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The Random Parameters AACD Models and Their Geometric Ergodicity *

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Abstract

This paper proposes a new type of random parameters AACD (RPAACD) models, which extends the AACD model. Depending on the state of the price process, the RPAACD models seem to be a valuable alternative to existing approaches and have the better overall performance. We give the transition probability of the process. Moreover by employing the transition probability, we obtain the probability properties of the RPACD model.

Keywords: AACD model, Markov chain, geometric ergodicity.

AMS Subject Classification: 60J99.

§1. Introduction

High-frequency financial time series have become widely available during the past decade or so. Engle and Russell (1998) developed the Autoregressive Conditional Duration (ACD) model whose explicit objective is the modeling of times between events. Since its introduction, the ACD model and its various extensions have become a leading tool in modeling the behavior of irregularly time-spaced financial data, and open the door to both theoretical and empirical developments. ACD models have been partly covered in books such as Bauwens and Giot (2001), Russell and Engle (2010), Tsay (2002) and Hautsch (2004). Recently, Pacurar (2008) reviews both the theoretical and empirical work that has been done on ACD models. In this article we follow the line of work originated by Engle and Russell (1998), where the durations between events (e.g., trades, quotes, price changes) are the quantities being modeled. Moreover, regime-switching ACD specifications have been introduced by Zhang et al. (2001) and recently extended by Meitz and Teräsvirta (2006) to allow for smooth transition specifications. Hujer et al. (2002) propose the Markov switching ACD model.

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However, in a great deal of research of ACD models, there is little work in the research of the probability properties and its limitation is obvious. These ACD models don't show up the disturbance from the environment to the system. In practice, the model parameters vary with the environment. The contribution of this paper is two-fold. First, we propose a new type of random parameters AACD (RPAACD) models. It turns out that flexible disturbance of the new impact function are necessary to model financial durations appropriately. So the RPAACD model seems to be a valuable alternative to existing approaches and has the better overall performance. Second, we get the probability properties of the RPAACD model, and give rigorous proofs of the probability properties.

§2. The RPAACD Model

Let $(\Omega, \mathcal{F}, \mathsf{P})$ be a probability space. Throughout this paper, all random variables and random vectors are assumed to be defined on this space. Let $\boldsymbol{E} = \{1, 2, \ldots, e\}$ be a finite set, $\boldsymbol{H}$ denote σ-algebra generated by all subsets of $\boldsymbol{E}$, $\{Z_k, k \geq 1\}$ be an irreducible and aperiodic Markov chain on $(\Omega, \mathcal{F}, \mathsf{P})$ with state space $(\boldsymbol{E}, \boldsymbol{H})$.

Let $x_k = t_k - t_{k-1}$ denote the time between two events occurring at time t_{k-1} and t_k, respectively. Engle and Russell (1998) proposed to specify the duration process based on a dynamic parameterization of the conditional mean function $\psi_k = \mathsf{E}(x_k|\mathcal{F}_{k-1})$, where $\mathcal{F}_{k-1}$ denotes the filtration up to period $k-1$. By defining ε_k as an i.i.d. random variable with positive support, the ACD model is given by $x_k = \psi_k \varepsilon_k$.

Hence, the specification of an ACD model includes (i) the choice of the functional form for the conditional mean function ψ_k and (ii) the choice of an appropriate distribution for ε_k.

The augmented autoregressive conditional duration (AACD) model was proposed by Fernandes and Grammig (2006) in their seminal paper and is given by

$$x_k = \psi_k \varepsilon_k, \tag{2.1}$$

$$\psi_k^\lambda = \omega + \alpha \psi_{k-1}^\lambda [|\varepsilon_{k-1} - b| - c(\varepsilon_{k-1} - b)]^v + \beta \psi_{k-1}^\lambda, \tag{2.2}$$

where $\omega, \alpha, \beta, b, c$ denote constants and $\omega > 0$, $\alpha > 0$, $\beta > 0$, $|c| \leq 1$.

