应用概率统计　第三十一卷
第五期　2015年10月

Chinese Journal of Applied Probability
and Statistics Vol.31 No.5 Oct. 2015

Smoothness of the Collision Local Times of Sub-Fractional Brownian Motions *

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Abstract

Let $S^{H_i} = \{S_t^{H_i}, t \geq 0\}$, $i = 1, 2$ be two independent, d-dimensional sub-fractional Brownian motions with respective indices $H_i \in (0, 1)$. Assume $d \geq 2$. Our principal results are the necessary and sufficient condition for the existence and smoothness of the collision local time and the intersection local time of S^{H_1} and S^{H_2} through chaos expansion and elementary inequalities.

Keywords: Sub-fractional Brownian motion, collision local time, intersection local time, chaos expansion.

AMS Subject Classification: 60G15, 60G18, 60J55.

§1. Introduction

Recently, the long-range dependence property has become an important aspect of stochastic models in various scientific areas including hydrology, telecommunication, turbulence, image processing and finance. The best known and most widely used process that exhibits the long-range dependence property is fractional Brownian motion (fBm in short). The fBm is a suitable generalization of the standard Brownian motion, which exhibits long-range dependence, self-similarity and has stationary increments. Some surveys and complete literature could be found in Biagini et al. (2008). On the other hand, many authors have proposed to use more general self-similar Gaussian processes and random fields as stochastic models. Such applications have raised many interesting theoretical questions about self-similar Gaussian processes and fields in general. Therefore, some generalizations of the fBm has been introduced such as bi-fractional Brownian motion, sub-fractional Brownian motion and the weighted fractional Brownian motion. However, in contrast to the extensive studies on fBm, there has been little systematic investigation on other self-similar Gaussian processes. The main reason for this is the complexity of

*The project was supported by the National Natural Science Foundation of China (11271020).
Received December 29, 2014.
doi: 10.3969/j.issn.1001-4268.2015.05.010

dependence structures for self-similar Gaussian processes which do not have stationary increments.

As an extension of Brownian motion, recently, Bojdecki et al. (2004) introduced the sub-fractional Brownian motion $S^H = \{S_t^H, t \geq 0\}$, $H \in (0,1)$ which is a mean zero Gaussian process with $S_0^H = 0$ and covariance

$$C_H(t,s) \equiv \mathsf{E}[S_t^H S_s^H] = s^{2H} + t^{2H} - \frac{1}{2}[(s+t)^{2H} + |t-s|^{2H}] \tag{1.1}$$

for all $s, t \geq 0$. For $H = 1/2$, S^H coincides with the standard Brownian motion B. S^H is neither a semimartingale nor a Markov process unless $H = 1/2$, so many of the powerful techniques from stochastic analysis are not available when dealing with S^H. Its increments are not stationary and satisfies the following estimates:

$$[(2-2^{2H-1}) \wedge 1](t-s)^{2H} \leq \mathsf{E}[(S_t^H - S_s^H)^2] \leq [(2-2^{2H-1}) \vee 1](t-s)^{2H}. \tag{1.2}$$

Therefore, it seems interesting to study sub-fBm. More results on sub-fBm can be found in Bojdecki et al. (2007), Shen (2011), Tudor (2007) and Yan and Shen (2010).

In this paper, we consider two independent sub-fBms S^{H_1} and S^{H_2} on $\mathbb{R}^d$, $d \geq 2$, with respective indices $H_i \in (0,1)$, $i = 1,2$. This means that we have two d-dimensional independent centered Gaussian processes $S^{H_1} = \{S_t^{H_1}, t \geq 0\}$ and $S^{H_2} = \{S_t^{H_2}, t \geq 0\}$ with covariance structure given by

$$\mathsf{E}[S_t^{H_1,i} S_s^{H_1,j}] = \delta_{ij} C_{H_1}(t,s); \qquad \mathsf{E}[S_t^{H_2,i} S_s^{H_2,j}] = \delta_{ij} C_{H_2}(t,s),$$

where $i, j = 1, 2, \ldots, d$, $s, t \geq 0$.

The object of study will be the collision local times and the intersection local times of S^{H_1} and S^{H_2}, whose definitions will be given in the next section. Using chaos expansion and elementary inequalities, we can give the necessary and sufficient condition for existence and smoothness of the collision local times and the intersection local times under some restrictive conditions.

For $H_i = 1/2$ $(i = 1,2)$, $d = 1$, the processes S^{H_1} and S^{H_2} are classical Brownian motions. The intersection local times of independent Brownian motions has been studied by several authors (Geman et al., 1984), and the self-intersection local times of Brownian motion has also been studied by many authors (Berman, 1991). Moreover, the collision local time for fBm has been studied by Jiang and Wang (2007), the self-intersection local time for fBm has been studied by Rosen (1987) for the planar case, and by Hu and Nualart (2005) for multidimensional case. On the other hand, Chen and Yan (2011), Nualart and

Ortiz-Latorre (2007), Wu and Xiao (2010) have studied the intersection local times of fractional Brownian motions.

The rest of this paper is organized as follows. In Section 2 we present the approach of chaos expansion of Gaussian process. In Section 3 we prove the collision local time of S^{H_1} and S^{H_2} exists in L^2 and it is smooth in the sense of the Meyer-Watanabe if and only if $\min\{H_1, H_2\} < 1/(d+2)$. In Section 4, as a related problem, we study the intersection local times of S^{H_1} and S^{H_2}. We show that it is smooth in the sense of the Meyer-Watanabe if and only if $\min\{H_1, H_2\} < 2/(d+2)$.

