and

$$\begin{aligned}
& \mathrm{ARL}_0(T(\delta_1, c'_1)) = \mathrm{ARL}_0(T(\delta_2, c'_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c'_m)) \\
>\ & \mathrm{ARL}_0(T(\delta_1, c_1)) = \mathrm{ARL}_0(T(\delta_2, c_2)) = \cdots = \mathrm{ARL}_0(T(\delta_m, c_m)) = M.
\end{aligned}$$

Proof It needs only to check that (3.2) of Theorem 3.3 holds for any two stopping times $T_i = T(\delta_i, c_i)$ and $T_j = T(\delta_j, c_j)$ for large M, that is

$$\frac{\mathrm{ARL}_\mu(T_i) + \mathrm{ARL}_\mu(T_j)}{2} \ge \mathrm{ARL}_\mu(\min\{T'_i, T'_j\}) \tag{3.8}$$

for large M, where $T_i' = T(\delta_i, c_i')$ and $T_j' = T(\delta_j, c_j')$. Without loss generality we take $i = 1$ and $j = 2$.

Let $T(\delta, c)$ denote the CUSUM chart with reference value δ and control limit c. It is known that (see Srivastava and Wu (1997))

$$\mathsf{E}_\mu(T(\delta, c)) = (1 + o(1)) \frac{e^{(\delta-2\mu)(c+2\delta\rho)/\delta} - 1 - (\delta - 2\mu)(c + 2\delta\rho)/\delta}{2(\mu - \delta/2)^2}$$

for $\delta > 2\mu$, $\mathsf{E}_\mu(T(\delta, c)) = (1 + o(1)) \cdot (c^2/\delta^2)$ as $\delta \to 2\mu$, and

$$\mathsf{E}_\mu(T(\delta, c)) = (1 + o(1)) \frac{2c}{\delta(2\mu - \delta)}$$

for $\delta < 2\mu$. Note that $c_i/c_i' \to 1$ as $M \to \infty$. It follows that

$$\mathsf{E}_\mu(T_i')) = (1 + o(1))\mathsf{E}_\mu(T_i)) \tag{3.9}$$

for large M and $i = 1, 2$.

If $\delta_i > 2\mu$, $1 \le i \le 2$, then $(\delta_2 - 2\mu)/\delta_2 > (\delta_1 - 2\mu)/\delta_1$ since $\delta_2 > \delta_1$, and therefore, by (3.9),

$$\begin{aligned} \mathsf{E}_\mu(T_i)) &= (1 + o(1)) \Big[\frac{e^{(\delta_i-2\mu)(c_i+2\delta_i\rho)/\delta_i} - 1 - (\delta_i - 2\mu)(c_i + 2\delta_i\rho)/\delta_i}{2(\mu - \delta_i/2)^2} \Big] \\ &> \mathsf{E}_\mu(T_1') + o(\mathsf{E}_\mu(T_1)) \end{aligned}$$

for $i = 1, 2$ and large M. Hence,

$$\frac{1}{2}(\mathsf{E}_\mu(T_1) + \mathsf{E}_\mu(T_2)) > \mathsf{E}_\mu(T_1') \ge \mathsf{E}_\mu(\min\{T_i', T_j'\})$$

for large M, that is, (3.8) holds for large M. Similarly, we can check that (3.8) holds for the other cases: $\delta_2 < 2\mu$ or $\delta_2 > 2\mu$, $\delta_1 \le 2\mu$ or $\delta_2 = 2\mu$, $\delta_1 < 2\mu$. Thus, we prove the corollary. 1cm□

Remark 2 The condition that the common ARL_0 is large in the corollary may not need in practical detection. In fact, the simulation examples in the next section have showed that (3.7) holds for $m = 2, 5$ when $\text{ARL}_0 = 500$. Moreover, the numerical simulations in the table 3 in the next section support that (3.3) of Theorem 3.3 can hold for many multi-charts.

We have shown that the CUSUM multi-chart has the better performance than that of its constituent CUSUM charts in the sense of (3.7). What is the result of comparing the CUSUM multi-chart with the GLR control chart? It has been shown that the GLR control chart has the best performance in detecting mean shift among the four control charts, the EWMA, optimal EWMA, GEWMA and CUSUM when the ARL_0 approaches to infinity (see Han and Tsung (2004)). But, when the ARL_0 is not large, the simulation results given in the next section show that the multi-chart has the better performance in detecting small mean shift than the GLR does.

