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Two-Limit Tobit-Autoregression-GARCH with Estimates of the Model of Stock Daily Return Rate with Price Limits

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Abstract

A stock daily return rate with price limits model, called two-limit Tobit-autoregression-GARCH (TLTARG) is introduced. Maximum likelihood estimation (MLE) for this model is constructed. With Monte Carlo experiments, the MLE is examined. An example of TLTARG model estimation on stock daily return rate in Shanghai stock market is given.

Keywords: Price limits, two-limit Tobit-autoregression-GARCH, maximum likelihood estimation, Monte Carlo experiments.

AMS Subject Classification: 60K30.

§ 1. Introduction

Daily price limits in stock market are supposed to be able to control market volatility and provide time for rational reassessment during times of panic trading. Despite the debate about their impact on market, the consequence is that they bound the price movements, truncate the distribution of true price changes and constrain the observed prices within a specified range based on the previous day's closing price.

Generally speaking, there are two kinds of daily price limits regulations in stock markets. One limits the range of the daily price movements according to the previous day's closing price, such as the price limits rule in Tokyo stock market. That is

$$p_t = \begin{cases} p_{t-1} + PL_1 & \text{if} \ \ p_t^* \geq p_{t-1} + PL_1 \\ p_t^* & \text{if} \ \ p_{t-1} + PL_2 < p_t^* < p_{t-1} + PL_1 \ , \\ p_{t-1} + PL_2 & \text{if} \ \ p_t^* \leq p_{t-1} + PL_2 \end{cases}$$

where p_t is the closing price observed on t-day, p_t^* is the latent closing price on t-day (p_t^* is unobservable if it hits the limits), PL_1 and PL_2 are the price upper limit and lower limit, respectively. The other sets the size of the daily return rate, such as the rule in Shanghai stock market. This is

$$r_t = \begin{cases} RL_1 & \text{if} \ \ r_t^* \geq RL_1 \\ r_t^* & \text{if} \ \ RL_2 < r_t^* < RL_2 \ , \\ RL_2 & \text{if} \ \ r_t^* \leq RL_2 \end{cases}$$

where $r_t = (p_t - p_{t-1})/p_{t-1}$ is the return rate observed on t-day, $r_t^* = (p_t^* - p_{t-1})/p_{t-1}$ is the latent return rate on t-day, RL_1 and RL_2 are, respectively, the return rate upper limit and lower limit. Perhaps, it is more appropriate to call the latter daily return rate limits rule. In this paper, we discuss the latter.

Although the topic about price limits has attracted a great deal attention since 1987 crash and price limits regulations exist actually in many stock markets in the world, an applicable econometric model has not been developed yet that explains the mechanism of price limits in stock market. The main difficulty to model the

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stock movements with price limits might be that in a market with price limits, when a shock occurs such that the price moves outside the daily maximum allowable range, it becomes unobservable. What is observed is merely a limit price, in which case, there is an imbalance between market demand and supply. Sometimes, the excess demand/supply will accumulate and be carried over to successive trading days until it is fully reflected at the trading price, at which time the price falls back within the limits again. When this case occurs, the analysis is more difficult.

§ 2. Model

Since the stock price will be censored according to price limits rule, as first step, it is rational to consider the two-limit Tobit regression model (Rosett (1975)). In this model with latent regression $y_t^* = \alpha' x_t + \varepsilon_t$ $(t = 1, \cdots, T)$, where α is a $K \times 1$ vector of parameters, x_t is a $K \times 1$ vector of independent variables and $\{\varepsilon_t\}$ are assumed to be i.i.d. $N(0, \sigma^2)$ series, the dependent variable is bounded below and above by L_1 and L_2. The dependent variable y_t is determined by

$$y_t = \begin{cases} L_1 & \text{if } y_t^* \geq L_1 \\ y_t^* & \text{if } L_2 < y_t^* < L_1 \\ L_2 & \text{if } y_t^* \leq L_2 \end{cases}, \tag{1}$$

where y_t^* is latent dependent variable and y_t is observable dependent variable. In following discussion, r_t will replace y_t denoting the daily return rate observed on t-day, r_t^* will replace y_t^* denoting the latent daily return rate on t-day, L_1 and L_2 are the return rate upper limit and lower limit, respectively.