§2. Preliminaries

Let Ω be the space of continuous $\mathbb{R}^1$-valued functions ω on $[0, T]$. Then Ω is a Banach space with respect to the supremum norm. Let $\mathscr{F}$ be the Borel σ-algebra on Ω. Let P be a probability measure on the measurable space $(\Omega, \mathscr{F})$. Let E denote the expectation on this probability space. The set of all square integrable functionals is denoted by $L^2(\Omega, \mathsf{P})$, i.e.

$$\mathsf{E}(F^2) = \int_\Omega |F(\omega)|^2 \mathsf{P}(\mathrm{d}\omega) < \infty. \tag{2.1}$$

We can introduce the chaos expansion, which is an orthogonal decomposition of $L^2(\Omega, \mathsf{P})$. We refer to Hu (2001) and the references therein for more details. Let $X := \{X_t, t \in [0, T]\}$ be a Gaussian process defined on the probability space $(\Omega, \mathscr{F}, \mathsf{P})$. If $p_n(x)$ is a polynomial of degree n in x, then we call $p_n(X_t)$ a polynomial function of X with $t \in [0, T]$. Let $\mathscr{P}_n$ be the completion with respect to the $L^2(\Omega, \mathsf{P})$ norm of the set $\{p_m(X_t) : 0 \leq m \leq n, t \in [0, T]\}$. Clearly, $\mathscr{P}_n$ is a subspace of $L^2(\Omega, \mathsf{P})$. If $\mathscr{C}_n$ denotes the orthogonal complement of $\mathscr{P}_{n-1}$ in $\mathscr{P}_n$, then $L^2(\Omega, \mathsf{P})$ is actually the direct sum of $\mathscr{C}_n$, i.e.,

$$L^2(\Omega, \mathsf{P}) = \bigoplus_{n=0}^{\infty} \mathscr{C}_n. \tag{2.2}$$

Namely, for any functional $F \in L^2(\Omega, \mathsf{P})$, there are F_n in $\mathscr{C}_n$, $n = 0, 1, 2, \ldots$, such that

$$F = \sum_{n=0}^{\infty} F_n, \tag{2.3}$$

The decomposition (2.3) is called the chaos expansion of F, and F_n is called the n-th chaos of F. Clearly, we have

$$\mathsf{E}(|F|^2) = \sum_{n=0}^{\infty} \mathsf{E}(|F_n|^2). \tag{2.4}$$

Recall that Meyer-Watanabe test function space $\mathscr{U}$ (Watanabe, 1984) is defined as

$$\mathscr{U} := \Big\{F \in L^2(\Omega, \mathsf{P}) : F = \sum_{n=0}^{\infty} F_n \text{ and } \sum_{n=0}^{\infty} n\mathsf{E}(|F_n|^2) < \infty\Big\},$$

and $F \in L^2(\Omega, \mathsf{P})$ is said to be smooth if $F \in \mathscr{U}$ (smoothness in the Meyer-Watanabe sense is the same as the functional being in the space $\mathbb{D}^{1,2}$ appearing in Malliavin calculus). Now, for $F \in L^2(\Omega, \mathsf{P})$, we define an operator Γ_u with $u \in [0,1]$ by

$$\Gamma_u F := \sum_{n=0}^{\infty} u^n F_n. \tag{2.5}$$

Set $\Theta(u) := \Gamma_{\sqrt{u}} F$. Then $\Theta(1) = F$. Define $\Phi_\Theta(u) := \mathrm{d}(\|\Theta(u)\|^2)/\mathrm{d}u$, where $\|F\|^2 := \mathsf{E}(|F|^2)$ for $F \in L^2(\Omega, \mathsf{P})$. We have

$$\Phi_\Theta(u) = \sum_{n=1}^{\infty} n u^{n-1} \mathsf{E}(|F_n|^2). \tag{2.6}$$

Note that $\|\Theta(u)\|^2 = \mathsf{E}(|\Theta(u)|^2) = \sum_{n=0}^{\infty} \mathsf{E}(u^n |F_n|^2)$.

Proposition 2.1 Let $F \in L^2(\Omega, \mathsf{P})$. Then $F \in \mathscr{U}$ if and only if $\Phi_\Theta(1) < \infty$.

Consider two d-dimensional independent sub-fBms $S^{H_i} = \{S_t^{H_i}, t \geq 0\}$, $i = 1, 2$, with respective indices $H_i \in (0,1)$. Let $H_n(x)$, $x \in \mathbb{R}$ be the Hermite polynomials of degree n. That is,

$$H_n(x) = (-1)^n \frac{1}{n!} \mathrm{e}^{x^2/2} \frac{\partial^n}{\partial x^n} \mathrm{e}^{-x^2/2}. \tag{2.7}$$

Then $\mathrm{e}^{tx - t^2/2} = \sum_{n=0}^{\infty} t^n H_n(x)$ for all $t \in \mathscr{C}$ and $x \in \mathbb{R}$, this implies that

$$\begin{aligned} &\exp\Big(iu\langle \xi, S_t^{H_1} - S_t^{H_2}\rangle + \frac{1}{2} u^2 \langle \xi, \mathsf{Var}\,(S_t^{H_1,1} - S_t^{H_2,2})\xi\rangle\Big) \\ &= \sum_{n=0}^{\infty} (iu)^n \sigma^n(t,\xi) H_n\Big(\frac{\langle \xi, S_t^{H_1} - S_t^{H_2}\rangle}{\sigma(t,\xi)}\Big), \end{aligned}$$

where $i = \sqrt{-1}$ and $\sigma(t,\xi) = \sqrt{\mathsf{Var}\,(S_t^{H_1,1} - S_t^{H_2,2})|\xi|^2}$ for $\xi \in \mathbb{R}^d$. Because of the orthogonality of $\{H_n(x), x \in \mathbb{R}\}_{n \in \mathbb{Z}_+}$, we see that $(iu)^n \sigma^n(t,\xi) H_n(\langle \xi, S_t^{H_1} - S_t^{H_2}\rangle / \sigma(t,\xi))$ is the n-th chaos of $\exp(iu\langle \xi, S_t^{H_1} - S_t^{H_2}\rangle + 2^{-1} u^2 |\xi|^2 \mathsf{Var}\,(S_t^{H_1,1} - S_t^{H_2,2}))$ for all $t \geq 0$. Similarly, we can obtain the same results if we use $S_t^{H_1} - S_s^{H_2}$ instead of $S_t^{H_1} - S_t^{H_2}$.

§3. Existence and Smoothness of the Collision Local Time

In this section, we consider the existence and smoothness of the collision local time process. Our main objective is to prove Theorem 3.2.

