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Strong Stability of Linear Forms of ψ-Mixing Random Variables *

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Abstract

The strong stability of linear forms had found many applications in the science and technology. In this paper, we investigate the strong stability of linear forms for ψ-mixing sequence. By using the termination, Borel-Cantelli lemma and properties of ψ-mixing sequence, the sufficient condition of the strong stability of linear forms for ψ-mixing sequence is given. Stability of other linear forms in ψ-mixing sequence are given at the same time.

Keywords: Strong stability, ψ-mixing, linear form.

AMS Subject Classification: 60F15.

§1. Introduction

Probability density estimation, nonparametric and nonlinear regression are probably the most widely studied nonparametric estimation problems. Many methods have been developed under the independent observations. In recent years, some papers have been developed to extending these method to dependent case due to the widely existence of dependent random variables, which arose lots of probability questions, such as the strong stability of linear forms. The strong stability of linear forms had found many applications in ecology, molecular biology, biochemistry etc. Study of the strong stability of linear forms is promoted by the large number law and is useful in compatibility of least square estimation in linear model. Therefore the research about the strong stability of linear forms is undoubtedly very important.

In 2004, Gan (2004)[1] studied the almost sure convergence of ρ-mixing random variables. For strictly stationary sequences, the ψ-mixing sequences was first introduced by Blum et al. (1963)[2]. ψ-mixing sequences include some widely used examples, such as

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countable state space Markov processes. More examples of ψ-mixing sequences can be found in Blum et al. (1963)[2]. As we have known, few studies can been found about the stability of ψ-mixing sequence.

In this paper, we first study the variables by using termination and then taken the sufficient condition of the strong stability of linear forms for ψ-mixing sequence in usual situation through Borel-Cantelli lemma and properties of ψ-mixing sequence. Based on above result, we give results on the stability of other linear forms in ψ-mixing sequence.

In the following, we present some results on the strong stability of linear forms in ψ-mixing sequence. The rest of the article is organized as follows: In Section 2, we state and prove the main result. Then in Section 3, we prove some theorems on the stability of other linear forms for ψ-mixing sequence.

§2. Strong Stability of Linear Forms for X_n

Before we state the main results, we recall some definitions which will be useful in the following.

Definition 2.1 Let $\{X_n, n \geq 1\}$ be a stationary random variables defined on probability space $(\Omega, \mathcal{F}, \mathsf{P})$. Denote by $\mathcal{F}_m$ the σ field generated by $\{X_i, 1 \leq i \leq m\}$ and by $\mathcal{F}^n$ the σ field generated by $\{X_i, i \geq n\}$. Let

$$\psi(r) = \sup_{p\geq 1} \sup_{\substack{A\in\mathcal{F}_p, B\in\mathcal{F}_{r+p},\\ \mathsf{P}(A)\mathsf{P}(B)\neq 0}} \Big|\frac{\mathsf{P}(AB)}{\mathsf{P}(A)\mathsf{P}(B)} - 1\Big|,$$

$\{X_n, n \geq 1\}$ is said to be a ψ-mixing random sequence if $\psi(r) \to 0$ as $r \to \infty$. $\psi(r)$ are the ψ-mixing coefficients.

Definition 2.2 A random variable sequence $\{X_n, n \geq 1\}$ is said to be *strongly stable* if there exist two constant sequences $\{b_n\}$ and $\{d_n\}$ with $0 < b_n \uparrow \infty$ such that

$$b_n^{-1} X_n - d_n \to 0 \qquad \text{a.s..} \tag{2.1}$$

Definition 2.3 A random variable sequence $\{X_n, n \geq 1\}$ is said to be *stochastically dominated* by a non-negative random variable X if there exists a positive constant c such that

$$\mathsf{P}(|X_n| > t) \leq c\,\mathsf{P}(X > t) \qquad \text{for any } t > 0,\ n \geq 1. \tag{2.2}$$

We denote this case by $\{X_n\} < X$.

Unless specially indicated, we suppose in the following that $\{X_n, n \geq 1\}$ is ψ-mixing and the corresponding mixing coefficients satisfy:

$$\sum_{r=1}^{\infty} \psi(r) < \infty. \tag{2.3}$$

The following theorems summarize the strong stability of linear forms for ψ-mixing sequence $\{X_n\}$.