Because of serial correlation of financial data, the explanatory item should include lagged auto-explanatory variables. Thus, the latent regression equation is written as

$$r_t^* = A_p(l) r_t^* + \alpha' x_t + \varepsilon_t, \tag{2}$$

where $A_p(l) = \alpha_1 l + \alpha_2 l^2 + \cdots + \alpha_p l^p$, l is the lag operator and x_t are other explanatory variables. Supposing that other explanatory variable x_t is only constant 1, equation (2) becomes a p-order autoregressive equation $r_t^* = \alpha_0 + \alpha_1 r_{t-1}^* + \cdots + \alpha_p r_{t-p}^* + \varepsilon_t$. Together with the price limits rule, it is defined the two-limit Tobit-autoregression model

$$\begin{aligned} & r_t^* = \alpha_0 + \alpha_1 r_{t-1}^* + \cdots + \alpha_p r_{t-p}^* + \varepsilon_t, \\ & r_t = \begin{cases} L_1 & \text{if } r_t^* \geq L_1 \\ r_t^* & \text{if } L_2 < r_t^* < L_1 \\ L_2 & \text{if } r_t^* \leq L_2 \end{cases}. \end{aligned} \tag{3}$$

This model has been used for futures market with price limits. The problem is how we estimate the model since some of latent r_{t-i}^* $(i = 1, 2, \cdots, p)$ are unobservable after they hit limits. In investigating futures market with price limits, Koderes (1993) and Yang (1995) coped with the problem using a proxy based on corresponding spot market price. However, the method is not suitable to stock market since stock market itself is the spot market of asset. While studying the effect of price limits on stock market, Lee (1996) suggested to use the opening price as the proxy of the previous day's limit-hit closing price. Since the market information will probably have changed greatly overnight, with this method, the estimates will be rough and intractable, especially when hitting limits continues for several days.

In fact, in stock market, most investors try to forecast stock price according to known information. For example, they predict a stock tomorrow's price on the basis of today's closing price, today's price change rate or other known market information but not unknown latent information. According to such a setting, we assume that in a stock market with price limits, investors' forecasting stock price is based on observable or known information and actual stock price movements should be appropriate reflection of investors' action. Based on this assumption, the autoregressive equation is redefined as $r_t^* = \alpha_0 + \alpha_1 r_{t-1} + \cdots + \alpha_p r_{t-p} + \varepsilon_t$. Thus, we have the modified two-limit Tobit-autoregression model

$$r_t^* = \alpha_0 + \alpha_1 r_{t-1} + \cdots + \alpha_p r_{t-p} + \varepsilon_t,$$
$$r_t = \begin{cases} L_1 & \text{if } r_t^* \geq L_1 \\ r_t^* & \text{if } L_2 < r_t^* < L_1 \\ L_2 & \text{if } r_t^* \leq L_2 \end{cases}, \tag{4}$$

where $L_2 \leq r_{t-i} \leq L_1, \quad i = 1, 2, \cdots, p$.

Much research on securities market has demonstrated that the data are potentially characterized by conditional heteroskedasticity. If ignored, it will cause inconsistent estimates in Tobit model (Hurd (1979), Arabmazar (1981), and Amemiya (1984)). Moreover, besides the rate of return, an asset holder would be interested in forecast of its risk (i.e., conditional variance) over the holding period. It is necessary, therefore, to specify the conditional variance-covariance structure for estimating the model consistently and forecasting the conditional variance. We model the disturbance ε_t in equation (4) following a generalized autoregressive conditional heteroskedastic, or GARCH(p, q), process (Bollerslev (1986)).

$$\varepsilon_t | \Theta_{t-1} \sim N(0, h_t),$$

where

$$h_t = \beta_0 + \sum_{j=1}^{q} \beta_j h_{t-j} + \sum_{j=1}^{r} \delta_j \varepsilon_{t-j}^2 \qquad \Big(\beta_0 > 0, \ \beta_j \geq 0, \ \delta_j \geq 0, \ \sum_{j=1}^{q} \beta_j + \sum_{j=1}^{r} \delta_j < 1\Big)$$

and Θ_{t-1} is the set of past information.