Let $S^{H_i} = \{S_t^{H_i}, t \geq 0\}$, $i = 1, 2$, be two independent, d-dimensional sub-fBms with respective indices $H_i \in (0,1)$. The so-called collision local times of S^{H_1} and S^{H_2} is formally defined as

$$\ell_T = \int_0^T \delta_0(S_s^{H_1} - S_s^{H_2})\mathrm{d}s, \qquad T \geq 0, \tag{3.1}$$

where δ_0 is the Dirac delta function. In order to give a rigorous meaning to ℓ_T, we approximate the Dirac delta function by the heat kernel

$$p_\varepsilon(x) = \frac{1}{(2\pi)^d}\int_{\mathbb{R}^d} \mathrm{e}^{i\langle\xi,x\rangle}\mathrm{e}^{-\varepsilon|\xi|^2/2}\mathrm{d}\xi, \qquad x \in \mathbb{R}^d. \tag{3.2}$$

For $\varepsilon > 0$, we define

$$\ell_{\varepsilon,T} = \int_0^T p_\varepsilon(S_s^{H_1} - S_s^{H_2})\mathrm{d}s = \frac{1}{(2\pi)^d}\int_0^T\int_{\mathbb{R}^d} \mathrm{e}^{i\langle\xi,S_s^{H_1}-S_s^{H_2}\rangle}\cdot \mathrm{e}^{-\varepsilon|\xi|^2/2}\mathrm{d}\xi\mathrm{d}s. \tag{3.3}$$

Firstly, we will prove the following theorem.

Theorem 3.1 (Existence of the collision local time) Let $H_i \in (0,1)$, $i = 1,2$. Assume $d \geq 2$, then $\ell_{\varepsilon,T}$ converges in L^2, as $\epsilon \to 0$ if and only if $H_1 \wedge H_2 < 1/d$. Moreover, the limit is denoted by ℓ_T, then $\ell_T \in L^2(\Omega, \mathsf{P})$.

Before proving Theorem 3.1, we need some preparations. Denote

$$\lambda_t = \mathsf{Var}\,(S_t^{H_1,1} - S_t^{H_2,2}), \qquad \rho_{s,t} = \mathsf{E}[(S_t^{H_1,1} - S_t^{H_2,2})(S_s^{H_1,1} - S_s^{H_2,2})]$$

for $s,t \geq 0$. Then it is easy to obtain

$$\mathsf{E}[\ell_{\varepsilon,T}] = \frac{1}{(2\pi)^{d/2}}\int_0^T (\lambda_s + \varepsilon)^{-d/2}\mathrm{d}s, \tag{3.4}$$

and

$$\mathsf{E}[\ell_{\varepsilon,T}^2] = \frac{1}{(2\pi)^d}\int_{[0,T]^2}[(\lambda_s+\varepsilon)(\lambda_t+\varepsilon) - \rho_{s,t}^2]^{-d/2}\mathrm{d}s\mathrm{d}t. \tag{3.5}$$

By symmetry one may assume $0 \leq s \leq t \leq T$, and we set $s = xt$, $0 \leq x \leq 1$. Thus we can rewrite λ_s and $\rho_{s,t}$ as

$$\lambda_s = (2 - 2^{2H_1-1})x^{2H_1}t^{2H_1} + (2 - 2^{2H_2-1})x^{2H_2}t^{2H_2}, \tag{3.6}$$

$$\begin{aligned}\rho_{s,t} = {} & t^{2H_1}\Big(x^{2H_1} + 1 - \frac{1}{2}(1+x)^{2H_1} - \frac{1}{2}(1-x)^{2H_1}\Big) \\ & + t^{2H_2}\Big(x^{2H_2} + 1 - \frac{1}{2}(1+x)^{2H_2} - \frac{1}{2}(1-x)^{2H_2}\Big).\end{aligned} \tag{3.7}$$

It follows that

$$\lambda_s\lambda_t - \rho_{s,t}^2 = t^{4H_1}f_1(x) + t^{4H_2}f_2(x) + t^{2H_1+2H_2}g(x), \tag{3.8}$$

where

$$f_i(x) := (2 - 2^{2H_i-1})^2x^{2H_i} - \Big(1 + x^{2H_i} - \frac{1}{2}(1+x)^{2H_i} - \frac{1}{2}(1-x)^{2H_i}\Big)^2$$

for $i=1,2$, and

$$g(x)=(2-2^{2H_1-1})(2-2^{2H_2-1})(x^{2H_1}+x^{2H_2})-2\Big(1+x^{2H_1}-\frac{1}{2}(1+x)^{2H_1}-\frac{1}{2}(1-x)^{2H_1}\Big)\Big(1+x^{2H_2}-\frac{1}{2}(1+x)^{2H_2}-\frac{1}{2}(1-x)^{2H_2}\Big). \tag{3.9}$$

For simplicity we assume that the notation $F\asymp G$ means that there are positive constants C_1 and C_2 so that $C_1G(x)\le F(x)\le C_2G(x)$ in the common domain for F and G. For $a,b\in\mathbb{R}$, $a\wedge b:=\min\{a,b\}$ and $a\vee b:=\max\{a,b\}$.

Lemma 3.1 (Yan and Shen, 2010) Let f_i and g be defined as above and let $0<H_i<1$, $i=1,2$. Then, for all $x\in[0,1]$, we have

$$f_i(x)\asymp x^{2H_i}(1-x)^{2H_i},\qquad i=1,2 \tag{3.10}$$

and

$$g(x)\asymp x^{2H_2}(1-x)^{2H_1}+x^{2H_1}(1-x)^{2H_2}. \tag{3.11}$$

Proof of Theorem 3.1 A slight extension of (3.5) yields

$$\mathsf{E}[\ell_{\varepsilon,T}\ell_{\eta,T}]=\frac{1}{(2\pi)^d}\int_{[0,T]^2}[(\lambda_s+\varepsilon)(\lambda_t+\eta)-\rho_{s,t}^2]^{-d/2}\mathrm{d}s\mathrm{d}t.$$

Consequently, a necessary and sufficient condition for the convergence in $L^2(\Omega,\mathsf{P})$ of $\ell_{\varepsilon,T}$ is that (see, for example, Nualart and Ortiz-Latorre (2007))

$$\Lambda_T\equiv\int_{[0,T]^2}(\lambda_s\lambda_t-\rho_{s,t}^2)^{-d/2}\mathrm{d}s\mathrm{d}t<\infty.$$

