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STATISTICAL INFERENCES BASED ON SAMPLE QUANTILES AND JACKKNIFE

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Abstract

The problem of statistical inferences about functions of population quantiles is considered. The asymptotic distributions of functions of sample quantiles are normal. We propose estimators of the variances and covariances of the asymptotic distribution by the jackknife method and establish their consistency. The results provide asymptotically valid procedures for statistical inferences.

§ 1. Introduction

In statistical applications, frequently the quantities of interest are the quantiles of the unknown population. Let F be a distribution function. Define $Q(t)=\inf\{x: F(x)\geqslant t\}$ for $0<t<1$. The p-quantile $(0<p<1)$ of F is then $Q(p)$. We consider the following general k-samples problem. Let F_j, $j=1, \cdots, k$, be unknown population distributions and μ_{lj} be the p_{lj}-quantile of F_j, $l=1, \cdots, h_j$, with $0<p_{lj}<1$. Denote $\sum_{j=1}^{k} h_j$ by K. The unknown parameter of interest is $\theta=g(\mu)$, where μ is the K-vector

$$(\mu_{11}, \cdots, \mu_{h_11}, \mu_{12}, \cdots, \mu_{h_22}, \cdots, \mu_{1k}, \cdots, \mu_{h_kk}) \tag{1.1}$$

and g is a continuous and differentiable function from $\mathbf{R}^K$ to $\mathbf{R}^L$ (L is a fixed integer). The simplest example of this type of problem is the comparison of two population medians μ_1 and μ_2 (assumed to be nonzero) and $\theta=\mu_1/\mu_2$. Let x_{ij}, $i=1, \cdots, n_j$ be independent and identically distributed (i. i. d.) samples from F_j. It is desired to make statistical inferences (such as construction of a confidence region for θ) based on the data x_{ij}.

For each j, denote the empirical distribution function corresponding to $x_{1j}, \cdots, x_{n_jj}$ by $\hat{F}_j$. The sample p_{lj}-quantile is defined to be $\hat{\mu}_{lj}=\inf\{x: \hat{F}_j(x)\geqslant p_{lj}\}$, $l=1, \cdots, h_j$. Let $\hat{\mu}$ be the vector given in (1.1) with μ_{lj} replaced by $\hat{\mu}_{lj}$. A point estimator of θ is then $\hat{\theta}=g(\hat{\mu})$. Under mild conditions (F_j' exists and is positive at each μ_{lj}),

$$n^{1/2}(\hat{\theta}-\theta)\to_d N(0, \Sigma), \tag{1.2}$$

where $\to_d$ denotes convergence in distribution, $\Sigma=[\nabla g(\mu)]^\tau V[\nabla g(\mu)]$, $\nabla g(\mu)$, is the

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gradient matrix of g at μ and $[\nabla g(\mu)]^{\tau}$ is its transpose, $V=$block diagonal $[V_1, \cdots, V_k]$ and for each j, V_j is an $h_j\times h_j$ symmetric matrix whose (l, t)th element is $p_{lj}(1-p_{tj})/[F'_j(\mu_{lj})F'_j(\mu_{tj})]$, $l\leqslant t$.

Since the variance-covariance matrix Σ is unknown, it is crucial to have a consistent estimator of Σ for the purposes of evaluating the accuracy of $\hat{\theta}$ and making other statistical inferences The jackknife (Quenouille, 1956; Tukey, 1958) provides a convenient and powerful method of estimating Σ. For the present problem, the customary delete-1 jackknife provides inconsistent estimator of Σ (Efron, 1982). For general delete-d jackknife estimator of Σ (see Section 2), Shao and Wu (1989) established its consistency for $d\to\infty$ as the sample size $n\to\infty$. However, a direct application of the result in Shao and Wu (1989, Example 1) to the present problem needs to assume that Var $\hat{\theta}$ exists and n Var $\hat{\theta}\to\Sigma$ as $n\to\infty$. The existence of Var $\hat{\theta}$ is a restrictive condition since it does not hold for commonly used functions such as the ratio of quantiles. Furthermore, the condition $n\operatorname{Var}\hat{\theta}\to\Sigma$ may not be easy to check even if Var $\hat{\theta}$ exists.