§ 3. Maximum Likelihood Estimation of TLTARG

Let $\Psi = \{1, 2, \cdots, T\}$ denote the full sample and then split it into three subsamples according to r_t

$$\Psi_0 = \{t | \forall t \ \text{ if } \ L_2 < r_t < L_1\}, \qquad \Psi_1 = \{t | \forall t \ \text{ if } \ r_t = L_1\}, \qquad \Psi_2 = \{t | \forall t \ \text{ if } \ r_t = L_2\}.$$

Since ε_t is assumed normally distributed with conditional heteroskedasticity, the conditional probability of upper limit moves is

$$\begin{aligned} & \mathsf{P}(r_t = L_1 | r_{t-1}, r_{t-2}, \cdots, r_{t-p}; \theta) \\ = \ & \mathsf{P}(\varepsilon_t \geq L_1 - \alpha_0 - A_p(l) r_t | r_{t-1}, r_{t-2}, \cdots, r_{t-p}; \theta) \\ = \ & 1 - \Phi\Big(\frac{L_1 - \alpha_0 - A_p(l) r_t}{\sqrt{h_t}}\Big) \end{aligned}$$

and the conditional probability of lower limit moves is

$$\begin{aligned} & \mathsf{P}(r_t = L_2 | r_{t-1}, r_{t-2}, \cdots, r_{t-p}; \theta) \\ = \ & \mathsf{P}(\varepsilon_t \leq L_2 - \alpha_0 - A_p(l) r_t | r_{t-1}, r_{t-2}, \cdots, r_{t-p}; \theta) \\ = \ & \Phi\Big(\frac{L_2 - \alpha_0 - A_p(l) r_t}{\sqrt{h_t}}\Big). \end{aligned}$$

The conditional density (Hamilton (1994, Chapter 5)) of no limit move is

$$f(r_t|r_{t-1},r_{t-2},\cdots,r_{t-p};\theta)=\frac{1}{\sqrt{h_t}}\phi\Big(\frac{r_t-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big).$$

Thus, the likelihood function may be written as

$$\mathcal{L}=\prod_{\Psi_0}\frac{1}{\sqrt{h_t}}\phi\Big(\frac{r_t-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big)\prod_{\Psi_1}\Big(1-\Phi\Big(\frac{L_1-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big)\Big)\prod_{\Psi_2}\Phi\Big(\frac{L_2-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big),\tag{5}$$

where θ is the vector of parameters, $A_p(l)=\alpha_1 l+\alpha_2 l^2+\cdots+\alpha_p l^p$, l is the lag operator, $\phi(x)=(1/\sqrt{2\pi})e^{-x^2/2}$ and $\Phi(x)=\int_{-\infty}^{x}(1/\sqrt{2\pi})e^{-\lambda^2/2}\mathrm{d}\lambda$ are the p.d.f. and c.d.f., respectively, of a standard normal random variable. The logarithm of (5) is

$$\begin{aligned}\ln\mathcal{L} &= -\frac{1}{2}\sum_{\Psi_0}\ln h_t+\sum_{\Psi_0}\ln\phi\Big(\frac{r_t-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big)+\sum_{\Psi_1}\ln\Big(1-\Phi\Big(\frac{L_1-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big)\Big)\\ &\quad+\sum_{\Psi_2}\ln\Phi\Big(\frac{L_2-\alpha_0-A_p(l)r_t}{\sqrt{h_t}}\Big).\end{aligned}\tag{6}$$

The normal equations are defined in vector notation as

$$\frac{\partial\ln\mathcal{L}}{\partial\theta}=0.\tag{7}$$

If $\widehat{\theta}$ is the solution of (7) and the second derivatives, $\partial^2\ln\mathcal{L}(\widehat{\theta})/\partial\theta\partial\theta'$ is a negative definite matrix, then $\widehat{\theta}$ is defined as the estimator of the maximum likelihood estimation. Since equation (7) is highly nonlinear, to solve it, an iterative numerical method must be used.