Thus, it is sufficient to prove that

$$\Lambda_T\equiv\int_{[0,T]^2}(\lambda_s\lambda_t-\rho_{s,t}^2)^{-d/2}\mathrm{d}s\mathrm{d}t<\infty$$

if and only if $H_1\wedge H_2<1/d$. It follows from Lemma 3.1 that

$$\begin{aligned}\lambda_s\lambda_t-\rho_{s,t}^2&\asymp(x^{2H_1}t^{2H_1}+x^{2H_2}t^{2H_2})[(1-x)^{2H_1}t^{2H_1}+(1-x)^{2H_2}t^{2H_2}]\\&\asymp(s^{2H_1}+s^{2H_2})[(t-s)^{2H_1}+(t-s)^{2H_2}]\end{aligned}$$

for all $0\le s\le t$ and $x=s/t$. We have

$$\begin{aligned}&\int_0^T\int_0^T(\lambda_t\lambda_s-\rho_{s,t}^2)^{-d/2}\mathrm{d}s\mathrm{d}t\\&\asymp C_{H_1,H_2}\int_0^T\mathrm{d}t\int_0^t[s^{2H_1}+s^{2H_2}]^{-d/2}[(t-s)^{2H_1}+(t-s)^{2H_2}]^{-d/2}\mathrm{d}s\\&\asymp C_{T,H_1,H_2}\int_0^T\mathrm{d}t\int_0^t\frac{1}{s^{d(H_1\wedge H_2)}(t-s)^{d(H_1\wedge H_2)}}\mathrm{d}s.\end{aligned}$$

It follows that

$$\int_0^T\int_0^T(\lambda_t\lambda_s-\rho_{s,t}^2)^{-d/2}\mathrm{d}s\mathrm{d}t<\infty \tag{3.12}$$

if and only if $H_1\wedge H_2<1/d$. $\square$

Now, we give the following proposition which is important for the proof of Theorem 3.2.

Proposition 3.1 Let $\lambda_t,\rho_{s,t}$ denote as above. Then for $T\geq 0$, $\ell_T\in\mathscr{U}$ if and only if

$$\int_0^T\int_0^T\rho_{s,t}^2(\lambda_t\lambda_s-\rho_{s,t}^2)^{-d/2-1}\mathrm{d}s\mathrm{d}t<\infty. \tag{3.13}$$

In order to prove Proposition 3.1, we need some preliminaries. Let X,Y be two random variables with joint Gaussian distribution such that $\mathsf{E}(X)=\mathsf{E}(Y)=0$ and $\mathsf{E}(X^2)=\mathsf{E}(Y^2)=1$. Then, for all $n,m\geq 0$, we have (see, for example, Nualart (2006))

$$\mathsf{E}(H_n(X)H_m(Y))=\begin{cases}0, & m\neq n,\\ \dfrac{1}{n!}[\mathsf{E}(XY)]^n, & m=n.\end{cases} \tag{3.14}$$

Lemma 3.2 (Chen and Yan, 2011) Suppose $d\geq 1$. For any $x\in[-1,1)$, we have

$$\sum_{n=1}^{\infty}\sum_{\substack{k_1,\ldots,k_d=0\\k_1+\cdots+k_d=n}}^{n}\frac{2n(2k_1-1)!!\cdots(2k_d-1)!!}{(2k_1)!!\cdots(2k_d)!!}x^n\asymp x(1-x)^{-(d/2+1)}.$$

It follows from $\rho_{s,t}^2\leq\lambda_s\lambda_t$ that

$$\frac{\rho_{s,t}^2}{(\lambda_s\lambda_t-\rho_{s,t}^2)^{d/2+1}}\asymp\sum_{n=1}^{\infty}\sum_{\substack{k_1,\ldots,k_d=0\\k_1+\cdots+k_d=n}}^{n}\frac{2n(2k_1-1)!!\cdots(2k_d-1)!!}{(2k_1)!!\cdots(2k_d)!!}\frac{\rho_{s,t}^{2n}}{(\lambda_s\lambda_t)^{n+d/2}}.$$

Proof of Proposition 3.1 For $\varepsilon>0$, $T\geq 0$, we denote $\Theta_\varepsilon(u,T,\ell_{\varepsilon,T}):=\mathsf{E}(|\Upsilon_{\sqrt{u}}\ell_{\varepsilon,T}|^2)$ and $\Theta(u,T,\ell_T):=\mathsf{E}(|\Upsilon_{\sqrt{u}}\ell_T|^2)$. Thus, by Proposition 2.1 to prove that (3.13) holds if and only if $\Phi_\Theta(1)<\infty$. Clearly, we have

$$\begin{aligned}\ell_{\varepsilon,T}&=\int_0^T p_\varepsilon(S_t^{H_1}-S_t^{H_2})\mathrm{d}t\\&=\frac{1}{(2\pi)^d}\int_0^T\int_{\mathbb{R}^d}\mathrm{e}^{-(\lambda_t+\varepsilon)|\xi|^2/2}\sum_{n=0}^{\infty}i^n\sigma^n(t,\xi)H_n\Big(\frac{\langle\xi,S_t^{H_1}-S_t^{H_2}\rangle}{\sigma(t,\xi)}\Big)\mathrm{d}\xi\mathrm{d}t\\&\equiv\sum_{n=0}^{\infty}F_n.\end{aligned}$$

Thus, by (3.14) and Lemma 3.2, we have

$$
\begin{aligned}
&\Phi_{\Theta_\varepsilon}(1)=\sum_{n=0}^{\infty} n\mathsf{E}(|F_n|^2)\\
&=\sum_{n=0}^{\infty}\frac{n}{(2\pi)^{2d}}\mathsf{E}\Big[\int_{[0,T]^2}\int_{\mathbb{R}^{2d}} \mathrm{e}^{-((\lambda_t+\varepsilon)|\xi|^2+(\lambda_s+\varepsilon)|\eta|^2)/2}\sigma^n(t,\xi)\sigma^n(s,\eta)\\
&\quad H_n\Big(\frac{\langle \xi, S_t^{H_1}-S_t^{H_2}\rangle}{\sigma(t,\xi)}\Big)H_n\Big(\frac{\langle \eta, S_s^{H_1}-S_s^{H_2}\rangle}{\sigma(s,\eta)}\Big)\mathrm{d}\xi\mathrm{d}\eta\mathrm{d}s\mathrm{d}t\Big]\\
&=\sum_{n=1}^{\infty}\frac{1}{(2\pi)^{2d}(2n-1)!}\int_{[0,T]^2}\rho_{s,t}^{2n}\mathrm{d}s\mathrm{d}t\\
&\quad\times\int_{\mathbb{R}^{2d}}\mathrm{e}^{-((\lambda_t+\varepsilon)(\xi_1^2+\cdots+\xi_d^2)+(\lambda_s+\varepsilon)(\eta_1^2+\cdots+\eta_d^2))/2}(\xi_1\eta_1+\cdots+\xi_d\eta_d)^{2n}\mathrm{d}\xi_1\cdots\mathrm{d}\xi_d\mathrm{d}\eta_1\cdots\mathrm{d}\eta_d\\
&=\frac{1}{(2\pi)^d}\sum_{n=1}^{\infty}\sum_{\substack{k_1,\ldots,k_d=0\\k_1+\cdots+k_d=n}}^{n}\frac{2n(2k_1-1)!!\cdots(2k_d-1)!!}{(2k_1)!!\cdots(2k_d)!!}\int_{[0,T]^2}\frac{\rho_{s,t}^{2n}}{((\lambda_t+\varepsilon)(\lambda_s+\varepsilon))^{n+d/2}}\mathrm{d}s\mathrm{d}t\\
&\asymp\int_{[0,T]^2}\rho_{s,t}^2((\lambda_t+\varepsilon)(\lambda_s+\varepsilon)-\rho_{s,t}^2)^{-d/2-1}\mathrm{d}s\mathrm{d}t.
\end{aligned}
$$