Here, we propose a new type of AACD model which nests a wide range of specifications and is given by

$$\psi_k^\lambda = \omega(Z_{k-1}) + \alpha(Z_{k-1}) \psi_{k-1}^\lambda [|\varepsilon_{k-1} - b| - c(\varepsilon_{k-1} - b)]^v + \beta(Z_{k-1}) \psi_{k-1}^\lambda, \tag{2.3}$$

where for any $i \in \boldsymbol{E}$, $\omega(i), \alpha(i), \beta(i), b, c$ denote constants and $\omega(i) > 0$, $\alpha(i) > 0$, $\beta(i) > 0$, $|c| \leq 1$.

We call this random parameters AACD (RPAACD) models. And $\{(\psi_k, Z_k), k \geq 1\}$ is called the derived sequence of the model (2.3). If $\boldsymbol{E}$ is a simple point set, the model (2.3) reduces to the model (2.2).

The model (2.3) indicates that the model can be interpreted as a nonlinear model with stochastic time-varying parameterization. The RPAACD model can better fit a system influenced by environment. The new model has broad application prospect. Extensions of this framework to the case of smooth transitions have been recently introduced by Meitz and Teräsvirta (2006). And extensions of this framework to the case of regime-switching ACD model have been recently introduced by Hautsch (2006).

§3. Properties of the RPAACD Model

In this paper, assume that the following three conditions hold

1. ε_k is an i.i.d. random variable with positive support and $\mathsf{E}(\varepsilon_k) = 1$.

2. $\{\psi_k, k \geq 1\}$ is independent of ε_k; For any $k \geq 1$, Z_k is independent of ψ_1 and $\{Z_k, k \geq 1\}$ is independent of $\{\varepsilon_k, k \geq 1\}$.

3. $\{\varepsilon_k, k \geq 1\}$ have density function $f(\cdot)$ that is positive and lower semi-continuous.

In the following, we are interested in the limit behavior of the sequence $\{(\psi_k, Z_k), k \geq 1\}$ of the model (2.3).

Lemma 3.1 The process $\{(\psi_k, Z_k), k \geq 1\}$ is a homogeneous Markov chain.

The proof is similar to that of Lemma 2.1 of Hou et al. (2005).

Let

$$\Phi(x, y, i) = \Big[\frac{y^\lambda - \omega(i) - \beta(i)x^\lambda}{\alpha(i)x^\lambda}\Big]^{1/v}$$

and μ be a Lebesgue measure on the space $(\boldsymbol{R}, \boldsymbol{B})$.

Lemma 3.2 The transition probability of the process $\{(\psi_k, Z_k), k \geq 1\}$ is as follows:

$$P((x, i), \Lambda \times j) = P_{ij}\Big[\int\limits_{\varepsilon_0 \geq b, y \in \Lambda} f\Big(b + \frac{1}{1-c}\Phi(x, y, i)\Big)\mu(\mathrm{d}y) + \int\limits_{\varepsilon_0 < b, y \in \Lambda} f\Big(b - \frac{1}{1+c}\Phi(x, y, i)\Big)\mu(\mathrm{d}y)\Big].$$

For any $n \geq 2$,

$$
\begin{aligned}
&P^{(n)}((x,i),\Lambda\times j)\\
=&\sum_{l_1,\ldots,l_{n-1}\in \boldsymbol{E}} P_{il_1}P_{l_1l_2}\cdots P_{l_{n-1}j}\Big[\int_{\varepsilon_0\geq b,y_1\in \boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x,y_1,i)\Big)\mu(\mathrm{d}y_1)\\
&+\int_{\varepsilon_0< b,y_1\in \boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x,y_1,i)\Big)\mu(\mathrm{d}y_1)\Big]\Big[\int_{\varepsilon_1\geq b,y_2\in \boldsymbol{R}} f\Big(b+\frac{1}{1-c}\\
&\Phi(y_1,y_2,l_1)\Big)\mu(\mathrm{d}y_2)+\int_{\varepsilon_1< b,y_2\in \boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(y_1,y_2,l_1)\Big)\mu(\mathrm{d}y_2)\Big]\cdots\\
&\Big[\int_{\varepsilon_{n-2}\geq b,y_{n-1}\in \boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(y_{n-2},y_{n-1},l_{n-2})\Big)\mu(\mathrm{d}y_{n-1})+\int_{\varepsilon_{n-2}< b,y_{n-1}\in \boldsymbol{R}}\\
&f\Big(b-\frac{1}{1+c}\Phi(y_{n-2},y_{n-1},l_{n-2})\Big)\mu(\mathrm{d}y_{n-1})\Big]\Big[\int_{\varepsilon_{n-1}\geq b,y_n\in \Lambda} f\Big(b+\frac{1}{1-c}\\
&\Phi(y_{n-1},y_n,l_{n-1})\Big)\mu(\mathrm{d}y_n)+\int_{\varepsilon_{n-1}< b,y_n\in \Lambda} f\Big(b-\frac{1}{1+c}\Phi(y_{n-1},y_n,l_{n-1})\Big)\mu(\mathrm{d}y_n)\Big].
\end{aligned}
$$