In this paper, we establish the consistency of the delete-d jackknife estimator $\hat{\Sigma}_d$ (2.1) and $\hat{\Sigma}_d(m)$ (2.2) (for $d\to\infty$at a certain rate) without assuming any moment condition on $\hat{\theta}$. The distribution of $\hat{\theta}-\theta$ can then be approximated by $N(0, \hat{\Sigma}_d/n)$ (or $N(0, \hat{\Sigma}_d(m)/n)$) and therefore statistical inferences can be made based on it. Note that this approach is nonparametric, since we do not assume that F_j belongs to a parametric family.

§ 2. The delete-d jackknife and its consistency

Let X be the $n\times k$ matrix whose (i, j)th element is the observation x_{ij}. Then the point estimator $\hat{\theta}=g(\hat{\mu})$ can be written as $\hat{\theta}(X)$. For given n, let $d=d_n<n$ be an integer and $r=n-d$. Let S_r be the colletion of subsets of $\{1, \cdots, n\}$ which have size r. The number of elements in S_r is $N=\binom{n}{d}$. For $s=\{i_1, \cdots, i_r\}\in S_r$, let X_s be the submatrix of X consisting of the i_1 th, $\cdots, i_r$ th rows of X and $\hat{\theta}_s=\hat{\theta}(X_s)$. The delete-$d$ jackknife estimator of Σ is

$$\hat{\Sigma}_d=\frac{nr}{dN}\sum_{s\in S_r}\Big(\hat{\theta}_s-\frac{1}{N}\sum_{s\in S_r}\hat{\theta}_s\Big)\Big(\hat{\theta}_s-\frac{1}{N}\sum_{s\in S_r}\hat{\theta}_s\Big)^{\tau}. \tag{2.1}$$

Note that when both n and d are large, $N=\binom{n}{d}$ is very large and the computation of $\hat{\Sigma}_d$ is cumbersome. In some cases, $\hat{\Sigma}_d$ can be represented as a function of the order statistics. For example, when $h_j\equiv 1$ and g is real-valued $(L=1)$,

$$\hat{\Sigma}_d=\frac{nr}{d}N^{-k}\Sigma\binom{i_j-1}{t_j-1}\binom{n-i_j}{r-t_j}\Big[g(y_{i_11}, \cdots, y_{i_kk})$$

$$-N^{-k}\Sigma\binom{i_j-1}{t_j-1}\binom{n-i_j}{r-t_j}g(y_{i_11},\cdots,y_{i_kk})\Big]^2,$$

where t_j= the integer part of rp_{1j}, y_{ij} is the i-th order statistic of $x_{1j},\cdots,x_{nj}$ and the summation is over all $i_1,\cdots,i_k$ satisfying $t_j\leqslant i_j\leqslant t_j+d$, $j=1,\cdots,k$. However, this is still not a convenient way for computing $\hat{\Sigma}_d$. We suggest the following approximation. For fixed n and X, randomly select m subsets $\{s_1,\cdots,s_m\}$ from S_r and approximate $\hat{\Sigma}_d$ by

$$\hat{\Sigma}_d(m)=\frac{nr}{dm}\sum_{i=1}^{m}\Big(\hat{\theta}_{s_i}-\frac{1}{m}\sum_{\nu=1}^{m}\hat{\theta}_{s_\nu}\Big)\Big(\hat{\theta}_{s_i}-\frac{1}{m}\sum_{\nu=1}^{m}\hat{\theta}_{s_\nu}\Big)^{\tau}. \tag{2.2}$$

The following result shows the consistency of $\hat{\Sigma}_d$.