Theorem 2.1 Let $\{b_n, n \geq 1\}$ be a sequence of positive numbers with $b_n \uparrow \infty$. Under the condition that $\mathsf{E}X_n = 0$ and $\sum\limits_{n=1}^{\infty} b_n^{-p} \mathsf{E}|X_n|^p < \infty$ for some $1 \leq p \leq 2$, we have

$$b_n^{-1} \sum_{i=1}^{n} X_i \to 0 \qquad \text{a.s.}.$$

To prove Theorem 2.1, we need the following lemmas.

Lemma 2.1 ([3], Lemma 1.2.11) Suppose $\{X_n, n \geq 1\}$ be a ψ-mixing random variable sequence, suppose $X \in \mathcal{F}^k$, $Y \in \mathcal{F}_{k+r}$, $\mathsf{E}|X| < \infty$, $\mathsf{E}|Y| < \infty$, then

$$\mathsf{E}|XY| < \infty, \qquad \text{and} \qquad |\mathsf{E}XY - \mathsf{E}X\mathsf{E}Y| \leq \psi(r)\mathsf{E}|X|\mathsf{E}|Y|.$$

Lemma 2.2 Let $\{X_n, n \geq 1\}$ be a mean zero ψ-mixing random sequences satisfying (2.3). Define $S_n = \sum\limits_{k=1}^{n} X_k$, and suppose $\mathsf{E}X_k^2 < \infty$ for any $k \geq 1$, then for any $\varepsilon > 0$, we have

$$\mathsf{P}\Big(\max_{1 \leq j \leq n} |S_j| > \varepsilon\Big) \leq \frac{\Big[1 + 4\sum\limits_{l=1}^{\infty} \psi(l)\Big] \sum\limits_{i=1}^{n} \mathsf{E}X_i^2}{\varepsilon^2}.$$

Proof For any $\varepsilon > 0$, let $\Lambda = \big\{\omega : \max\limits_{1 \leq j \leq n} |S_j(\omega)| > \varepsilon\big\}$. For any $\omega \in \Lambda$, define

$$\nu(\omega) = \min\{j : 1 \leq j \leq n, |S_j(\omega)| > \varepsilon\},$$
$$\Lambda_k = \{\omega : \nu(\omega) = k\},$$

where $\max\limits_{1 \leq j \leq k} |S_j(\omega)|$ is taken to be zero for $k = 1$. Thus the Λ_k are disjoint and $\Lambda = \bigcup\limits_{k=1}^{n} \Lambda_k$. It follows that

$$\int_{\Lambda} S_n^2 \mathrm{dP} = \sum_{k=1}^{n} \int_{\Lambda_k} S_n^2 \mathrm{dP} = \sum_{k=1}^{n} \int_{\Lambda_k} [S_k^2 + 2S_k(S_n - S_k) + (S_n - S_k)^2] \mathrm{dP},$$

by same argument as [3] and Lemma 2.1, we have

$$\sum_{k=1}^{n} |\mathsf{E}S_k(S_n - S_k)| \leq \sum_{l=1}^{\infty} \psi(l) \sum_{i=1}^{n} \mathsf{E}X_i^2,$$

it follows

$$\mathsf{E}S_n^2+2\sum_{l=1}^{\infty}\psi(l)\sum_{i=1}^{n}\mathsf{E}X_i^2\geq\int_{\Lambda}S_n^2\mathrm{dP}+2\sum_{l=1}^{\infty}\psi(l)\sum_{i=1}^{n}\mathsf{E}X_i^2\geq\sum_{k=1}^{n}\int_{\Lambda_k}S_k^2\mathrm{dP}\geq\varepsilon^2\mathsf{P}(\Lambda). \quad (2.4)$$

By Lemma 2.1, we have

$$\mathsf{E}S_n^2\leq\sum_{i=1}^{n}\mathsf{E}X_i^2+2\sum_{i<j}\psi(j-i)\mathsf{E}|X_i|\mathsf{E}|X_j|\leq[1+2\sum_{l=1}^{\infty}\psi(l)]\sum_{i=1}^{n}\mathsf{E}X_i^2,$$

together with (2.4), we have

$$\mathsf{P}(\Lambda)\leq\frac{\Big[1+4\sum\limits_{l=1}^{\infty}\psi(l)\Big]\sum\limits_{i=1}^{n}\mathsf{E}X_i^2}{\varepsilon^2}. \qquad \square$$

Lemma 2.3 Let $\{X_n,n\geq1\}$ be a ψ-mixing random sequence satisfying (2.3). If the following conditions are satisfied:

(i) $\sum\limits_{n=1}^{\infty}\mathsf{E}X_n<\infty$;

(ii) $\sum\limits_{n=1}^{\infty}\mathsf{Var}(X_n)<\infty$,

then the series $\sum\limits_{k=1}^{n}X_k$ converges almost surely.