Remark 1 Upon noting $\{Z_k\}$ is irreducible, then for any measure λ on $(\boldsymbol{E},\boldsymbol{H})$, $\{Z_k\}$ is λ-irreducible. Choose a suitable measure still denoted by λ, which satisfies $\lambda\{i\}>0$, $i\in \boldsymbol{E}$. Thus, we can induce a measure $\mu\times\lambda$ on $(\boldsymbol{R}\times \boldsymbol{E},\boldsymbol{B}\times \boldsymbol{H})$. And $\mu(A)>0$ implies $\mu\times\lambda(A\times\{j\})>0$, for any $A\in \boldsymbol{B}$, $j\in \boldsymbol{E}$.

Lemma 3.3 The process $\{(\psi_k,Z_k),k\geq 1\}$ is $\mu\times\lambda$ irreducible and aperiodic.

Proof For all $\Lambda\times\{j\}\in \boldsymbol{B}\times \boldsymbol{H}$, $\mu\times\lambda(\Lambda\times\{j\})>0$. Fix any $(x,i)\in \boldsymbol{R}\times \boldsymbol{E}$, since $\{Z_n\}$ is irreducible, there exists a positive constant n_0 such that

$$P_{ij}^{(n)}>0 \qquad \text{for all } \ n\geq n_0.$$

Then there exist $k_1,k_2,\ldots,k_{n-1}$ such that $P_{ik_1}P_{k_1k_2}\cdots P_{k_{n-1}j}>0$.

By Lemma 3.2, we obtain

$$
\begin{aligned}
&P^{(n)}((x,i),\Lambda\times j)\\
\geq& P_{ik_1}P_{k_1k_2}\cdots P_{k_{n-1}j}\Big[\int_{\varepsilon_0\geq b,y_1\in \boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x,y_1,i)\Big)\mu(\mathrm{d}y_1)\\
&+\int_{\varepsilon_0< b,y_1\in \boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x,y_1,i)\Big)\mu(\mathrm{d}y_1)\Big]\Big[\int_{\varepsilon_1\geq b,y_2\in \boldsymbol{R}} f\Big(b+\frac{1}{1-c}\\
&\Phi(y_1,y_2,k_1)\Big)\mu(\mathrm{d}y_2)+\int_{\varepsilon_1< b,y_2\in \boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(y_1,y_2,k_1)\Big)\mu(\mathrm{d}y_2)\Big]\cdots
\end{aligned}
$$

$$\Big[\int\limits_{\varepsilon_{n-2}\geq b, y_{n-1}\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(y_{n-2},y_{n-1},k_{n-2})\Big)\mu(\mathrm{d}y_{n-1})+\int\limits_{\varepsilon_{n-2}<b, y_{n-1}\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(y_{n-2},y_{n-1},k_{n-2})\Big)\mu(\mathrm{d}y_{n-1})\Big]\Big[\int\limits_{\varepsilon_{n-1}\geq b, y_n\in\Lambda} f\Big(b+\frac{1}{1-c}\Phi(y_{n-1},y_n,k_{n-1})\Big)\mu(\mathrm{d}y_n)+\int\limits_{\varepsilon_{n-1}<b, y_n\in\Lambda} f\Big(b-\frac{1}{1+c}\Phi(y_{n-1},y_n,k_{n-1})\Big)\mu(\mathrm{d}y_n)\Big]>0.$$

Thus $\{(\psi_k,Z_k),k\geq 1\}$ is $\mu\times\lambda$ irreducible.