Theorem 1. Assume that g is differentiable in a neighborhood of μ and ∇g is continuous at μ and that for each j, F_j is continuous, F'_j exists in a neighborhood of μ_{lj} with $F'_j(\mu_{lj})>0$ and is continuous at μ_{lj}, $l=1,\cdots,h_j$. If $d=d_n$ is selected so that

$$n^{1/2}/d\to 0 \text{ and } d/n\to 0, \tag{2.3}$$

then as $n\to\infty$, $\hat{\Sigma}_d$ converges to Σ in probability, i.e.,

$$\hat{\Sigma}_d\to_p\Sigma.$$

Note that $\hat{\Sigma}_d(m)$ is random for given X. Let P^* be the probability corresponding to the random seletion of s_ν. Then we have

Theorem 2. Assume the conditions in Theorem 1 and

$$n\leqslant cm \text{ for a positive constant } c. \tag{2.4}$$

Then

$$\hat{\Sigma}_d(m)\to_p\Sigma$$

in the sense that for any $\varepsilon>0$, as $n\to\infty$,

$$P(\|\hat{\Sigma}_d(m)-\Sigma\|>\varepsilon)=E[P^*(\|\hat{\Sigma}_d(m)-\Sigma\|>\varepsilon\,|\,X)]\to 0,$$

where $\|\ \|$ is the Euclidean norm of a $K\times K$ matrix.

The following results are needed for the proofs of Theorems 1 and 2. The proof of Lemma 1 is straightforward and is omitted. The proofs of other lemmas are given in Section 3.

Lemma 1. Let $a_i\in\mathbf{R}^q$, $i=1,\cdots,n$. Then for any integer $d<n$,

$$\frac{nr}{dN}\sum_{s\in S_r}(\bar{a}_s-\bar{a})(\bar{a}_s-\bar{a})^{\tau}=\frac{1}{n-1}\sum_{i=1}^{n}(a_i-\bar{a})(a-\bar{a})^{\tau},$$

where $\bar{a}=n^{-1}\sum_{i=1}^{n}a_i$ and $\bar{a}_s=r^{-1}\sum_{i\in s}a_i$ for $s\in\mathbf{S}_r$.

Lemma 2. Let $z_1,\cdots,z_n$ be i. i. d. from a distribution F which is continuous and strictly increasing in a neighborhood of $\mu=Q(p)$ $(0<p<1)$. For $s\in\mathbf{S}_r$, let $\hat{\mu}_s$ be the sample p-quantile based on z_i, $i\in s$. Suppose that $d/n\to 0$. Then

$$\max_{s\in\mathbf{S}_r}|\hat{\mu}_s-\mu|\to 0 \text{ a.s.}$$

Lemma 3. Let $\xi_s=\xi(X_s)$ be q-vectors whose components are functions of X_s. Suppose that as $n\to\infty$, there are q-vector c and $q\times q$ matrix B such that

$$\max_{s\in S_r}\|\xi_s-c\|\to_p 0,$$

where $\|\ \|$ is the Euclidean norm, and

$$\frac{nr}{dN}\sum_{s\in S_r}\Big[\xi_s-\frac{1}{N}\sum_{s\in S_r}\xi_s\Big]\Big[\xi_s-\frac{1}{N}\sum_{s\in S_r}\xi_s\Big]^\tau\to_p B. \tag{2.5}$$

If G is a function from $\mathbf{R}^q$ to $\mathbf{R}^r$ and ∇G is continuous at c, then

$$\frac{nr}{dN}\sum_{s\in S_r}\Big[G(\xi_s)-\frac{1}{N}\sum_{s\in S_r}G(\xi_s)\Big]\Big[G(\xi_s)-\frac{1}{N}\sum_{s\in S_r}G(\xi_s)\Big]^\tau\to_p[\nabla G(c)]^\tau B[\nabla G(c)]. \tag{2.6}$$

Furthermore, the result still holds if N and S_r in (2.5)—(2.6) are replaced by m and a random sample $\{s_1,\cdots,s_m\}$ from S_r.