Proof By applying Lemma 2.2 to the series $\{X_j-\mathsf{E}X_j\}$, we have for any positive integers m, $n_1\leq n_2$

$$\mathsf{P}\Big\{\max_{n_1\leq k\leq n_2}\Big|\sum_{j=n_1}^{k}(X_j-\mathsf{E}X_j)\Big|\geq\frac{1}{m}\Big\}\leq m^2\Big[1+4\sum_{l=1}^{\infty}\psi(l)\Big]\sum_{i=n_1}^{n_2}\mathsf{Var}(X_i),$$

it follows from the convergence of (ii) that for each m:

$$\lim_{n_1\to\infty,n_2\to\infty}\mathsf{P}\Big\{\max_{n_1\leq k\leq n_2}\Big|\sum_{j=n_1}^{k}(X_j-\mathsf{E}X_j)\Big|\geq\frac{1}{m}\Big\}=0.$$

Therefore the tail of $\sum\limits_n\{X_n-\mathsf{E}X_n\}$ converges to zero a.s., it follows $\sum\limits_n\{X_n-\mathsf{E}X_n\}$ converges, and so dose $\sum\limits_n X_n$ in view of (i). $\square$

Proof of Theorem 2.1 For each n, let $F_n(x)$ be the distribution function of X_n. Define $Y_n=X_nI(|X_n|\leq b_n)$, where $I(\cdot)$ is the indicator function. Then

$$\sum_n\mathsf{E}\Big(\frac{Y_n^2}{b_n^2}\Big)=\sum_n\int_{|x|\leq b_n}\frac{x^2}{b_n^2}\mathrm{d}F_n(x)\leq\sum_n\int_{|x|\leq b_n}\frac{|x|^p}{b_n^p}\mathrm{d}F_n(x)\leq\sum_n\frac{\mathsf{E}|X_n|^p}{b_n^p},$$

it follows

$$\sum_n\mathsf{Var}\Big(\frac{Y_n}{b_n}\Big)\leq\sum_n\mathsf{E}\Big(\frac{Y_n^2}{b_n^2}\Big)\leq\sum_n\frac{\mathsf{E}|X_n|^p}{b_n^p}<\infty.$$

Therefore, by Lemma 2.3, we have

$$\sum_n \frac{Y_n - \mathsf{E}Y_n}{b_n} \qquad \text{converges a.s..} \tag{2.5}$$

Note that $\mathsf{E}X_n = 0$, we have

$$\begin{aligned}\sum_n \frac{|\mathsf{E}Y_n|}{b_n} &= \sum_n \frac{\Big|\int_{|x|\le b_n} x\mathrm{d}F_n(x)\Big|}{b_n} = \sum_n \frac{\Big|\int_{|x|>b_n} x\mathrm{d}F_n(x)\Big|}{b_n} \\ &\le \sum_n \int_{|x|>b_n} \frac{|x|}{b_n}\mathrm{d}F_n(x) \le \sum_n \int_{|x|>b_n} \frac{|x|^p}{b_n^p}\mathrm{d}F_n(x) \\ &\le \sum_n \frac{\mathsf{E}|X_n|^p}{b_n^p} < \infty. \end{aligned} \tag{2.6}$$

It follows from (2.5) and (2.6) that $\sum\limits_n Y_n/b_n$ converges a.s.. On the other hand,

$$\sum_n \mathsf{P}(X_n \ne Y_n) = \sum_n \int_{|x|>b_n} \mathrm{d}F_n(x) \le \sum_n \int_{|x|>b_n} \frac{|x|^p}{b_n^p}\mathrm{d}F_n(x) \le \sum_n \frac{\mathsf{E}|X_n|^p}{b_n^p} < \infty.$$

By Borel-Cantelli lemma, we have $\sum\limits_n X_n/b_n$ converges a.s..