It follows that

$$\lim_{\varepsilon\to0}\Phi_{\Theta_\varepsilon}(1)\asymp\int_0^T\int_0^T\rho_{s,t}^2(\lambda_t\lambda_s-\rho_{s,t}^2)^{-d/2-1}\mathrm{d}s\mathrm{d}t$$

for all $T\geq 0$. This completes the proof. □

Theorem 3.2 Let $\ell_T, T\geq 0$ be the collision local time process of two independent, d-dimensional sub-fBms $S^{H_i}=\{S_t^{H_i}, t\geq 0\}$, $i=1,2$ with respective indices $H_i\in(0,1)$. Then for every $T>0$, the random variable ℓ_T is smooth in the sense of the Meyer-Watanabe if and only if $\min\{H_1,H_2\}<1/(d+2)$.

Proof By Proposition 3.1, it is sufficient to prove that

$$\int_0^T\int_0^T\rho_{s,t}^2(\lambda_t\lambda_s-\rho_{s,t}^2)^{-d/2-1}\mathrm{d}s\mathrm{d}t<\infty$$

if and only if $\min\{H_1,H_2\}<1/(d+2)$. Without loss of generality we may assume $s\leq t$ and $s=xt$, where $x\in[0,1]$. It follows from Lemma 3.1 that

$$\lambda_s\lambda_t-\rho_{s,t}^2\asymp(s^{2H_1}+s^{2H_2})[(t-s)^{2H_1}+(t-s)^{2H_2}]$$

for all $0\leq s\leq t$ and $x=s/t$. On the other hand, an elementary calculus can show that

$$x^{2H}\leq 1+x^{2H}-\frac{1}{2}(1+x)^{2H}-\frac{1}{2}(1-x)^{2H}\leq(2-2^{2H-1})x^{2H}$$

for all $x,H\in(0,1)$. Hence

$$\rho_{s,t}\asymp(t^{2H_1}x^{2H_1}+t^{2H_2}x^{2H_2}).$$

Firstly, we give the proof of the sufficient condition. Since

$$\rho_{s,t} \le (2 - 2^{2H_1-1})T^{2H_1} + (2 - 2^{2H_2-1})T^{2H_2}.$$

We have, when $H_1 \wedge H_2 < 1/(d+2)$,

$$\begin{aligned}
&\int_0^T \int_0^T \rho_{s,t}^2 (\lambda_t\lambda_s - \rho_{s,t}^2)^{-(d+2)/2} \mathrm{d}s\mathrm{d}t \\
\le\ & C_{T,H_1,H_2} \int_0^T \mathrm{d}t \int_0^t \frac{1}{[s^{2H_1} + s^{2H_2}]^{(d+2)/2}[(t-s)^{2H_1} + (t-s)^{2H_2}]^{(d+2)/2}} \mathrm{d}s \\
\le\ & C_{T,H_1,H_2} \int_0^T \mathrm{d}t \int_0^t \frac{1}{s^{(d+2)(H_1\wedge H_2)}(t-s)^{(d+2)(H_1\wedge H_2)}} \mathrm{d}s < \infty.
\end{aligned}$$

In the following, we give the proof of the necessary condition. Since

$$\rho_{s,t} \ge C_{H_1,H_2}(t^{2H_1}x^{2H_1} + t^{2H_2}x^{2H_2}).$$

These deduce for $T > 0$,

$$\begin{aligned}
&\int_0^T \int_0^T \rho_{s,t}^2 (\lambda_t\lambda_s - \rho_{s,t}^2)^{-(d+2)/2} \mathrm{d}s\mathrm{d}t \\
\ge\ & C_{H_1,H_2} \int_0^T \mathrm{d}t \int_0^t \frac{(s^{2H_1} + s^{2H_2})^2}{[s^{2H_1} + s^{2H_2}]^{(d+2)/2}[(t-s)^{2H_1} + (t-s)^{2H_2}]^{(d+2)/2}} \mathrm{d}s \\
\ge\ & C_{T,H_1,H_2} \int_0^T \mathrm{d}t \int_0^t \frac{1}{s^{(d-2)(H_1\wedge H_2)}(t-s)^{(d+2)(H_1\wedge H_2)}} \mathrm{d}s.
\end{aligned}$$

It follows that

$$\int_0^T \int_0^T \rho_{s,t}^2 (\lambda_t\lambda_s - \rho_{s,t}^2)^{-d/2-1} \mathrm{d}s\mathrm{d}t < \infty \tag{3.15}$$

if and only if $\min\{H_1, H_2\} < 1/(d+2)$. □

§4. Existence and Smoothness of the Intersection Local Time

In this section, we study the intersection local time of two independent, d-dimensional sub-fBms S^{H_1} and S^{H_2} with indices $H_i \in (0,1)$, $i = 1, 2$, which is formally defined as

$$I(S^{H_1}, S^{H_2}) = \int_0^T \int_0^T \delta(S_t^{H_1} - S_s^{H_2}) \mathrm{d}s\mathrm{d}t,$$

it is a measure of the amount of time that the trajectories of the two processes, S^{H_1} and S^{H_2} intersect on the time interval $[0, T]$. The objective of study in this section is the smoothness of the intersection local time of S^{H_1} and S^{H_2}. This improve the corresponding results obtained by Shen (2011). For $\varepsilon > 0$, we define

$$\begin{aligned} I_\varepsilon(S^{H_1}, S^{H_2}) &= \int_0^T \int_0^T p_\varepsilon(S_t^{H_1} - S_s^{H_2}) \mathrm{d}s\mathrm{d}t \\ &= \frac{1}{(2\pi)^d} \int_0^T \int_0^T \int_{\mathbb{R}^d} \mathrm{e}^{i\langle \xi, S_t^{H_1} - S_s^{H_2} \rangle} \mathrm{e}^{-\varepsilon|\xi|^2/2} \mathrm{d}\xi \mathrm{d}s\mathrm{d}t. \end{aligned} \tag{4.1}$$

First, we consider the existence of the intersection local time process, we will use elementary method to prove the following theorem.