By Lemma 4.1.6 of An and Chen (1998) and the preceding conclusion, we have the process $\{(\psi_k,Z_k),k\geq 1\}$ is $\mu\times\lambda$ aperiodic. □

Lemma 3.4 If $\boldsymbol{K}$ is a bounded set in $\boldsymbol{R}$ and $\mu(\boldsymbol{K})>0$, then for any $i\in\boldsymbol{E}$, $\boldsymbol{K}\times\{i\}$ is a small set of $\{(\psi_k,Z_k),k\geq 1\}$. It follows that $\boldsymbol{K}\times\boldsymbol{E}$ is a small set of $\{(\psi_k,Z_k),k\geq 1\}$.

Proof Let $\Lambda\times\{j\}\in\boldsymbol{B}\times\boldsymbol{H}$, and $\mu\times\lambda(\Lambda\times\{j\})>0$. By the proof of Lemma 3.3, there exists a positive constant n_0, such that for all $n\geq n_0$ there exist positive constants $k_1,k_2,\ldots,k_{n-1}$ such that $P_{ik_1}P_{k_1k_2}\cdots P_{k_{n-1}j}>0$.

Thus, for all $(x,i)\in\boldsymbol{R}\times\boldsymbol{E}$,

$$\begin{aligned}&P^{(n)}((x,i),\Lambda\times j)\\ \geq{}& P_{ik_1}P_{k_1k_2}\cdots P_{k_{n-1}j}\Big[\int\limits_{\varepsilon_0\geq b, y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x,y_1,i)\Big)\mu(\mathrm{d}y_1)\\ &+\int\limits_{\varepsilon_0<b, y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x,y_1,i)\Big)\mu(\mathrm{d}y_1)\Big]\Big[\int\limits_{\varepsilon_1\geq b, y_2\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\\ &\Phi(y_1,y_2,k_1)\Big)\mu(\mathrm{d}y_2)+\int\limits_{\varepsilon_1<b, y_2\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(y_1,y_2,k_1)\Big)\mu(\mathrm{d}y_2)\Big]\cdots\\ &\Big[\int\limits_{\varepsilon_{n-2}\geq b, y_{n-1}\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(y_{n-2},y_{n-1},k_{n-2})\Big)\mu(\mathrm{d}y_{n-1})+\int\limits_{\varepsilon_{n-2}<b, y_{n-1}\in\boldsymbol{R}}\\ &f\Big(b-\frac{1}{1+c}\Phi(y_{n-2},y_{n-1},k_{n-2})\Big)\mu(\mathrm{d}y_{n-1})\Big]\Big[\int\limits_{\varepsilon_{n-1}\geq b, y_n\in\Lambda} f\Big(b+\frac{1}{1-c}\\ &\Phi(y_{n-1},y_n,k_{n-1})\Big)\mu(\mathrm{d}y_n)+\int\limits_{\varepsilon_{n-1}<b, y_n\in\Lambda} f\Big(b-\frac{1}{1+c}\Phi(y_{n-1},y_n,k_{n-1})\Big)\mu(\mathrm{d}y_n)\Big].\end{aligned}$$

Let

$$\overline{W}_\Lambda(y_1)=\Big[\int\limits_{\varepsilon_1\geq b, y_2\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(y_1,y_2,k_1)\Big)\mu(\mathrm{d}y_2)$$

$$+\int\limits_{\varepsilon_1<b,y_2\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(y_1,y_2,k_1)\Big)\mu(\mathrm{d}y_2)\Big]\cdots$$

$$\Big[\int\limits_{\varepsilon_{n-2}\geq b,y_{n-1}\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(y_{n-2},y_{n-1},k_{n-2})\Big)\mu(\mathrm{d}y_{n-1})$$

$$+\int\limits_{\varepsilon_{n-2}<b,y_{n-1}\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(y_{n-2},y_{n-1},k_{n-2})\Big)\mu(\mathrm{d}y_{n-1})\Big]$$

$$\Big[\int\limits_{\varepsilon_{n-1}\geq b,y_n\in\Lambda} f\Big(b+\frac{1}{1-c}\Phi(y_{n-1},y_n,k_{n-1})\Big)\mu(\mathrm{d}y_n)$$

$$+\int\limits_{\varepsilon_{n-1}<b,y_n\in\Lambda} f\Big(b-\frac{1}{1+c}\Phi(y_{n-1},y_n,k_{n-1})\Big)\mu(\mathrm{d}y_n)\Big].$$