§4. Monte Carlo Experiment

Amemiya (1973) proved that the MLE (i.e., ML estimator) of a standard Tobit model is strongly consistent and asymptotically normal with the asymptotic variance-covariance matrix equal to $-(\partial^2\ln\mathcal{L}/\partial\theta\partial\theta')^{-1}$. At present, however, we can not conclude whether the MLE of TLTARG maintains these properties since the TLTARG model is different from a standard Tobit model. For this reason, it is necessary to examine whether the procedure discussed in Section 3 works well or not. Since the solution of equation (7) is complicated, the analytical evaluation is difficult. Therefore, we evaluate the MLE by a numerical procedure (i.e., Monte Carlo experiments). A TLTAR(1)G(1,1) model

$$r_t^*=\alpha_0+\alpha_1 r_{t-1}+\varepsilon_t,$$

$$r_t=\begin{cases}L_1 & \text{if } r_t^*\ge L_1\\ r_t^* & \text{if } L_2<r_t^*<L_1\\ L_2 & \text{if } r_t^*\le L_2\end{cases},\tag{8}$$

$$\varepsilon_t|\Theta_{t-1}\sim N(0,h_t),\qquad h_t=\beta_0+\beta_1 h_{t-1}+\beta_2\varepsilon_{t-1}^2\tag{9}$$

is adopted in experiments. The true values of five parameters are taken as $\alpha_0=0.5$, $\alpha_1=0.5$, $\beta_0=1$, $\beta_1=0.5$ and $\beta_2=0.4$. In our experiments, $L_1=|L_2|=L$, i.e., the limits are symmetric. We select $L=2(34.5\%)$, $4(12.7\%)$, $6(5.3\%)$ and $L=\infty(0\%)$, where the percentage in parentheses is called " ALS ratio " defined as: ALS ratio = average limit hit number / sample size[1]. For each L, we select the sample size $T=250$, 500, 1000

[1] We take sample size $n=1000$, simulate the TLTAR(1)G(1,1) process and then count up the limit hit number. In order to obtain the ALS ratio, this procedure is repeated 1000 times.

and then use these generated data to estimate the model with ML method discussed in Section 3. These steps are repeated 1000 times. Thus, for each given L and T, we obtain 1000 estimates of α_0, α_1, β_0, β_1 and β_2.

Table 1.a Monte Carlo experiments with the estimation of TLTAR(1)G(1,1) model

Sample Size T	Estimator (True Value)	Limit $L = 2(34.5\%)^*$			Limit $L = 4(12.7\%)$		
		Mean	SD	Bias	Mean	SD	Bias
250	$\alpha_0(0.500)$	0.514	0.134	0.014	0.498	0.137	0.002
	$\alpha_1(0.500)$	0.490	0.092	0.010	0.499	0.070	0.001
	$\beta_0(1.000)$	1.325	0.939	0.325	1.267	0.733	0.267
	$\beta_1(0.500)$	0.390	0.323	0.110	0.431	0.209	0.069
	$\beta_2(0.400)$	0.432	0.249	0.032	0.415	0.162	0.015
500	$\alpha_0(0.500)$	0.509	0.093	0.009	0.504	0.096	0.004
	$\alpha_1(0.500)$	0.495	0.065	0.005	0.500	0.047	0.000
	$\beta_0(1.000)$	1.217	0.735	0.217	1.112	0.497	0.112
	$\beta_1(0.500)$	0.430	0.248	0.070	0.477	0.137	0.023
	$\beta_2(0.400)$	0.418	0.174	0.018	0.396	0.111	0.004
1000	$\alpha_0(0.500)$	0.502	0.067	0.002	0.502	0.067	0.002
	$\alpha_1(0.500)$	0.498	0.045	0.002	0.497	0.033	0.003
	$\beta_0(1.000)$	1.099	0.453	0.099	1.071	0.299	0.071
	$\beta_1(0.500)$	0.470	0.167	0.030	0.481	0.088	0.019
	$\beta_2(0.400)$	0.400	0.124	0.000	0.404	0.076	0.004

* The percentage in parentheses is ALS ratio.