Theorem 4.1 (Existence of the intersection local time) Let $H_i \in (0,1)$, $i = 1, 2$. Assume $d \geq 2$, then $I_\varepsilon(S^{H_1}, S^{H_2})$ converges in L^2, as $\epsilon \to 0$ if and only if $H_1 d \wedge H_2 d < 2$. Moreover, the limit is denoted by $I(S^{H_1}, S^{H_2})$, then $I(S^{H_1}, S^{H_2}) \in L^2(\Omega, \mathsf{P})$.

Denote

$$\begin{aligned} a_{s,t} &\equiv \mathsf{Var}\,(S_t^{H_1,1} - S_s^{H_2,2}) = (2 - 2^{2H_1-1}) t^{2H_1} + (2 - 2^{2H_2-1}) s^{2H_2}, \\ a_{u,v} &\equiv \mathsf{Var}\,(S_v^{H_1,1} - S_u^{H_2,2}) = (2 - 2^{2H_1-1}) v^{2H_1} + (2 - 2^{2H_2-1}) u^{2H_2}, \\ \rho_{s,t,u,v} &= \mathsf{E}[(S_t^{H_1,1} - S_s^{H_2,2})(S_v^{H_1,1} - S_u^{H_2,2})] \\ &= t^{2H_1} + v^{2H_1} - \frac{1}{2}[(t+v)^{2H_1} + |t-v|^{2H_1}] \\ &\quad + s^{2H_2} + u^{2H_2} - \frac{1}{2}[(s+u)^{2H_2} + |s-u|^{2H_2}] \end{aligned} \tag{4.2}$$

for all $s, t, u, v \geq 0$. Elementary calculus can show that

$$\mathsf{E}[I_\varepsilon(S^{H_1}, S^{H_2}] = \frac{1}{(2\pi)^{d/2}} \int_0^T \int_0^T (a_{s,t} + \varepsilon)^{-d/2} \mathrm{d}s\mathrm{d}t. \tag{4.3}$$

$$\mathsf{E}[I_\varepsilon^2(S^{H_1}, B^{H_2})] = \frac{1}{(2\pi)^d} \int_{[0,T]^4} ((a_{s,t} + \varepsilon)(a_{u,v} + \varepsilon) - \rho_{s,t,u,v}^2)^{-d/2} \mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v. \tag{4.4}$$

Without loss of generality, we may assume $v \leq t$, $u \leq s$ and $v = xt$, $u = ys$ with $x, y \in [0,1]$. Then we can rewrite $a_{u,v}$ and $\rho_{s,t,u,v}$ as follows

$$\begin{aligned} a_{u,v} &= (2 - 2^{2H_1-1}) x^{2H_1} t^{2H_1} + (2 - 2^{2H_2-1}) y^{2H_2} s^{2H_2}, \\ \rho_{s,t,u,v} &= t^{2H_1} \Big\{ 1 + x^{2H_1} - \frac{1}{2}[(1+x)^{2H_1} + (1-x)^{2H_1}] \Big\} \\ &\quad + s^{2H_2} \Big\{ 1 + y^{2H_2} - \frac{1}{2}[(1+y)^{2H_2} + (1-y)^{2H_2}] \Big\}. \end{aligned} \tag{4.5}$$

It follows that

$$a_{s,t} a_{u,v} - \rho_{s,t,u,v}^2 = t^{4H_1} f(x) + s^{4H_2} f(y) + t^{2H_1} s^{2H_2} g(x,y), \tag{4.6}$$

where

$$f(x) := (2 - 2^{2H_1-1})^2 x^{2H_1} - \Big(1 + x^{2H_1} - \frac{1}{2}(1+x)^{2H_1} - \frac{1}{2}(1-x)^{2H_1}\Big)^2,$$
$$f(y) := (2 - 2^{2H_2-1})^2 y^{2H_2} - \Big(1 + y^{2H_2} - \frac{1}{2}(1+y)^{2H_2} - \frac{1}{2}(1-y)^{2H_2}\Big)^2,$$

and

$$\begin{aligned} g(x,y) &= (2 - 2^{2H_1-1})(2 - 2^{2H_2-1})(x^{2H_1} + y^{2H_2}) \\ &\quad - 2\Big(1 + x^{2H_1} - \frac{1}{2}(1+x)^{2H_1} - \frac{1}{2}(1-x)^{2H_1}\Big) \\ &\quad \times \Big(1 + y^{2H_2} - \frac{1}{2}(1+y)^{2H_2} - \frac{1}{2}(1-y)^{2H_2}\Big). \end{aligned} \tag{4.7}$$

Thus, by Lemma 3.1, we get

$$f(x) \asymp x^{2H_1}(1-x)^{2H_1}, \qquad f(y) \asymp y^{2H_2}(1-y)^{2H_2} \tag{4.8}$$

and

$$g(x,y) \asymp x^{2H_1}(1-y)^{2H_2} + y^{2H_2}(1-x)^{2H_1} \tag{4.9}$$

for all $x, y \in [0,1]$.