Lemma 4. Let $z_1,\cdots,z_n$ be i.i.d. random vectors with $E\|z_1\|^4<\infty$. Let $D=\mathrm{Var}(z_1)$, $\bar z=n^{-1}\sum_{i=1}^n z_i$ and $\bar z_s=r^{-1}\sum_{i\in s}z_i$, $s\in\mathbf{S}_r$. For given $z_1,\cdots,z_n$, let $s_1,\cdots,s_m$ be a random sample from $\mathbf{S}_r$. Assume (2.3)—(2.4). Then

$$\frac{nr}{dm}\sum_{\nu=1}^m\Big(\bar z_{s_\nu}-\frac{1}{m}\sum_{\nu=1}^m\bar z_{s_\nu}\Big)\Big(\bar z_{s_\nu}-\frac{1}{m}\sum_{\nu=1}^m\bar z_{s_\nu}\Big)^\tau\to_p D.$$

Proof of Theorem 1. Let $\hat\mu^{(s)}=\hat\mu(X_s)$ be the vector of sample quantiles based on sample X_s. From Lemmas 2 and 3, we only need to show

$$\frac{nr}{dN}\sum_{s\in S_r}(\hat\mu^{(s)}-\hat\mu)(\hat\mu^{(s)}-\hat\mu)^\tau\to_p V. \tag{2.7}$$

Let $u_{ij}=F_j(x_{ij})$. Then u_{ij} are i.i.d. with uniform distribution on [0, 1]. Let $\hat\nu^{(s)}$ and $\hat\nu$ be defined the same as $\hat\mu^{(s)}$ and $\hat\mu$ with x_{ij} replaced by u_{ij} and $\hat\nu_{lj}^{(s)}$ and $\hat\nu_{lj}$ be the components of $\hat\nu^{(s)}$ and $\hat\nu$. Then $\hat\mu_{lj}=Q_j(\hat\nu_{lj})$ and $\hat\mu_{lj}^{(s)}=Q_j(\hat\nu_{lj}^{(s)})$, where $Q_j(t)=\{x\colon F_j(x)\geqslant t\}$ is continuously differentiable at p_{lj} under the assumptions on F_j. A further application of Lemmas 2 and 3 shows that (2.7) holds if

$$\frac{nr}{dN}\sum_{s\in S_r}(\hat\nu^{(s)}-\hat\nu)(\hat\nu^{(s)}-\hat\nu)^\tau\to_p W,$$

where W equals V with F_j, $j=1,\cdots,k$, replaced by the uniform distribution function on [0, 1]. From Theorem 1 in Duttweiler (1973),

$$\hat\nu_{lj}^{(s)}=p_{lj}+r^{-1}\sum_{i\in s}(p_{lj}-I_{(u_{ij}\le p_{lj})})+R_{lj}^{(s)}$$

with $E[R_{lj}^{(s)}]^2=O(r^{-3/2})$. Let $z_{ilj}=p_{lj}-I_{(u_{ij}\le p_{lj})}$, $z_i=(z_{i11},\cdots,z_{ih_11},\cdots,z_{i1k},\cdots,z_{ih_kk})^\tau$, $\bar z=n^{-1}\sum_{i=1}^n z_i$, $R^{(s)}=(R_{11}^{(s)},\cdots,R_{h_11}^{(s)},\cdots,R_{1k}^{(s)},\cdots,R_{h_kk}^{(s)})^\tau$ and $\bar R=N^{-1}\sum_{s\in S_r}R^{(s)}$. Using Lemma 1,

$$\frac{nr}{dN}\sum_{s\in S_r}(\hat\nu^{(s)}-\hat\nu)(\hat\nu^{(s)}-\hat\nu)^\tau=A_n+B_n+2C_n.$$

where $A_n=(n-1)^{-1}\sum_{i=1}^n(z_i-\bar z)(z_i-\bar z)^\tau$, $B_n=\frac{nr}{dN}\sum_{s\in S_r}(R^{(s)}-\bar R)(R^{(s)}-\bar R)^\tau$ and C_n satisfies $\|C_n\|\leqslant\|A_n\|\|B_n\|$. From the law of large numbers, $A_n\to_p W$. The result follows from for (l,j) and (l',j'),

$$E\left|\frac{nr}{dN}\sum_{s\in S_r}R_{lj}^{(s)}R_{l'j'}^{(s)}\right|=\frac{nr}{d}O(r^{-3/2})=O\Big(\frac{n^{1/2}}{d}\Big)=o(1)$$

under (2.3).