Table 1.b Monte Carlo experiments with the estimation of TLTAR(1)G(1,1) model

Sample Size T	Estimator (True Value)	Limit $L = 6(5.3\%)$			Limit $L = \infty(0\%)$		
		Mean	SD	Bias	Mean	SD	Bias
250	$\alpha_0(0.500)$	0.516	0.143	0.016	0.500	0.140	0.000
	$\alpha_1(0.500)$	0.495	0.063	0.005	0.499	0.056	0.001
	$\beta_0(1.000)$	1.217	0.649	0.217	1.169	0.572	0.169
	$\beta_1(0.500)$	0.456	0.163	0.044	0.470	0.138	0.030
	$\beta_2(0.400)$	0.406	0.134	0.006	0.393	0.108	0.007
500	$\alpha_0(0.500)$	0.508	0.095	0.008	0.500	0.101	0.000
	$\alpha_1(0.500)$	0.495	0.044	0.005	0.497	0.044	0.003
	$\beta_0(1.000)$	1.105	0.399	0.105	1.095	0.343	0.095
	$\beta_1(0.500)$	0.477	0.101	0.023	0.482	0.082	0.018
	$\beta_2(0.400)$	0.404	0.086	0.004	0.402	0.078	0.002
1000	$\alpha_0(0.500)$	0.503	0.069	0.003	0.500	0.072	0.000
	$\alpha_1(0.500)$	0.499	0.030	0.001	0.500	0.030	0.000
	$\beta_0(1.000)$	1.037	0.246	0.037	1.026	0.214	0.026
	$\beta_1(0.500)$	0.493	0.067	0.007	0.498	0.054	0.002
	$\beta_2(0.400)$	0.401	0.062	0.001	0.398	0.051	0.002

The results are shown as Table 1.a and Table 1.b, where Mean, SD and Bias denote the estimates' average, the standard deviation and the bias, which is defined as: Bias = |mean − true value|. From Table 1.a and Table 1.b, we can see that in small sample, the estimates of both α_0 and α_1, the parameters in mean equation (8) are unbiased, but the estimates of β_0, β_1 and β_2, the parameters in variance equation (9) are biased. With the sample size T increasing, the Mean's of all estimates approach their corresponding true parameter values and SD's also decrease. Accordingly, it might be concluded that the MLE's are consistent for large sample. In addition, the results show that with the limit size decreasing (i.e., the ALS ratio increasing), the speed of the

Mean's converging to the true parameter values slows down. Intuitively, since the information included in limit-hit data is incomplete, the ALS ratio increasing means that the proportion of incomplete information increases. As a result, for the same sample size, the greater the ALS ratio is, the greater the bias of estimates is. Thus, with the ALS ratio increasing, it is necessary to take larger sample size for estimation in order to make up lost information.