Proof of Theorem 4.1 A slight extension of (4.4) yields

$$\mathsf{E}[I_\varepsilon(S^{H_1}, S^{H_2}) I_\eta(S^{H_1}, S^{H_2})] = \frac{1}{(2\pi)^d} \int_{[0,T]^4} ((a_{s,t}+\varepsilon)(a_{u,v}+\eta) - \rho^2_{s,t,u,v})^{-d/2} \mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v.$$

Consequently, a necessary and sufficient condition for the convergence in $L^2(\Omega, \mathsf{P})$ of $I_\varepsilon(S^{H_1}, S^{H_2})$ is that

$$\Lambda_T \equiv \int_{[0,T]^4} (a_{s,t}a_{u,v} - \rho^2_{s,t,u,v})^{-d/2} \mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v < \infty.$$

Thus, it is sufficient to prove that

$$\Lambda_T \equiv \int_{[0,T]^4} (a_{s,t}a_{u,v} - \rho^2_{s,t,u,v})^{-d/2} \mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v < \infty$$

if and only if $H_1 d \wedge H_2 d < 2$. By symmetry we have

$$\Lambda_T = 4\int_0^T \int_0^t \int_0^T \int_0^s (a_{s,t}a_{u,v} - \rho^2_{s,t,u,v})^{-d/2} \mathrm{d}u\mathrm{d}s\mathrm{d}v\mathrm{d}t.$$

By (4.8) and (4.9) we have

$$\begin{aligned} a_{s,t}a_{u,v} - \rho^2_{s,t,u,v} &\asymp t^{4H_1} x^{2H_1}(1-x)^{2H_1} + s^{4H_2} y^{2H_2}(1-y)^{2H_2} \\ &\quad + t^{2H_1} s^{2H_2}(x^{2H_1}(1-y)^{2H_2} + y^{2H_2}(1-x)^{2H_1}) \\ &= (v^{2H_1} + u^{2H_2})[(t-v)^{2H_1} + (s-u)^{2H_2}] \end{aligned}$$

for all $0 \le v < t$, $0 \le u < s$ and $x = v/t$, $y = u/s$. These deduce for all $H_i \in (0,1)$, $i = 1, 2$ and $T > 0$,

$$\begin{aligned}\Lambda_T &\le C\int_0^T \mathrm{d}t\int_0^t (v^{H_1}(t-v)^{H_1})^{-d/2}\mathrm{d}v\int_0^T \mathrm{d}s\int_0^s (u^{H_2}(s-u)^{H_2})^{-d/2}\mathrm{d}u\\ &= C\Big(\int_0^T t^{1-H_1d}\mathrm{d}t\int_0^1 x^{-H_1d/2}(1-x)^{-H_1d/2}\Big)\Big(\int_0^T s^{1-H_2d}\mathrm{d}t\int_0^1 y^{-H_2d/2}(1-y)^{-H_2d/2}\Big)\\ &< \infty\end{aligned}$$

if $H_1d \wedge H_2d < 2$. On the other hand, making a change to spherical coordinates, as the integrand in A_T is always positive, we have

$$\begin{aligned}\Lambda_T &\ge \int_{D_T} [(v^{2H_1} + u^{2H_2})((t-v)^{2H_1} + (s-u)^{2H_2})]^{-d/2}\mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v\\ &\ge \int_0^T r^{3-2(H_1\wedge H_2)d}\mathrm{d}r\int_\Theta \varphi(\theta)\mathrm{d}\theta,\end{aligned}$$

where $D_T := \{(s,t,u,v) \in \mathbb{R}_+^4 | s^2 + t^2 + u^2 + v^2 \le T^2\}$. Note that the angular integral is different from zero thanks to the positivity of the integrand. It follows that

$$\int_0^T\int_0^T\int_0^T\int_0^T (a_{s,t}a_{u,v} - \rho_{s,t,u,v}^2)^{-d/2}\mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v < \infty \tag{4.10}$$

if and only if $H_1d \wedge H_2d < 2$. □

Next we establish the smoothness of random variable $I(S^{H_1}, S^{H_2})$ under some restrictive indices.

Theorem 4.2 Suppose that $d \ge 2$. Let $I(S^{H_1}, S^{H_2})$ be the intersection local time of two independent, d-dimensional subfBms S^{H_1} and S^{H_2} with $H_i \in (0,1)$, $i = 1, 2$. Then for every $T > 0$, the random variable $I(S^{H_1}, S^{H_2})$ is smooth in the sense of the Meyer-Watanabe if and only if $H_1 \wedge H_2 < 2/(d+2)$.

In order to prove Theorem 4.2, we need the following Proposition. It could be proved along the line of the proof of Proposition 3.1. We omit the proof.

Proposition 4.1 Let $a_{s,t}, a_{u,v}, \rho_{s,t,u,v}$ denote as above. For all $T \ge 0$, $I(S^{H_1}, S^{H_2}) \in \mathscr{U}$ if and only if

$$\int_{[0,T]^4} \rho_{s,t,u,v}^2 (a_{s,t}a_{u,v} - \rho_{s,t,u,v}^2)^{-d/2-1}\mathrm{d}u\mathrm{d}v\mathrm{d}s\mathrm{d}t < \infty. \tag{4.11}$$

Proof of Theorem 4.2 By Proposition 4.1 it suffices to show that

$$\int_{[0,T]^4} \rho_{s,t,u,v}^2 (a_{s,t}a_{u,v} - \rho_{s,t,u,v}^2)^{-d/2-1}\mathrm{d}u\mathrm{d}v\mathrm{d}s\mathrm{d}t < \infty \tag{4.12}$$

if and only if $H_1 \wedge H_2 < 2/(d+2)$. By (4.8) and (4.9), we have

$$a_{s,t}a_{u,v} - \rho^2_{s,t,u,v} \asymp [x^{2H_1}t^{2H_1} + y^{2H_2}s^{2H_2}][(1-x)^{2H_1}t^{2H_1} + (1-y)^{2H_2}s^{2H_2}].$$

On the other hand,

$$(t^{2H_1}x^{2H_1}+s^{2H_2}y^{2H_2})^2 \le \rho^2_{s,t,u,v} \le (2-2^{2H_1-1})(2-2^{2H_2-1})(t^{2H_1}x^{2H_1}+s^{2H_2}y^{2H_2})^2. \quad (4.13)$$