§4. Numerical Illustration

The purpose of this section is to illustrate some simulation results of ARL's of the two-sided CUSUM, EWMA, CUSUM multi-chart, EWMA multi-chart and the GLR charts. The numerical results of ARL's were obtained based on 10000-repetition experiment. The common ARL_0 here is taken to be 500. We compare the simulation results for 10 mean shifts ($\mu_1 = 0.1, \mu_2 = 0.25, \cdots, \mu_{10} = 4$) listed in the first column of Tables' with change point $\tau = 1$.

In order to compare the averages of ARLs' for the CUSUM and EWMA charts with the ARL of the CUSUM and EWMA multi-charts, we list the simulation results of the CUSUM and EWMA charts with the parameters, $\{\delta_1 = 0.1, \delta_2 = 0.5, \delta_3 = 1, \delta_4 = 1.5, \delta_5 = 2\}$ and $\{r_1 = 0.1, r_2 = 0.3, r_3 = 0.5, r_4 = 0.7, r_5 = 0.9\}$, in Table 1 and 2, respectively. Table 3 compares the simulation results of the ARL_μs' for the GLR, CUSUM multi-chart, EWMA multi-chart and the averages of the ARL_μs' for five CUSUM and five EWMA charts corresponding to the cases, $\{\delta_1 = 0.1, \delta_2 = 0.5, \delta_3 = 1, \delta_4 = 1.5, \delta_5 = 2\}$ and $\{r_1 = 0.1, r_2 = 0.3, r_3 = 0.5, r_4 = 0.7, r_5 = 0.9\}$, respectively. The values in the parenthesis in every column of Tables' are the standard deviation of the simulation results of the stopping times. In the first two rows of Tables 1 and 2, c denotes various values of the width of the control limits, δ and r are the parameters of the CUSUM and EWMA charts, respectively. The Aver. CUSUM and Aver. EWMA both in the second and forth column of Table 3 denote respectively, the average of ARL's for the constituent CUSUM and EWMA charts. The sizes of the mean shifts (μ) are listed in the first column of Tables'. We list the simulation results of the GLR (T_{GL}) with the control limit $c = 3.494$ such that $\mathrm{ARL}_0(T_{\mathrm{GL}}) = 500$ in the last column of Table 3. It should be explained that to obtain the $\mathrm{ARL}_0(T^*_{\mathrm{C}}) = 500$ for the CUSUM multi-chart, T^*_{C} in Table 3 we take the control limits, $c'_1 = 2.71$, $c'_2 = 5.22$, $c'_3 = 6.029$, $c'_4 = 6.282$ and $c'_5 = 6.301$ such that $\mathrm{ARL}_0(T(\delta_1, c'_1)) = 1297.4$, $\mathrm{ARL}_0(T(\delta_2, c'_2)) = 1298.5$, $\mathrm{ARL}_0(T(\delta_3, c'_3)) = 1298.6$, $\mathrm{ARL}_0(T(\delta_4, c'_4)) = 1297.2$ and $\mathrm{ARL}_0(T(\delta_5, c'_5)) = 1298.1$. Similarly, we can choose the control limits of the EWMA multi-chart T^*_{E} such that $\mathrm{ARL}_0(T^*_{\mathrm{E}}) = 500$.

Usually, each chart has its strong points in detecting different sizes of the mean shift. In order to compare their whole performance of control charts in detecting various sizes of the mean shift, we use the number,

$$\mathrm{ETD} = \sum_{k=1}^{10} \mu_k \mathrm{ARL}_{\mu_k} \Big/ \sum_{k=1}^{10} \mu_k$$

listed in the last row of Tables', as a standard to judge who is well on the whole. Obviously, the smaller the number is, the better the control chart performs. We may call the number as an expectation of the time for detecting the mean shifts (ETD). The calculation of the number ETD is based on the consideration that in order to sum up the ARLs' in some reasonable way one may reduce the ARLs of detecting small shifts and enlarge the ARLs of detecting large shifts such that the resulting ARLs' can added up in the "same" order of quantity. As can be seen that the ETD is a result of weakening the strong point of control chart in detecting small shifts, at the same time, magnifying its weak point in detecting large shifts.