Proof of Theorem 2. Using Lemma 4, the proof of Theorem 2 is similar to that of Theorem 1.

§ 3. Proofs of lemmas

Proof of Lemma 2. Let N_μ be a neighborhood of μ such that $F(x)$ is continuous and strictly increasing on N_μ. Then $a(x)=p-F(x)$ is continuous and strictly decreasing on N_μ. Let $A=\{a(x):\ x\in N_\mu\}$. Then $a^{-1}(u)$ is continuous on A. For any $\varepsilon>0$, there is a $\delta>0$ such that

$$|t-\mu|<\varepsilon \text{ if } |a(t)-a(\mu)|=|a(t)|<\delta.$$

Let $\hat{F}$ be the empirical distribution based on z_i, $i=1, \cdots, n$, and $\hat{F}^{(s)}$ be the empirical distribution based on z_i, $i\in s$. For almost all $z_1, z_2, \cdots$, there is an n_0 such that for $n>n_0$, $r^{-1}<\delta/3$, $d/n<\delta/3$ and $\|\hat{F}-F\|_\infty<\delta/3$. Then for all $s\in S_r$,

$$\begin{aligned}|a(\hat{\mu}_s)|&=|p-F(\hat{\mu}_s)|\leqslant|F(\hat{\mu}_s)-\hat{F}^{(s)}(\hat{\mu}_s)|+r^{-1}\\&\leqslant\|\hat{F}^{(s)}-F\|_\infty+r^{-1}\leqslant\|\hat{F}^{(s)}-\hat{F}\|_\infty+\|\hat{F}-F\|_\infty+r^{-1}\\&\leqslant d/n+\|\hat{F}-F\|_\infty+r^{-1}<\delta.\end{aligned}$$

Thus, for all $s\in \mathbf{S}_r$, $|\hat{\mu}_s-\mu|<\varepsilon$. This completes the proof.

Proof of Lemma 3. Let $\bar{\xi}=N^{-1}\sum_{s\in \mathbf{S}_r}\xi_s$. From the mean-value theorem,

$$G(\xi_s)-G(\bar{\xi})=[\nabla G(t_s)]^\tau(\xi_s-\bar{\xi})$$

where t_s satisfying $\|t_s-\bar{\xi}\|\leqslant\|\xi_s-\bar{\xi}\|$. Let $R_s=[\nabla G(t_s)-\nabla G(\bar{\xi})]^\tau(\xi_s-\bar{\xi})$ and $\bar{R}=N^{-1}\sum_{s\in \mathbf{S}_r}R_s$. Then

$$\begin{aligned}&\frac{nr}{dN}\sum_{s\in S_r}\Big[G(\xi_s)-\frac{1}{N}\sum_{s\in S_r}G(\xi_s)\Big]\Big[G(\xi_s)-\frac{1}{N}\sum_{s\in S_r}G(\xi_s)\Big]^\tau\\&=\frac{nr}{dN}[\nabla G(\bar{\xi})]^\tau\sum_{s\in S_r}(\xi_s-\bar{\xi})(\xi_s-\bar{\xi})^\tau[\nabla G(\bar{\xi})]\\&\quad+\frac{nr}{dN}\sum_{s\in S_r}(R_s-\bar{R})(R_s-\bar{R})^\tau\\&\quad+\frac{2nr}{dN}[\nabla G(\bar{\xi})]^\tau\sum_{s\in S_r}(\xi_s-\bar{\xi})(R_s-\bar{R})^\tau.\end{aligned}$$

From the continuity of ∇G at c and $\max_{s\in S_r}\|\xi_s-\bar{\xi}\|\leqslant2\max_{s\in S_r}\|\xi_s-c\|\to_s 0$, $\nabla G(\bar{\xi})\to_s\nabla G(c)$ and for any $\varepsilon>0$,

$$\frac{nr}{dN}\sum_{s\in S_r}R_s^\tau R_s\leqslant\varepsilon^2\frac{nr}{dN}\sum_{s\in S_r}\|\xi_s-\bar{\xi}\|^2$$

when n is sufficiently large. Thus (2.6) follows from (2.5). The proof for the second assertion is the same.