Table 2.a Monte Carlo experiments with the estimation of AR(1)G(1,1) model

Sample	Estimator	Limit $L=2(34.5\%)$			Limit $L=4(12.7\%)$		
Size T	(True Value)	Mean	SD	Bias	Mean	SD	Bias
250	$\alpha_0(0.500)$	0.386	0.100	0.114	0.509	0.145	0.009
	$\alpha_1(0.500)$	0.347	0.066	0.153	0.434	0.065	0.066
	$\beta_0(1.000)$	0.770	0.600	0.230	1.177	0.808	0.177
	$\beta_1(0.500)$	0.402	0.382	0.098	0.492	0.236	0.008
	$\beta_2(0.400)$	0.121	0.063	0.279	0.198	0.079	0.202
500	$\alpha_0(0.500)$	0.397	0.076	0.103	0.524	0.103	0.024
	$\alpha_1(0.500)$	0.345	0.046	0.155	0.435	0.047	0.065
	$\beta_0(1.000)$	0.648	0.399	0.352	1.066	0.476	0.066
	$\beta_1(0.500)$	0.482	0.258	0.018	0.527	0.148	0.027
	$\beta_2(0.400)$	0.118	0.045	0.282	0.190	0.054	0.210
1000	$\alpha_0(0.500)$	0.397	0.049	0.103	0.517	0.062	0.017
	$\alpha_1(0.500)$	0.345	0.032	0.155	0.439	0.030	0.061
	$\beta_0(1.000)$	0.600	0.266	0.400	1.005	0.284	0.005
	$\beta_1(0.500)$	0.512	0.174	0.012	0.542	0.095	0.042
	$\beta_2(0.400)$	0.119	0.030	0.281	0.194	0.037	0.206

Table 2.b Monte Carlo experiments with the estimation of AR(1)G(1,1) model

Sample	Estimator	Limit $L=6(5.3\%)$			Limit $L=\infty(0\%)$		
Size T	(True Value)	Mean	SD	Bias	Mean	SD	Bias
250	$\alpha_0(0.500)$	0.535	0.147	0.035	0.506	0.141	0.006
	$\alpha_1(0.500)$	0.468	0.059	0.032	0.491	0.059	0.009
	$\beta_0(1.000)$	1.286	0.788	0.286	1.161	0.509	0.161
	$\beta_1(0.500)$	0.487	0.190	0.013	0.465	0.129	0.035
	$\beta_2(0.400)$	0.281	0.092	0.119	0.402	0.108	0.002
500	$\alpha_0(0.500)$	0.525	0.103	0.025	0.495	0.103	0.005
	$\alpha_1(0.500)$	0.470	0.044	0.030	0.499	0.041	0.001
	$\beta_0(1.000)$	1.155	0.401	0.155	1.091	0.358	0.091
	$\beta_1(0.500)$	0.512	0.105	0.012	0.488	0.085	0.012
	$\beta_2(0.400)$	0.283	0.061	0.117	0.390	0.078	0.010
1000	$\alpha_0(0.500)$	0.522	0.068	0.022	0.502	0.071	0.002
	$\alpha_1(0.500)$	0.474	0.030	0.026	0.498	0.032	0.002
	$\beta_0(1.000)$	1.076	0.269	0.076	1.029	0.200	0.029
	$\beta_1(0.500)$	0.533	0.073	0.033	0.494	0.052	0.006
	$\beta_2(0.400)$	0.275	0.044	0.125	0.399	0.052	0.001

In contrast, Table 2.a and Table 2.b report also the estimates of AR(1)G(1,1) (i.e., autoregression(1)-GARCH(1,1)) model, in which estimation, the price limits are ignored. In this case, the results show that MLE's are biased and inconsistent. With the size of limit L increasing (i.e., ALS ratio decreasing), the bias of estimates decreases. When $L\to\infty$ (i.e., no price limit exists), both methods' estimates are almost the same. The Monte Carlo experiment procedure of TLTAR(1)G(1,1) is written by TSP code.

§ 5. Estimation of Shanghai Stock Daily Return Rate

With only 8 listed stocks, the Shanghai stock market commenced on 19 December 1990 and there had been nearly 600 listed stocks by the end of 2000. Initially, an upper and lower bound of 5% daily change rate of stock price was set. The price restriction was finally lifted on 21 May 1992. Without daily price limits, the stock price moved violently at times. For this reason, the daily price limits regulation was introduced again in December 1996. It sets that the size of both upper and lower limits of daily price change rates based on previous closing price is 10%.