Table 1 ARL's of the CUSUM control chart with $\text{ARL}_0 = 500$

SHIFTS (μ)	$\delta_1 = 0.1$ $c_1 = 1.979$	$\delta_2 = 0.5$ $c_2 = 4.29$	$\delta_3 = 1$ $c_3 = 5.075$	$\delta_4 = 1.5$ $c_4 = 5.337$	$\delta_5 = 2$ $c_5 = 5.355$
0	500 (414)	500 (491)	500 (502)	500 (490)	500 (498)
0.1	239 (169)	301 (284)	369 (366)	417 (416)	439 (433)
0.25	91.7 (42.7)	94.2 (77.2)	144 (135)	202 (198)	252 (250)
0.5	44.2 (14.3)	31.0 (17.7)	38.9 (31.8)	58.1 (53.7)	81.9 (78.8)
0.75	28.9 (7.46)	17.5 (7.55)	17.2 (11.1)	22.0 (17.9)	30.7 (27.6)
1	21.5 (4.74)	12.2 (4.45)	10.5 (5.56)	11.6 (7.92)	14.6 (11.9)
1.25	17.2 (3.43)	9.34 (2.96)	7.52 (3.36)	7.53 (4.37)	8.56 (6.04)
1.5	14.3 (2.59)	7.59 (2.16)	5.83 (2.29)	5.50 (2.72)	5.80 (3.59)
2	10.8 (1.69)	5.55 (1.32)	4.07 (1.30)	3.56 (1.41)	3.43 (1.63)
3	7.27 (0.93)	3.68 (0.72)	2.60 (0.66)	2.19 (0.64)	1.95 (0.71)
4	5.54 (0.64)	2.84 (0.51)	2.03 (0.38)	1.64 (0.51)	1.39 (0.51)
$\sum_{k=1}^{10} \mu_k \text{ARL}_{\mu_k} \Big/ \sum_{k=1}^{10} \mu_k$	15.38	10.53	11.01	13.05	15.55

Table 2 ARL's of the EWMA control chart with $\text{ARL}_0 = 500$

SHIFTS (μ)	$r_1 = 0.1$ $c_1 = 2.818$	$r_2 = 0.3$ $c_2 = 3.026$	$r_3 = 0.5$ $c_3 = 3.073$	$r_4 = 0.7$ $c_4 = 3.085$	$r_5 = 0.9$ $c_5 = 3.089$
0	500 (497)	500 (495)	500 (492)	500 (504)	500 (502)
0.1	320 (316)	403 (398)	438 (431)	455 (458)	470 (473)
0.25	106 (95.8)	187 (181)	256 (255)	308 (311)	354 (355)
0.5	31.2 (22.2)	55.4 (51.6)	88.7 (87.4)	128 (128)	176 (178)
0.75	15.8 (8.85)	22.5 (18.9)	36.0 (33.9)	55.5 (54.6)	84.6 (85.3)
1	10.3 (4.78)	11.9 (8.61)	17.4 (15.3)	26.9 (25.5)	42.7 (42.1)
1.25	7.68 (3.05)	7.65 (4.73)	10.0 (8.01)	14.7 (13.4)	23.5 (22.9)
1.5	6.10 (2.15)	5.55 (3.00)	6.53 (4.64)	8.90 (7.72)	13.7 (13.1)
2	4.36 (1.25)	3.55 (1.48)	3.64 (2.04)	4.30 (3.12)	5.80 (5.01)
3	2.87 (0.67)	2.16 (0.66)	1.92 (0.78)	1.86 (0.95)	1.98 (1.28)
4	2.19 (0.42)	1.61 (0.52)	1.33 (0.49)	1.23 (0.45)	1.21 (0.48)
$\sum_{k=1}^{10} \mu_k \text{ARL}_{\mu_k} \Big/ \sum_{k=1}^{10} \mu_k$	9.83	12.64	16.53	21.32	28.01

Table 3 illustrates that both the CUSUM and EWMA multi-charts have the better performance in detecting all size of the mean shifts than that of its constituent charts in the sense that the average of the ARLs of the constituent charts is large than the ARLs of the multi-charts. Comparisons of the numbers ETD in Table 1, 2 and 3 show that the ETD (9.27) of the CUSUM multi-chart with five constituent CUSUM charts is smallest among all tests, though the value ETD weakens the strong point, at the same time, magnifies the weak point of the CUSUM multi-chart. That is to say the CUSUM multi-chart is superior on the whole (in the sense of the number ETD) to a single CUSUM, EWMA, EWMA multi-chart and GLR control charts in detecting various mean shifts when the in-control average run length is not large enough.