Proof of Lemma 4. Let

$$A_m=\frac{nr}{dm}\sum_{\nu=1}^{m}(\bar{z}_{s_\nu}-\bar{z})(\bar{z}_{s_\nu}-\bar{z})^\tau$$

and

$$B_m=\frac{nr}{d}\Big(\bar{z}-m^{-1}\sum_{\nu=1}^{m}\bar{z}_{s_\nu}\Big)\Big(\bar{z}-m^{-1}\sum_{\nu=1}^{m}\bar{z}_{s_\nu}\Big)^\tau.$$

Then

$$\frac{nr}{dm}\sum_{\nu=1}^{m}\Big(\bar{z}_{s_\nu}-\frac{1}{m}\sum_{\nu=1}^{m}\bar{z}_{s_\nu}\Big)\Big(\bar{z}_{s_\nu}-\frac{1}{m}\sum_{\nu=1}^{m}\bar{z}_{s_\nu}\Big)^\tau=A_m+B_m+2C_m,$$

where $\|C_m\|\leqslant\|A_m\|\ \|B_m\|$. Hence it suffices to show $A_m\to_p D$ and $B_m\to_p 0$. Let E^* and var* be the conditional expectation and variance taken under P^* for given $z_1, \cdots, z_n$. By Lemma 1,

$$E^*\Big[\frac{nr}{dm}\sum_{\nu=1}^{m}(\bar{z}_{s_\nu}-\bar{z})(\bar{z}_{s_\nu}-\bar{z})^\tau\Big]=\frac{nr}{dN}\sum_{s\in S_r}(\bar{z}_s-\bar{z})(\bar{z}_s-\bar{z})^\tau=\frac{1}{n-1}\sum_{i=1}^{n}(z_i-\bar{z})(z_i-\bar{z})^\tau.$$

Let a_{pq} be the (p, q)t element of A_m. Then

$$\mathrm{Var}^*(a_{pq})\leqslant\Big(\frac{nr}{d^2}\Big)^2\frac{1}{mN}\sum_{s\in S_r}\|\bar{z}_s-\bar{z}\|^4.$$

From the result in Lehmann (1983, p. 138), $r^2E\|\bar{z}_s-\bar{z}\|^4=O(1)$. Hence $\mathrm{Var}^*(a_{pq})\to_p 0$ since $n^2/d^2m\to 0$ under (2.3)—(2.4). Since $(n-1)^{-1}\sum_{i=1}^{n}(z_i-\bar{z})(z_i-\bar{z})^\tau\to_p D$, we have $A_m\to_p D$. Then the result follows from

$$E(B_m)=E[E^*(B_m)]\leqslant E\Big[\frac{nr}{mdN}\sum_{s\in S_r}(\bar{z}_s-\bar{z})(\bar{z}_s-\bar{z})^\tau\Big]$$
$$=\frac{1}{m(n-1)}E\sum_{i=1}^{n}(z_i-\bar{z})(z_i-\bar{z})^\tau=\frac{1}{m}D\to 0.$$

§ 4. Some monte carlo results

As an example, we examine the performance of the jackknife estimator and the confidence interval based on the jackknife via a Monte Carlo study. We consider a two sample problem: x_{ij}, $i=1, \cdots, n$, $j=1, 2$, are independent and x_{ij} has distribution F_j for each i and j. Let μ_j be the median of F_j and $\hat{\mu}_j$ be the sample median besed on x_{ij}, $i=1, \cdots, n$. The quantity of interest is the ratio $\theta=\mu_1/\mu_2$ and we estimate θ by $\hat{\theta}=\hat{\mu}_1/\hat{\mu}_2$. Using the proposed jackknife method, we estimate the variance of $\hat{\theta}$ by $n^{-1}\hat{\Sigma}_d(m)$, where $\hat{\Sigma}_d(m)$ is given by (2.2), and obtain an approximate 90% confidence interval for θ:

$$CI=[\hat{\theta}-1.645[\hat{\Sigma}_d(m)/n]^{1/2},\ \hat{\theta}+1.645[\hat{\Sigma}_d(m)/n]^{1/2}]. \tag{4.1}$$

According to (2.3), we select $d=$the integer part of $n^{2/3}$. For m, we select $m=$the integer part of $n^{3/2}$ as suggested in Shao (1987).