Applying Kronecker lemma to $\sum\limits_n X_n/b_n$, for each ω in a set of probability one, we obtain the desired result $b_n^{-1}\sum\limits_{i=1}^n X_i \to 0$ almost surely. □

§3. Stability of Other Linear Forms

In this section, we give results on the stability of other linear forms in ψ-mixing random variables. All the proofs are based on the result of Theorem 2.1.

Theorem 3.1 Let $\{a_n\}$ and $\{b_n\}$ be two sequences of positive numbers with $b_n \uparrow \infty$. Suppose that $\{X_n\}$ is stochastically dominated by a non-negative random variable X, i.e. $\{X_n\} < X$. Define $N(x) = \mathrm{Card}\{n : c_n \le x\}$, where $c_n = b_n/a_n$. If the following conditions are satisfied:

(1) $\mathsf{E}N(X) < \infty$;

(2) $\int_0^\infty t^{p-1}\mathsf{P}(X > t)\int_t^\infty N(y)/y^{p+1}\mathrm{d}y\mathrm{d}t < \infty$, $(1 \le p \le 2)$,

then there exist a sequence d_n such that

$$b_n^{-1}\sum_{i=1}^n a_iX_i - d_n \to 0 \qquad \text{a.s..}$$

Proof Define $Y_n = X_n I(|X_n| \le c_n)$, $S_n = \sum_{i=1}^{n} a_i X_i$, $T_n = \sum_{i=1}^{n} a_i Y_i$. Then

$$\sum_{n=1}^{\infty} \mathsf{P}(X_n \ne Y_n) = \sum_{n=1}^{\infty} \mathsf{P}(|X_n| > c_n) \le c \sum_{n=1}^{\infty} \mathsf{P}(X > c_n) \le c\,\mathsf{E}N(X) < \infty.$$

By Borel-Cantelli lemma, for any real sequence $\{d_n\}$, $\{b_n^{-1}T_n - d_n\}$ and $\{b_n^{-1}S_n - d_n\}$ converge on the same set and to the same limit. We prove next that $b_n^{-1}\sum_{i=1}^{n} a_i(Y_i - \mathsf{E}Y_i) \to 0$ a.s., which is the theorem with $d_n = b_n^{-1}\sum_{i=1}^{n} a_i \mathsf{E}Y_i$.

Note that $\{a_n(Y_n - \mathsf{E}Y_n), n \ge 1\}$ is ψ-mixing sequence with mean zero and satisfying (2.3), we have

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{\mathsf{E}|a_n(Y_n - \mathsf{E}Y_n)|^p}{b_n^p} &\le c \sum_{n=1}^{\infty} c_n^{-p}\mathsf{E}|Y_n|^{-P} \\
&\le c \sum_{n=1}^{\infty} p c_n^{-p} \int_0^{c_n} t^{p-1}\mathsf{P}(|X_n| > t)\mathrm{d}t \\
&\le cp \int_0^{\infty} t^{p-1}\mathsf{P}(X > t) \sum_{\{n: c_n > t\}} c_n^{-p}\mathrm{d}t \\
&\le cp^2 \int_0^{\infty} t^{p-1}\mathsf{P}(X > t) \int_t^{\infty} N(y)/y^{p+1}\mathrm{d}y\mathrm{d}t.
\end{aligned}$$

The last inequality follows from the fact that

$$\begin{aligned}
\sum_{\{n: c_n > t\}} c_n^{-p} &= \lim_{u\to\infty} \sum_{\{n: t < c_n < u\}} c_n^{-p} = \lim_{u\to\infty} \int_t^u y^{-p}\mathrm{d}N(y) \\
&= \lim_{u\to\infty} \Big[u^{-p}N(u) - t^{-p}N(t) + p\int_{t<y\le u} N(y)/y^{p+1}\mathrm{d}y\Big]
\end{aligned}$$

and

$$u^{-p}N(u) \le p\int_u^{\infty} N(y)/y^{p+1}\mathrm{d}y \to 0 \qquad \text{as} \qquad u \to \infty.$$

By condition (2) and Theorem 2.1, we get the desired result. □

Theorem 3.2 If we replace the condition (1) and (2) of Theorem 3.1 by the following conditions:

(3) $\mathsf{E}N(X) < \infty$;

(4) $\int_1^{\infty} \mathsf{E}N(X/s)\mathrm{d}s < \infty$;

(5) $\max_{1\le j\le n} c_j^p \sum_{j=n}^{\infty} c_j^{-p} = O(n)$,

and furthermore assume that $\mathsf{E}X_n = 0$, we have $b_n^{-1}\sum_{i=1}^{n} a_i X_i \to 0$ a.s..