Table 3 Comparison of the averages of ARL's of the CUSUM and EWMA charts with the ARL's of the multi-chart and GLR control charts with $\mathrm{ARL}_0 = 500$

SHIFTS (μ)	Aver. CUSUM $T_{\mathrm{C}}(\delta_1), \cdots, T_{\mathrm{C}}(\delta_5)$	Multi-chart T_{C}^*	Aver. EWMA $T_{\mathrm{E}}(r_1), \cdots, T_{\mathrm{E}}(r_5)$	Multi-chart T_{E}^*	GLR(T_G) $c = 3.494$
0	500 (479)	500 (460)	500 (498)	500 (499)	500 (492)
0.1	353 (334)	262 (201)	417 (415)	381 (374)	324 (288)
0.25	157 (141)	97.0 (60.5)	242 (240)	146 (135)	114 (83.1)
0.5	50.8 (39.3)	35.2 (20.9)	95.8 (93.5)	40.1 (31.0)	37.4 (23.8)
0.75	23.2 (14.3)	18.2 (9.73)	42.9 (40.3)	18.2 (11.3)	18.6 (10.8)
1	14.1 (6.92)	11.6 (5.98)	21.8 (19.2)	11.2 (6.08)	11.4 (6.24)
1.25	10.0 (4.03)	8.08 (3.98)	12.7 (10.4)	7.81 (3.95)	7.83 (4.11)
1.5	7.81 (2.67)	6.03 (2.82)	8.15 (6.13)	5.85 (2.91)	5.77 (2.92)
2	5.48 (1.47)	3.83 (1.61)	4.33 (2.58)	3.68 (1.77)	3.58 (1.66)
3	3.54 (0.73)	2.20 (0.73)	2.16 (0.87)	1.92 (0.89)	1.94 (0.81)
4	2.69 (0.51)	1.58 (0.53)	1.52 (0.47)	1.28 (0.49)	1.31 (0.49)
$\sum_{k=1}^{10} \mu_k \mathrm{ARL}_{\mu_k} \Big/ \sum_{k=1}^{10} \mu_k$	13.1	9.27	17.67	10.89	9.87

§5. Conclusion and Discussion

We know that the CUSUM or EWMA chart can not give play to its strong point in detecting more than one mean shifts or the mean shift which is unknown, and the GLR and GEWMA require a complex computing in detecting the size of a mean shift. To remedy these defects of the control charts we consider a multi-chart in this paper.

It is shown that the multi-chart has not only the merit in the computational issue but also can quickly detect the size of the mean shift. We proved that the CUSUM multi-chart performs better than its constituent CUSUM charts which consist of the CUSUM multi-chart in the sense that the average of the ARLs of the constituent charts is large than that of the multi-chart. Moreover, the numerical simulation results show that the CUSUM multi-chart is superior on the whole (in the sense of the ETD) to the CUSUM, EWMA, GLR and EWMA multi-charts in detecting various mean shifts when the in-control average run length is not large.

In this paper we only discuss the problem of detecting the mean shift. In fact, we can use the multi-chart to detect simultaneously a change of several values. It often needs for us to detect simultaneously a change of two characteristic values, μ and γ, of a vector random process $Z_n = (X_n, Y_n)$, $n \geq 1$. To detect the change of a pair of (μ, γ), we can choose the parameters $\delta_1 = (\mu_1, \gamma_1)$, $\delta_2 = (\mu_1, \gamma_2)$, $\delta_3 = (\mu_2, \gamma_1)$, $\delta_4 = (\mu_2, \gamma_2)$, $\cdots$, $\delta_k = (\mu_i, \gamma_j)$, $\cdots$, $\delta_m = (\mu_l, \gamma_s)$ to constitute a multi-chart as follows:

$$T^*(\Delta_m) = \min(T(\delta_1), T(\delta_2), \cdots, T(\delta_m)),$$

where

$$T(\delta_k) = \min\{n > 0, |h_n(\mu_i, \sigma_j; Z_1, \cdots, Z_n)| \in D_{ij}\},$$

where $\delta_k = (\mu_i, \sigma_j)$, D_{ij} is a out-control domain of (μ_i, σ_j) and $h_n(\mu_i, \sigma_j; Z_1, \cdots, Z_n)$ is a test statistic function with two parameters μ_i and σ_j.