Obviously, for all $y_1\in\boldsymbol{R}^m$, $\overline{W}_\Lambda(y_1)>0$.

$$\inf_{(x,i)\in\boldsymbol{K}\times\{i\}} P^{(n)}((x,i),\Lambda\times j)=\inf_{x\in\boldsymbol{K}} P^{(n)}((x,i),\Lambda\times j)$$

$$\geq P_{ik_1}P_{k_1k_2}\cdots P_{k_{n-1}j}\inf_{x\in\boldsymbol{K}}\Big[\int\limits_{\varepsilon_0\geq b,y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)$$

$$+\int\limits_{\varepsilon_0<b,y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big].$$

Let

$$H(\boldsymbol{K},\Lambda)=\inf_{x\in\boldsymbol{K}}\Big[\int\limits_{\varepsilon_0\geq b,y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)$$

$$+\int\limits_{\varepsilon_0<b,y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big].$$

Obviously, $H(\boldsymbol{K},\Lambda)\geq 0$.

If $H(\boldsymbol{K},\Lambda)=0$, moreover $\boldsymbol{K}$ is a bounded set, there exists $x_n\in\boldsymbol{K}$ such that

$$\lim_{n\to+\infty}\Big[\int\limits_{\varepsilon_0\geq b,y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x_n,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)$$

$$+\int\limits_{\varepsilon_0<b,y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x_n,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big]=0.$$

Since $x_n\in\boldsymbol{K}$ is a bounded sequence, there exists $x_{n_k}\subset x_n$ such that $\lim\limits_{k\to+\infty} x_{n_k}=x_0\in\boldsymbol{R}$.

Therefore, by Fatou lemma and the lower semicontinuity of $f(\cdot)$, we have

$$\begin{aligned}
&\Big[\int_{\varepsilon_0\geq b, y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x_0,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\\
&+\int_{\varepsilon_0<b, y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x_0,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big]\\
\leq&\Big[\int_{\varepsilon_0\geq b, y_1\in\boldsymbol{R}} \varliminf_{k\to+\infty} f\Big(b+\frac{1}{1-c}\Phi(x_{n_k},y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\\
&+\int_{\varepsilon_0<b, y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x_{n_k},y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big]\\
\leq&\varliminf_{k\to+\infty}\Big[\int_{\varepsilon_0\geq b, y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x_{n_k},y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\\
&+\int_{\varepsilon_0<b, y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x_{n_k},y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big]=0.
\end{aligned}$$

This contradicts

$$\begin{aligned}
&\Big[\int_{\varepsilon_0\geq b, y_1\in\boldsymbol{R}} f\Big(b+\frac{1}{1-c}\Phi(x_0,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\\
&+\int_{\varepsilon_0<b, y_1\in\boldsymbol{R}} f\Big(b-\frac{1}{1+c}\Phi(x_0,y_1,i)\Big)\overline{W}_\Lambda(y_1)\mu(\mathrm{d}y_1)\Big]>0.
\end{aligned}$$

Hence $H(\boldsymbol{K},\Lambda)>0$.

Therefore $\inf\limits_{(x,i)\in\boldsymbol{K}\times\{i\}} P^{(n)}((x,i),\Lambda\times j)>0$.

By Lemma 4.1.8 of An and Chen (1998), we obtain $\boldsymbol{K}\times\{i\}$ is a small set of $\{(\psi_k, Z_k), k\geq 1\}$. It follows that $\boldsymbol{K}\times\boldsymbol{E}$ is a small set of $\{(\psi_k,Z_k),k\geq1\}$. □

Theorem 3.1 If for any $i\in\boldsymbol{E}$, $\mathsf{E}\{\alpha(i)[|\varepsilon_0-b|-c(\varepsilon_0-b)]^v+\beta(i)\}<1$ and there exists an $M\in\boldsymbol{R}$ such that $\omega(i)<M$, then $\{(\psi_k,Z_k),k\geq1\}$ is geometrically ergodic Markov chain and model (2.3) is adjoint geometrically ergodic.