The data (company code: 600637) are downloaded from http://www.stockstar.com.cn (Estimation Period: 1996/12/9–2000/8/10). According to Akaike's (1981) information criterion, the mean equation includes one lagged observable dependent variable. We restrict the variance equation to a GARCH(1,1) specification since it has been shown to be a parsimonious representation of conditional variance that adequately fits many economic time series (e.g., Bollerslev (1987)). Therefore, the model to be estimated is TLTAR(1)G(1,1) written as

$$R_t^* = \alpha_0 + \alpha_1 R_{t-1} + \varepsilon_t \qquad (L_2 \le R_{t-1} \le L_1),$$
$$R_t = \begin{cases} L_1 & \text{if } R_t^* \ge L_1 \\ R_t^* & \text{if } L_2 < R_t^* < L_1 \\ L_2 & \text{if } R_t^* \le L_2 \end{cases}, \tag{10}$$
$$\varepsilon_t|\Theta_{t-1} \sim N(0, h_t), \qquad h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 \varepsilon_{t-1}^2,$$

where $R_t = 100 \times r_t$, $r_t = (P_t - P_{t-1})/P_{t-1}$, $R_t^* = 100 \times r_t^*$, $r_t^* = (P_t^* - P_{t-1})/P_{t-1}$, P_t is the closing price observed on t-day, P_t^* is the latent closing price on t-day, upper limit $L_1 = 10$ and lower limit $L_2 = -10$. This study uses the BHHH algorithm, an algorithm attributed to Berndt, Hall, Hall and Hausman (1974). The estimate procedure is written by TSP code.

Figure 1 shows the plot of R_t versus day t. Table 3 reports the summary statistics of sample data R_t. Table 4.a and Table 4.b report the estimates of TLTAR(1)G(1,1) and AR(1)G(1,1) models respectively. Figure 2.a and Figure 2.b show TLTAR(1)G(1,1) and AR(1)G(1,1) conditional heteroskedasticity h_t (i.e., volatility) versus day t, respectively. We can see that though the ALS ratio is merely 5%, averagely, the conditional variance in TLTAR(1)G(1,1) model is over 10 greater than that in AR(1)G(1,1) model, which ignores price limits.

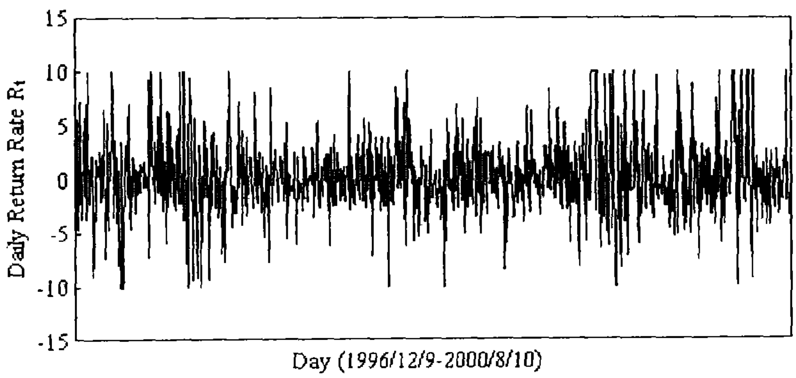

Figure 1 Stock (Code: 600637) daily return rate R_t

Table 3 Summary statistics of stock daily return rate R_t

Company Code	Sample Size	Limit-hit Number	ALS Ratio	Mean	Variance	Skewness	Kurtosis
600637	882	44	5.0%	0.230	14.633	0.329	3.997

Table 4.a TLTAR(1)G(1,1) estimates of stock daily return rate

Parameter	Estimate	Error	T-statistic	P-value
α_0	0.128	0.122	1.047	0.295
α_1	0.048	0.041	1.163	0.245
β_0	1.550	0.325	4.774	0.000
β_1	0.733	0.035	20.706	0.000
β_2	0.196	0.031	6.412	0.000
$\log L$		-2348.39		

Table 4.b AR(1)G(1,1) estimates of stock daily return rate

Parameter	Estimate	Error	T-statistic	P-value
α_0	0.135	0.119	1.133	0.257
α_1	0.047	0.038	1.237	0.216
β_0	1.421	0.342	4.151	0.000
β_1	0.773	0.040	19.482	0.000
β_2	0.128	0.026	4.866	0.000
$\log L$		-2378.43		