It follows that

$$\begin{aligned}
&\int_0^T\int_0^T\int_0^T\int_0^T (a_{s,t}a_{u,v} - \rho^2_{s,t,u,v})^{-d/2-1}\rho^2_{s,t,u,v}\mathrm{d}s\mathrm{d}t\mathrm{d}u\mathrm{d}v \\
&\ge C_{H_1,H_2,T}\int_0^T\int_0^1\int_0^T\int_0^1 \frac{(t^{2H_1}x^{2H_1}+s^{2H_2}y^{2H_2})st}{((1-x)^{2H_1}t^{2H_1}+(1-y)^{2H_2}s^{2H_2})^{1+d/2}}\mathrm{d}y\mathrm{d}s\mathrm{d}x\mathrm{d}t \\
&\ge C_{H_1,H_2,T}\int_0^1\mathrm{d}y\int_0^y\mathrm{d}x\int_0^x\mathrm{d}t\int_0^t\mathrm{d}s\frac{s^{2(H_1\wedge H_2)+1}x^{2(H_1\wedge H_2)}}{t^{2(H_1\wedge H_2)(1+d/2)-1}(1-x)^{2(H_1\wedge H_2)(1+d/2)}} \\
&\ge C_{H_1,H_2,T}\int_0^1\mathrm{d}y\int_0^y\frac{x^{4-(H_1\wedge H_2)(d-2)}}{(1-x)^{2(H_1\wedge H_2)(1+d/2)}}\mathrm{d}x \\
&= C_{H_1,H_2,T}\int_0^1 x^{4-(H_1\wedge H_2)(d-2)}(1-x)^{1-2(H_1\wedge H_2)(1+d/2)}\mathrm{d}x,
\end{aligned}$$

where we have used the fact: $u^\alpha + u^\beta = u^{\alpha\wedge\beta}(1+u^{\alpha\vee\beta-\alpha\wedge\beta})$, $u,\alpha,\beta>0$. Which implies that $H_1\wedge H_2 < 2/(d+2)$ if the convergence (ii) holds.

On the other hand,

$$\begin{aligned}
&\int_0^T\int_0^T\int_0^T\int_0^T (a_{s,t}a_{u,v} - \rho^2_{s,t,u,v})^{-d/2-1}\rho^2_{s,t,u,v}\mathrm{d}u\mathrm{d}s\mathrm{d}v\mathrm{d}t \\
&\le C_{H_1,H_2,T}\int_0^T\int_0^1\int_0^T\int_0^1 \frac{(t^{2H_1}x^{2H_1}+s^{2H_2}y^{2H_2})^2 st}{[(x^{H_1}t^{H_1}y^{H_2}s^{H_2})((1-x)^{H_1}t^{H_1}(1-y)^{H_2}s^{H_2})]^{d/2+1}}\mathrm{d}y\mathrm{d}s\mathrm{d}x\mathrm{d}t \\
&\le C_{H_1,H_2,T}\int_0^T\int_0^1\int_0^T\int_0^1 \\
&\qquad \frac{1}{x^{(d+2)H_1/2}y^{(d+2)H_2/2}(1-x)^{(d+2)H_1/2}(1-y)^{(d+2)H_2/2}t^{(d+2)H_1-1}s^{(d+2)H_2-1}}\mathrm{d}y\mathrm{d}s\mathrm{d}x\mathrm{d}t \\
&< \infty
\end{aligned}$$

if $H_1\wedge H_2 < 2/(d+2)$. Thus, the proof is completed. □

References

[1] Berman, S.M., Self-intersections and local nondeterminism of Gaussian processes, *The Annals of Probability*, **19(1)**(1991), 160–191.

[2] Biagini, F., Hu, Y., Øksendal, B. and Zhang, T., *Stochastic Calculus for Fractional Brownian Motion and Applications*, Springer-Verlag, London, 2008.
[3] Bojdecki, T., Gorostiza, L.G. and Talarczyk, A., Sub-fractional Brownian motion and its relation to occupation times, *Statistics and Probability Letters*, **69(4)**(2004), 405–419.
[4] Bojdecki, T., Gorostiza, L.G. and Talarczyk, A., Some extensions of fractional Brownian motion and sub-fractional Brownian motion related to particle systems, *Electronic Communications in Probability*, **12**(2007), 161–172.
[5] Chen, C. and Yan, L., Remarks on the intersection local time of fractional Brownian motions, *Statistics and Probability Letters*, **81(8)**(2011), 1003–1012.
[6] Geman, D., Horowitz, J. and Rosen, J., A local time analysis of intersections of Brownian paths in the plane, *The Annals of Probability*, **12(1)**(1984), 86–107.
[7] Hu, Y., Self-intersection local time of fractional Brownian motions-via chaos expansion, *Journal of Mathematics of Kyoto University*, **41(2)**(2001), 233–250.
[8] Hu, Y. and Nualart, D., Renormalized self-intersection local time for fractional Brownian motion, *The Annals of Probability*, **33(3)**(2005), 948–983.
[9] Jiang, Y. and Wang, Y., On the collision local time of fractional Brownian motions, *Chinese Annals of Mathematics, Series B*, **28(3)**(2007), 311–320.
[10] Nualart, D., *The Malliavin Calculus and Related Topics* (*Second Edition*), Springer, New York, 2006.
[11] Nualart, D. and Ortiz-Latorre, S., Intersection local time for two independent fractional Brownian motions, *Journal of Theoretical Probability*, **20(4)**(2007), 759–767.
[12] Rosen, J., The intersection local time of fractional Brownian motion in the plane, *Journal of Multivariate Analysis*, **23(1)**(1987), 37–46.
[13] Shen, G., Necessary and sufficient condition for the smoothness of intersection local time of subfractional Brownian motions, *Journal of Inequalities and Applications*, **2011(1)**(2011), Article: 139 (16 pages).
[14] Tudor, C., Some properties of the sub-fractional Brownian motion, *Stochastics*, **79(5)**(2007), 431–448.
[15] Watanabe, S., *Lectures on Stochastic Differential Equations and Malliavin Calculus*, Tata Institute of Fundamental Research, Springer-Verlag, New York, 1984.
[16] Wu, D. and Xiao, Y., Regularity of intersection local times of fractional Brownian motions, *Journal of Theoretical Probability*, **23(4)**(2010), 972–1001.
[17] Yan, L. and Shen, G., On the collision local time of sub-fractional Brownian Motions, *Statistics and Probability Letters*, **80(5-6)**(2010), 296–308.

次分数布朗运动相遇局部时的光滑性

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令$S^{H_i}=\{S_t^{H_i},t\geq 0\}$, $i=1,2$是指标分别为$H_i\in(0,1)$的两个独立的$d\geq 2$维次分数布朗运动. 本文利用混沌展开与初等的不等式给出了S^{H_1}与S^{H_2}的相遇局部时、相交局部时存在性, 光滑性的充分必要条件.

关键词: 次分数布朗运动, 相遇局部时, 相交局部时, 混沌展开.
学科分类号: O211.