In this example, we consider three different sample sizes $n=20$, 30 and 40. For each sample size, we consider two kinds of distributions F_1 and F_2:

(a) Normal Case.

$F_1(t)=\Phi[(t-1)/0.25]$ and $F_2(t)=\Phi[(t-2)/0.5]$,

where Φ is the standard normal distribution function.

(b) Exponential Case.

$$F_1(t)=1-e^{-t} \text{ and } F_2(t)=1-e^{-t/2},\ t>0.$$

In both cases, the true value of θ is 0.5.

The results in Table 1 is based on 2000 Monte Carlo simulations. We summarize the Monte Carlo results as follows.

(1) The confidence interval CI given by (4.1) has very accurate coverage probability, even when n is as small as 20.

(2) The variance estimator $n^{-1}\hat{\Sigma}_d(m)$ is not accurate when $n\leqslant 30$. It is better when $n=40$. This indicates that in this example the variance estimation problem requires a large sample size.

(3) From the good performance of CI, the selection of d and m are adequate.

Table 1. Monte Carlo Approximations of the Bias, Vairance and Coverage Probability

		Normal Case		
		$\hat{\theta}$	$n^{-1}\hat{\Sigma}_d(m)$	CI
$n=20$	Bias	0.0014	0.0006	
$d=7$	Variance	0.0025	3.7×10^{-6}	
$m=90$	Coverage Prob.			0.913
$n=30$	Bias	0.0022	0.0004	
$d=9$	Variance	0.0016	1.5×10^{-6}	
$m=165$	Coverage Prob.			0.912
$n=40$	Bias	0.0017	0.0002	
$d=12$	Variance	0.0012	4.9×10^{-7}	
$m=253$	Coverage Prob.			0.896
		Exponential Case		
		$\hat{\theta}$	$n^{-1}\hat{\Sigma}_d(m)$	CI
$n=20$	Bias	0.0552	0.0449	
$d=7$	Variance	0.0678	0.0349	
$m=90$	Coverage Prob.			0.930
$n=30$	Bias	0.0356	0.0193	
$d=9$	Variance	0.0431	0.0061	
$m=165$	Coverage Prob.			0.904
$n=40$	Bias	0.0350	0.0074	
$d=12$	Variance	0.0301	0.0012	
$m=253$	Coverage Prob			0.894

References

[1] Duttweiler, D. L. The mean-square error of Bahadur's order-statistic approximation. *Ann. Statist.* **1** (1973), 446—453.

[2] Efron, B. *The Jackknife, the Bootstrap, and Other Resampling Plans*. SIAM, Philadelphia. (1982).

[3] Lehmann, E. L. *Theory of point estimation*. Wiley, New York. (1983).

[4] Quenouille, M Notes on bias in estimation. *Biometrika* **43** (1956), 353—360.

[5] Shao, J Sampling and resampling: An efficient approximation to jackknife variance estimators in linear

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models. *Chinese J. Appl. Prob. and Statist.* 3, (1987). 应用概率统计, 368—379.
[6] Shao, J., Wu, C. F. J. A general theory for jackknife variance estimation. *Ann. Statist.* **17** (1989), 1176—1197.
[7] Tukey, J. Bias and confidence in not quite large samples. *Ann. Math. Statist*, **29**, (1958), 614.

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基于子样分位数和刀切法的统计推断

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本文考虑对母体分位数之函数作统计推断的问题. 子样分位数之函数的渐近分布为正态. 使用刀切法，我们给出了渐近分布的方差与协方差的估计量并建立了它们的一致性. 这些结果提供了一些在渐近意义下正确的统计推断方法.