Proof Y_n, S_n and T_n are defined as in Theorem 3.1. Similarly we have $\sum_{n=1}^{\infty} \mathsf{P}(X_n \neq Y_n) \leq c\mathsf{E}N(X) < \infty$. In order to prove the desired result, it suffices to prove that $b_n^{-1}\sum_{i=1}^{n} a_i Y_i \to 0$. By condition (3) and (4), we can easy prove $b_n^{-1}\sum_{i=1}^{n} a_i \mathsf{E}Y_i \to 0$. Therefore we only need to prove $b_n^{-1}\sum_{i=1}^{n} a_i(Y_i - \mathsf{E}Y_i) \to 0$.

Since $\{a_n(Y_n - \mathsf{E}Y_n)\}$ is also a ψ-mixing sequence with satisfying (2.3), therefore,

$$\begin{aligned}
\sum_{n=1}^{\infty} \mathsf{E}|a_n(Y_n - \mathsf{E}Y_n)|^p/b_n^p &\leq c\sum_{n=1}^{\infty} c_n^{-p}\mathsf{E}|X_n|^p I(|X_n| \leq c_n) \\
&\leq c\sum_{n=1}^{\infty} c_n^{-p}(c_n^p \mathsf{P}(X > c_n) + \mathsf{E}X^p I(X \leq c_n)) \\
&= c\sum_{n=1}^{\infty} \mathsf{P}(X > c_n) + c\sum_{n=1}^{\infty} c_n^{-p}\mathsf{E}X^p I(X \leq c_n).
\end{aligned}$$

Define $d_n = \max_{1\leq j\leq n} c_j$, $d_0 = 0$, then

$$\begin{aligned}
\sum_{n=1}^{\infty} c_n^{-p}\mathsf{E}X^p I(X \leq c_n) &\leq \sum_{n=1}^{\infty} c_n^{-p}\mathsf{E}X^p I(X \leq d_n) \\
&= \sum_{n=1}^{\infty}\sum_{j=1}^{n} c_n^{-p}\mathsf{E}X^p I(d_{j-1} < X \leq d_j) \\
&= \sum_{j=1}^{\infty} \mathsf{E}X^p I(d_{j-1} < X \leq d_j)\sum_{n=j}^{\infty} c_n^{-p} \\
&\leq \sum_{j=1}^{\infty} \mathsf{P}(d_{j-1} < X \leq d_j) d_j^p \sum_{n=j}^{\infty} c_n^{-p} \\
&\leq c\sum_{j=1}^{\infty} j\mathsf{P}(d_{j-1} < X \leq d_j) \\
&= c\sum_{j=1}^{\infty} \mathsf{P}(X > d_{j-1}) \\
&\leq c\Big(1 + \sum_{j=1}^{\infty} \mathsf{P}(X > c_j)\Big) < \infty.
\end{aligned}$$

By Theorem 2.1, we have $b_n^{-1}\sum_{i=1}^{n} a_i(Y_i - \mathsf{E}Y_i) \to 0$ and the proof is complete. □

In what follows, let $\alpha(x) : R_+ \to R_+$ be a positive, non-increasing function. Define $a_n = \alpha(n)$, $b_n = \sum_{i=1}^{n} a_i$, $c_n = b_n/a_n$ and suppose that

(I) $0 \leq \liminf_{n\to\infty} n^{-1}c_n\alpha(\log c_n) \leq \limsup_{n\to\infty} n^{-1}c_n\alpha(\log c_n) < \infty$;

(II) $x\alpha(\log^+ x)$ is non-decreasing for $x > 0$.