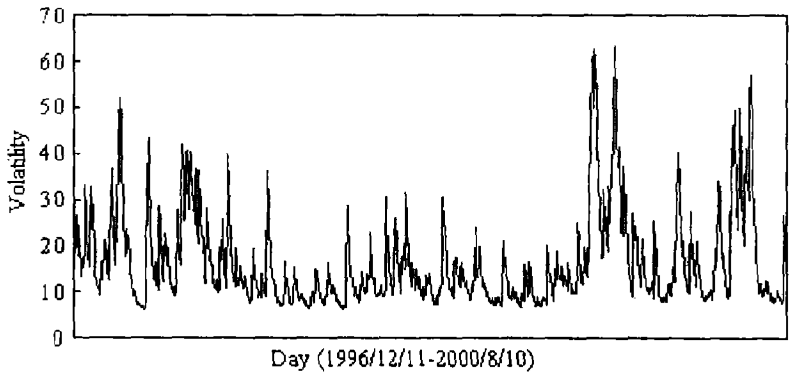

Figure 2.a Stock (600637) daily return rate volatility estimate using TLTAR(1)G(1,1) model

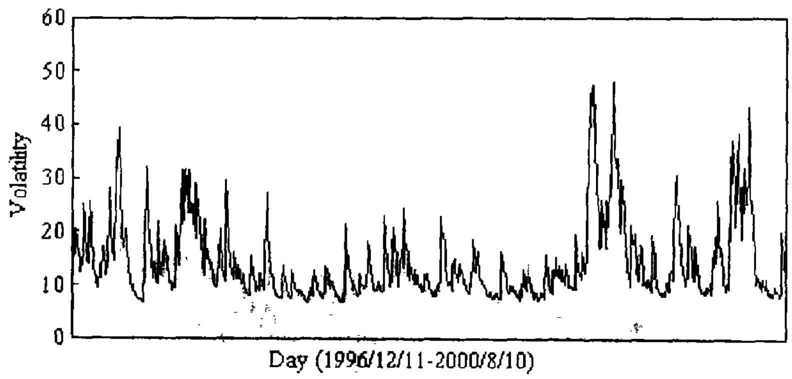

Figure 2.b Stock (600637) daily return rate volatility estimates using AR(1)G(1,1) model

§6. Conclusions

In this paper, a stock return rate with price limits model, called two-limit Tobit-auregression-GARCH (TLTARG) is introduced. Maximum likelihood estimation for this model is constructed. The Monte Carlo experiments show that the MLE's of mean equation are unbiased and consistent. However, the MLE's of variance equation are biased though consistent. With the limit L decreasing (i.e., the ALS ratio increasing), the speed that the MLE's converge to the corresponding true parameter values slows down. These experiments show also that if price limits are ignored (i.e., ARG model), the MLE's are bias and inconsistent. With the ALS ratio decreasing, however, the bias decreases.

Finally, as an empirical application, we estimated a TLTAR(1)G(1,1), which is specified as the stock (company code: 600637) return rate model in Shanghai stock market. Meanwhile, we also estimated a AR(1)G(1,1) model using the same data. It is shown that though the ALS ratio is 5% only, the average conditional variance of TLTAR(1)G(1,1) model is over 10 greater than that of AR(1)G(1,1) model. It implies that though for some stocks' price movements, price limits may be hit so seldom that ignoring them makes no discernible difference, the TLTARG model is more accurate than ARG.

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双限制 Tobit 自回归 GARCH 模型和价格限制下股票日收益率模型估计

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本文提出了一个在价格限制即涨跌停板制度存在的情况下，股票日收益率可能遵循的时间序列模型 — 双限制 Tobit 自回归 GARCH 模型，建立了此模型的最大似然估计法 (MLE)，用 Monte Carlo 实验研究了最大似然估计量性质．作为此模型应用，我们对一个上海股市的股票日收益率模型参数进行了估计．

关 键 词: 价格限制，双限制 Tobit 自回归 GARCH 模型，最大似然估计， Monte Carlo 实验.

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