However, the detection performance of a multi-chart depends on the choice of the test statistic functions and its parameters. In order to constitute a good multi-chart one must solve three problems: What is the form of the test functions to be taken? How many the constituent charts should be? What are the reference values to be chosen? It is worth to further study the three problems.

References

[1] Apley, D.W. and Tsung, F.G., The autoregressive T-squared chart for monitoring univariate autocorrelated processes, *Journal of Quality Technology*, **34**(2002), 80–96.

[2] Baum, C.W. and Veeravalli, V.V., A sequential procedure for multi hypothesis testing, *IEEE Trans. Inform. Theory*, **40**(1994), 1994–2007.

[3] Baxley, R.V., JR., An application of variable sampling interval control charts, *Journal of Quality Technology*, **27**(1995), 275–282.

[4] Box, G.E.P. and Luceño, A., *Statistical Control by Monitoring and Feedback Adjustment*, Wiley, New York, 1997.

[5] Chernoff, H., Sequential design of experiments, *Ann. Math. Statist.*, **30**(1959), 755–770.

[6] Crowder, S.V., A simple method for studying run length distributions of exponentially weighted moving average control charts, *Technometrics*, **29**(1987), 401–407.

[7] Crowder, S.V., Design of exponentially weighted moving average schemes, *Journal of Quality Technology*, **21**(1989), 155–162.

[8] Dragalin, V., The optimality of generalized CUSUM procedure in quickest detection problem, Proceedings of Steklov Math. Inst.: *Statistics and Control of Stochastic Processes*, **202**(1993), 132–148.

[9] Dragalin, V., The design and analysis of 2-CUSUM procedure, *Commun. Statist. – Simula.*, **26**(1997), 67–81.

[10] Dragalin, V.P., Tartakovsky, A.G., and Veeravalli, V.V., Multihypothesis sequential probability ratio tests — Part I: Asymptotic optimality, *IEEE Trans. Inform. Theory*, **45**(1999), 2448–2461.

[11] Han, D. and Tsung, F.G., A generalized EWMA control chart and its comparison with the optimal EWMA, CUSUM and GLR schemes, *Ann. Statist.*, **32**(2004), 316–339.

[12] Hawkins, D.M. and Olwell, D.H., *Cummulative Sum Charts and Charting for Quality Improvement*, Springer-Verlag, New York, 1998.

[13] Hotelling, H., "Multivariate quality control", in *Techniques of Statitical Analysis*, eds. C. Eisenhart, M.W. Hastay, and W.A. Wallis, New York, McGraw-Hill, 1974.

[14] Jiang, W., Tsui, K.L. and Woodall, W.H., A new SPC monitoring method: the ARMA chart, *Technometrics*, **42**(2000), 399–416.

[15] Jones, L.A., Champ, C.W. and Rigdon, S.T., The performance of exponentially weighted moving average charts with estimated parameters, *Technometrics*, **43**(2001), 156–167.

[16] Lai, T.L., Sequential change-point detection in quality control and dynamic systems, *J. Roy. Statist. Soc. Ser. B*, **57**(1995), 613–658.

[17] Lai, T.L., Information bounds and quick detection of parameter changes in stochastic systems, *IEEE Trans. Inform. Theory*, **44**(1998), 2917–2929.

[18] Lai, T.L. and Shan, J.Z., Efficient recursive algorithms for detection for abrupt changes in signals and control systems, *IEEE Trans. Automat Control*, **44**(1999), 952–966.

[19] Lai, T.L., Sequential multiple hypothesis testing and efficient fault detection-Isolation in stochastic systems, *IEEE Trans. Inform. Theory*, **46**(2000), 595–608.