Proof Let $\phi(i)=\mathsf{E}\{\alpha(i)[|\varepsilon_0-b|-c(\varepsilon_0-b)]^v+\beta(i)\}$. Since $\phi(i)<1$ for any $i\in\boldsymbol{E}$, then there is a nonnegative constant ρ such that $\max\limits_{i\in\boldsymbol{E}}\phi(i)<\rho<1$. Let $g(y,u)=|y|^\lambda$, $\overline{W}=\max\limits_{i\in\boldsymbol{E}}\omega(i)/(\rho-\phi(i))+1$, $\boldsymbol{B}=\{(y,u):(y,u)\in\boldsymbol{R}\times\boldsymbol{E},|y^\lambda|\leq\overline{W}\}$, $C_1=\min\limits_{i\in\boldsymbol{E}}[(\rho-\phi(i))\overline{W}-\omega(i)]$, $C_2=\max\limits_{i\in\boldsymbol{E}}[\omega(i)+\phi(i)\overline{W}]$. Notice $g(y,u)$ is a nonnegative measurable function, $\boldsymbol{B}$ is a small set for $\{(\psi_k, Z_k), k \geq 1\}$, $C_1 > 0$, and $C_2 > 0$, then

$$\begin{aligned}
&\mathsf{E}\{g(\psi_{k+1}, Z_{k+1})|(\psi_k, Z_k) = (x, i)\} \\
&= \mathsf{E}\{\omega(i) + \alpha(i)x^{\lambda}[|\varepsilon_k - b| - c(\varepsilon_k - b)]^{v} + \beta(i)x^{\lambda}\} \\
&= \omega(i) + \phi(i)x^{\lambda}.
\end{aligned}$$

For any $(x, i) \notin \boldsymbol{B}$,

$$\begin{aligned}
\mathsf{E}\{g(\psi_{k+1}, Z_{k+1})|(\psi_k, Z_k) = (x, i)\} &= \rho x^{\lambda} - [(\rho - \phi(i))x^{\lambda} - \omega(i)] \\
&\leq \rho x^{\lambda} - [(\rho - \phi(i))\overline{W} - \omega(i)] \\
&\leq \rho x^{\lambda} - C_1.
\end{aligned}$$

For any $(x, i) \in \boldsymbol{B}$,

$$\begin{aligned}
\mathsf{E}\{g(\psi_{k+1}, Z_{k+1})|(\psi_k, Z_k) = (x, i)\} &= \omega(i) + \phi(i)x^{\lambda} \\
&\leq \omega(i) + \phi(i)\overline{W} \leq C_2.
\end{aligned}$$

By Theorem 4.1.12 of An and Chen (1998), $\{(\psi_k, Z_k), k \geq 1\}$ is a geometrically ergodic Markov chain.

By the property of conditional probability and the already obtained result, note that $|\boldsymbol{E}|$ is finite, model (2.3) is adjoint geometrically ergodic. □

§4. Conclusion

In this work, we introduces a type of random parameters AACD (RPAACD) models. The parameters of RPAACD models is driven by a hidden Markov chain. The RPAACD models has a broader foreground. Moreover, we give the transition probability of the process $\{(\psi_k, Z_k), k \geq 1\}$. By the transition probability, we demonstrate the irreducibility and aperiodicity of the process $\{(\psi_k, Z_k), k \geq 1\}$ and construct the small set of the process $\{(\psi_k, Z_k), k \geq 1\}$. Finally, Lyapunov functional can be proposed, we give the proof of geometric ergodicity of $\{(\psi_k, Z_k), k \geq 1\}$. But whether the RPAACD models have good statistical properties seems to be a promising direction for further analysis.

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带随机参数的AACD模型及其几何遍历性

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在本论文中提出了一类新的AACD模型——带随机参数的AACD模型. 带随机参数的AACD模型是AACD模型的拓展, 该模型能更好的模拟实际情况, 有更广泛的应用领域. 并且给出该模型的转移概率, 通过转移概率, 得到了该模型的一系列概率性质.

关键词: AACD模型, 马尔可夫链, 几何遍历性.

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