Under the above conditions (I) and (II), we have the following theorem:

Theorem 3.3 Suppose $\{X_n\}$ is identical distributed with $\mathsf{E}|X_1|\alpha(\log^+|X_1|) < \infty$, then there exist d_n, such that

$$b_n^{-1}\sum_{i=1}^{n} a_iX_i - d_n \to 0 \qquad \text{a.s..}$$

Proof By the definition of a_n, b_n, c_n and assumption (II), there exist $m_0 \in N$, $\alpha > 0$, $\beta > 0$ such that for any $n \geq m_0$,

$$\alpha n \leq c_n\alpha(\log c_n) \leq \beta n.$$

Therefore $c_n \geq \alpha n(\alpha(\log c_n))^{-1}$, which guarantees that for any $m \geq m_0$,

$$\sum_{j=m}^{\infty} c_j^{-2} \leq \alpha^2(\log c_m)/\alpha^2 m,$$

Define $Y_n = X_nI(|X_n| \leq c_n)$ as in Theorem 3.1, then for $m \geq m_0$,

$$\begin{aligned}
\sum_{j=m}^{\infty}\mathsf{E}|a_j(Y_j-\mathsf{E}Y_j)|^2/b_j^2 &\leq c\sum_{j=m}^{\infty}c_j^{-2}\mathsf{E}|X_j|^2I(|X_j|\leq c_j)\\
&= c\sum_{j=m}^{\infty}c_j^{-2}\mathsf{E}|X_1|^2I(|X_1|\leq c_j)\\
&= c\sum_{j=m}^{\infty}c_j^{-2}[\mathsf{E}X_1^2I(|X_1|\leq c_{m-1})+\sum_{i=m}^{j}\mathsf{E}X_1^2I(c_{i-1}<|X_1|\leq c_i)]\\
&\leq O(1)+c\sum_{j=m}^{\infty}c_j^{-2}\sum_{i=m}^{j}\mathsf{E}X_1^2I(c_{i-1}<|X_1|\leq c_i)\\
&\leq O(1)+c\sum_{i=m}^{\infty}\alpha^{-2}i^{-1}\alpha^2(\log c_i)\mathsf{E}X_1^2I(c_{i-1}<|X_1|\leq c_i)\\
&\leq O(1)+c\beta\alpha^{-2}\sum_{i=m}^{\infty}\alpha(\log c_i)\mathsf{E}|X_1|I(c_{i-1}<|X_1|\leq c_i)\\
&\leq O(1)+c\beta\alpha^{-2}\sum_{i=m}^{\infty}\mathsf{E}|X_1|\alpha(\log^+|X_1|)I(c_{i-1}<|X_1|\leq c_i)\\
&< \infty.
\end{aligned}$$

By Theorem 2.1, we have $b_n^{-1}\sum_{i=1}^{n}a_i(Y_i-\mathsf{E}Y_i)\to 0$ a.s..

On the other hand,

$$\begin{aligned}
\sum_{i=1}^{\infty}\mathsf{P}(X_i\neq Y_i) &= \sum_{i=1}^{\infty}\mathsf{P}(|X_i|>c_i)\\
&= \sum_{i=1}^{m_0-1}\mathsf{P}(|X_i|>c_i)+\sum_{i=m_0}^{\infty}\mathsf{P}(|X_i|>c_i)\\
&\leq (m_0-1)+\sum_{i=m_0}^{\infty}\mathsf{P}(|X_i|\alpha(\log^+|X_i|)\geq c_i\alpha(\log c_i))\\
&\leq (m_0-1)+\sum_{i=m_0}^{\infty}\mathsf{P}(|X_i|\alpha(\log^+|X_i|)\geq \alpha i)<\infty.
\end{aligned}$$

Borel-Cantelli lemma gives that the conclusion of the Theorem 3.3 is true, taking $d_n = b_n^{-1} \sum_{i=1}^{n} a_i \mathsf{E} Y_i$. □

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ψ-混合相依变量线性形式的强稳定性

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线性形式的强稳定性在科学技术上存在着广泛应用. 本文讨论了ψ-混合随机变量列线性形式的强稳定性. 通过对ψ-混合随机变量列运用截尾术, 借助于ψ-混合随机变量的性质以及Borel-Cantelli引理, 得到了ψ-混合随机变量线性形式具有强稳定性的充分条件. 同时也给出了一些其它形式的结果.

关键词: 强稳定性, ψ-混合, 线性形式.

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