[20] Lorden, G., Procedures for reacting to a change in distribution, *Ann. Math. Statist.*, **42**(1971), 520–527.

[21] Lorden, G. and Eisenberger, I., Detection of failure rate increases, *Technometrics*, **15**(1973), 167–175.

[22] Lowry, C.A., Woodall, W.H., Champ, C.W. and Rigdon, S.E., A multivariate exponentially weighted moving average control chart, *Technometrics*, **34**(1992), 46–53.

[23] Lucas, J.M. and Saccucci, M.S., Exponentially weighted moving average control schemes: properties and enhancements, *Technometrics*, **32**(1990), 1–16.

[24] Luceño, A., Average run lengths and run length probability distributions for Cuscore charts to control normal mean, *Computational Statistics & Data Analysis*, **32**(1999), 177–195.

[25] Mason, R.L., Champ, C.W., Tracy, N.D., Wierda, S.J. and Young, J.C., Assessment of multivariate process control techniques, *Journal of Quality Technology*, **29**(1997), 140–143.

[26] Mastrangelo, C.M. and Brown, E.C., Shift detection properties of moving centerline control chart schemes, *Journal of Quality Technology*, **32**(2000), 67–74.

[27] Mastrangelo, C.M. and Montgomery, D.C., SPC with correlated observations for the chemical and process industries, *Quality and Reliability Engineering International*, **11(2)**(1995), 79–89.

[28] Montgomery, D.C. and Mastrangelo, C.M., Some statistical process control charts methods for auto-correlated data, *Journal of Quality Technology*, **23**(1991), 179–193.

[29] Moustakides, G.V., Optimal stopping times for detecting changes in distribution, *Ann. Statist.*, **14**(1986), 1379–1387.

[30] Murray-Smith, R. and Johansen, T.A., *Multiple Model Approaches to Modelling and Control*, Taylor & Francis, London, 1997.

[31] Nikiforo, I., A suboptimal quadratic change detection scheme, *IEEE Trans. Inform. Theory*, **46**(2000), 2095–2107.

[32] Ramírez, G.J., Monitoring clean room air using Cuscore charts, *Qual. Reliab. Engng. Int.*, **14**(1998), 281–189.

[33] Reynolds, M.R., JR., Shewhart and EWMA control charts using variable sampling intervals with sampling at fixed times, *Journal of Quality Technology*, **28**(1996a), 199–212.

[34] Reynolds, M.R., JR., Variable sampling-interval control charts with sampling at fixed times, *IIE Transactions*, **28**(1996b), 497–510.

[35] Ritov, Y., Decision theoretic optimality of the CUSUM procedure, *Ann. Statist.*, **18**(1990), 1464–1469.

[36] Shu, L.J., Apley, D.W. and Tsung, F., Autocorrelated process monitoring using triggered Cuscore charts, *Quality and Reliability Engineering International*, **18**(2002), 411–421.

[37] Siegmund, D. and Venkatraman, E.S., Using the generalized likelihood ratio statistic for sequential detection of a change-point, *Ann. Statist.*, **23**(1995), 255–271.

[38] Sparks, R.S., CUSUM charts for signalling varying location shifts, *Journal of Quality Technology*, **32**(2000), 157–171.

[39] Willsky, A.S. and Jones, H.L., A generalized likelihood ratio approach to detection and estimation of jumps in linear systems, *IEEE Trans. Automat. Contr.*, **21**(1976), 108–112.

[40] Srivastava, M.S. and Wu, Y.H., Evaluation of optimum weights and average run lengths in EWMA control schemes, *Commun. Statist. – Theory Meth.*, **26(5)**(1997), 1253–1267.

监测均值变动的多重控制图方法

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本文证明了当受控平均运行长度充分大时, 多重控制图有两个优点: 一是相比较GLR (广义似然比)和GEWMA (广义指数权重移动平均)控制图它可以大大降低运算的复杂性; 二是能够较快地监测均值变化的大小. 数值模拟也表明: 多重控制图不仅优于其构成的单个控制图, 而且在监测未知的均值变动方面也优于单个的CUSUM, EWMA, 多重EWMA和GLR控制图.

关键词: 统计过程控制, 变点监测, 平均运行长度.

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